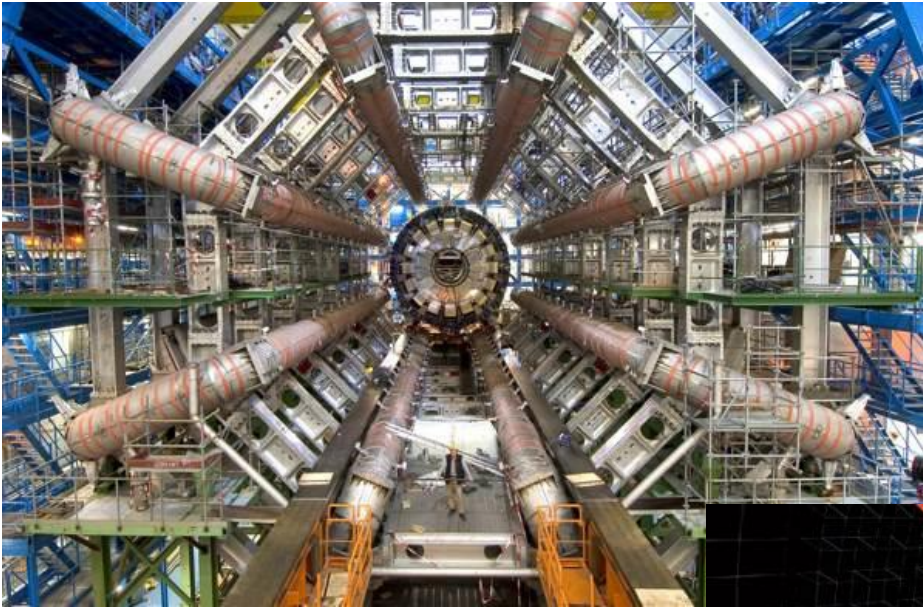




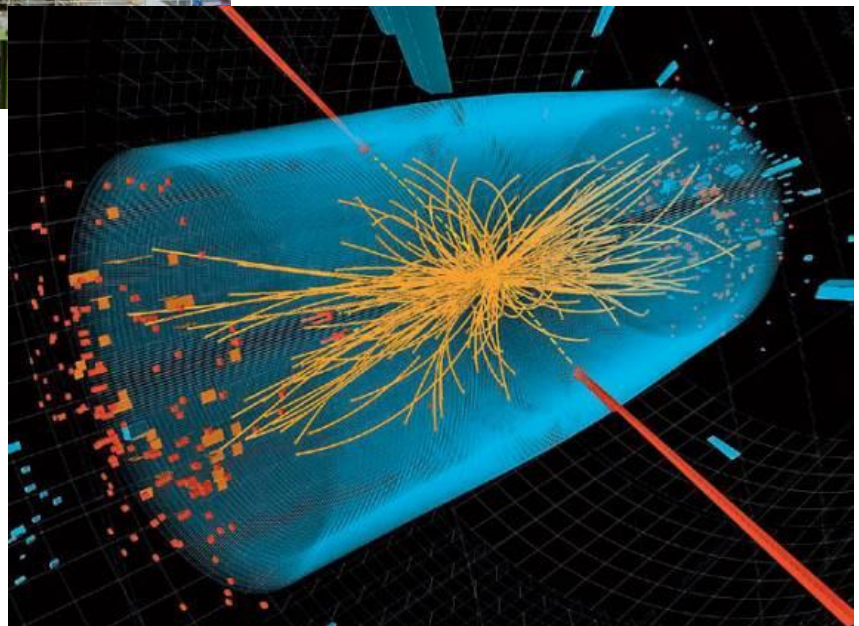
# James M. Hill

## Physics Handbook



- Equations, Variables, and Units
- Physical Constants
- Solar System Data
- Metric System
- Last Edited: Fall 2018

Mr. P. MacDonald





\*\*In the equations that follow all variables that are vectors could have a horizontal and vertical component. You have to remember to analyze each dimension independently if the problem warrants.\*\*

### Mathematical Addition of Vectors

| Symbol                    | Quantity (Unit)                                 | Symbol                             | Quantity (Unit)   |
|---------------------------|---|------------------------------------|---|
| $\vec{R}$                 | Resultant                                       | $\vec{A}, \vec{B}$                 | Arbitrary Vectors                                       |
| $R_E$ or $R_x$            | Horizontal component of resultant (E = Eastern) | $A_E$ or $A_x$                     | Horizontal component of arbitrary vectors (E = Eastern) |
| $R_N$ or $R_y$            | Vertical component of resultant (N = Northern)  | $A_N$ or $A_y$                     | Vertical component of arbitrary vectors (N = Northern)  |
| $\theta$                  | Angle made with horizontal ( $^\circ$ )         |                                    |   |
| $R_x = A_x + B_x + \dots$ |   | $R_y = A_y + B_y + \dots$          |   |
|                           |   | $ \vec{R}  = \sqrt{R_x^2 + R_y^2}$ | $\theta = \tan^{-1} \left  \frac{R_y}{R_x} \right $     |

! Look very carefully at the trigonometry when calculating vector components.

### Kinematics - Mathematical Analysis & Projectile Motion

| Symbol  | Quantity (Unit)                  | Symbol  | Quantity (Unit)                                     | Symbol  | Quantity (Unit)   |
|---|----------------------------------|---|---|---|---|
| <i>anything<sub>f</sub></i>                       | Final value                      | <i>anything</i>                                   | Magnitude   | $d$   | Distance (m)  |
| <i>anything<sub>o</sub></i>                       | Initial Value                    | $\vec{d}$   | Displacement (m)                                    | $v_{sp}$  | Average Speed (m/s)   |
| <i>anything<sub>x</sub></i>                       | Horizontal component             | $\vec{v}_{avg}$                                   | Average velocity (m/s)                              | $t$   | time (s; refers to a time interval)                           |
| <i>anything<sub>y</sub></i>                       | Vertical component               | $\vec{v}$   | Velocity (m/s)                                      | $\theta$  | Angle made with horizontal (degrees, $^\circ$ )               |
| <i>anything<sub>E</sub></i>                       | Eastern component                | $\vec{a}$   | Acceleration (m/s <sup>2</sup> )                    | $\Delta$  | Change in (final - initial)                                   |
| <i>anything<sub>N</sub></i>                       | Northern component               | $\vec{g}$   | 9.81 (m/s <sup>2</sup> ; surface of the Earth)      |   |   |
| $\vec{v}_{avg} = \frac{\vec{d}_f - \vec{d}_o}{t}$ | $v_{sp} = \frac{d}{t}$           | $\vec{v}_{avg} = \frac{\vec{v}_f + \vec{v}_o}{2}$ | $\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{t}$         | $\vec{d}_f = \vec{d}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$ | $\vec{v}_f^2 = \vec{v}_o^2 + 2\vec{a}(\vec{d}_f - \vec{d}_o)$ |
| $v_{ox} =  \vec{v}  \cos \theta$                  | $v_{oy} =  \vec{v}  \sin \theta$ | $ \vec{v}  = \sqrt{v_{fx}^2 + v_{fy}^2}$          | $\theta = \tan^{-1} \left  \frac{v_y}{v_x} \right $ |   |   |

### Dynamics - Forces, Impulse, Torque, Momentum, & Circular Motion

| Symbol                        | Quantity (Unit)                              | Symbol                        | Quantity (Unit)                      | Symbol   | Quantity (Unit)                  |                      |                                   |                             |                               |
|-------------------------------|--|-------------------------------|--------------------------------------|--|----------------------------------|----------------------|-----------------------------------|-----------------------------|-------------------------------|
| $\vec{F}_{net}$               | Net force (N)                                | $\vec{F}_T$                   | Force of Tension (N)                 | $\vec{J}$  | Impulse (N·s)                    |                      |                                   |                             |                               |
| $\vec{F}_A$                   | Force applied (N)                            | $\vec{F}_N$                   | Normal Force (N)                     | $\vec{p}$  | Momentum (kg·m/s)                |                      |                                   |                             |                               |
| $\vec{F}_g$                   | Force of gravity (N)                         | $\vec{F}_c$                   | Centripetal Force (N)                | $\vec{p}_{oT}$                                     | Initial total momentums (kg·m/s) |                      |                                   |                             |                               |
| $\vec{F}_f$                   | Force of friction (N)                        | $m$                           | Mass (kg)                            | $\vec{p}_{fT}$                                     | Final total momentums (kg·m/s)   |                      |                                   |                             |                               |
| $\vec{F}_s$                   | Restoring Force (N)                          | $\mu$                         | Coefficient of friction (no units)   | $f$  | Frequency (Hz)                   |                      |                                   |                             |                               |
| $\vec{a}_c$                   | Centripetal Acceleration (m/s <sup>2</sup> ) | $T$                           | Period (s)                           | $r$  | Circular & Orbital Radius (m)    |                      |                                   |                             |                               |
| $\tau$                        | Torque (N·m)                                 | $F_\perp$                     | Perpendicular Component of Force (N) | $v$  | Circular Speed (m/s)             |                      |                                   |                             |                               |
| $k$                           | Spring Constant (N/m)                        |                               |                                      |  |                                  |                      |                                   |                             |                               |
| $\vec{F}_{net} = \sum Forces$ | $\sum \vec{F} = \sum m \times \vec{a}$       | $\vec{F}_{net} = m\vec{a}$    | $\vec{F}_g = m\vec{g}$               | $ \vec{F}_f  = \mu  \vec{F}_N $                    | $F_s = -kx$                      | $\vec{p} = m\vec{v}$ | $\vec{J} = \Delta\vec{p}$         | $\vec{F}t = m\Delta\vec{v}$ | $\vec{p}_{oT} = \vec{p}_{fT}$ |
| $v = \frac{2\pi r}{T}$        | $a_c = \frac{v^2}{r}$                        | $F_c = \frac{mv^2}{r}$        | $f = \frac{1}{T}$                    | $v = \sqrt{rg\mu_s}$                               | $v = \sqrt{rg \tan \theta}$      | $\tau = rF_\perp$    | $\vec{\tau}_{net} = \sum Torques$ |                             |                               |
| $\vec{J} = \vec{F}t$          |  | $T = 2\pi \sqrt{\frac{m}{k}}$ |                                      | $\vec{J} = \text{Area under } F - t \text{ curve}$ |                                  |                      |                                   |                             |                               |

**Work, Energy, & Power**

| Symbol   | Quantity (Unit)                    | Symbol | Quantity (Unit)                   |
|----------|------------------------------------|--------|-----------------------------------|
| $W$      | Work (J)                           | $P$    | Power (W; Watts)                  |
| $W_{nc}$ | Work by non-conservative force (J) | $h$    | Height above reference (m)        |
| $E_k$    | Kinetic energy (J)                 | $d$    | Distance (m)                      |
| $E_g$    | Gravitational potential energy (J) | $r$    | Radius of circle (m)              |
| $E_e$    | Elastic potential energy (J)       | $x$    | Stretch/compression of spring (m) |
| $E_T$    | Total mechanical energy (J)        |        |                                   |

|  |             |                         |                         |                  |                  |                  |                  |                       |                   |
|--|-------------|-------------------------|-------------------------|------------------|------------------|------------------|------------------|-----------------------|-------------------|
| $W = F_{\parallel} \Delta d$<br>W = Area under F – d curve | $E_g = mgh$ | $E_k = \frac{1}{2}mv^2$ | $E_e = \frac{1}{2}kx^2$ | $W = \Delta E_k$ | $W = \Delta E_g$ | $W = \Delta E_e$ | $\Delta E_T = 0$ | $W_{nc} = \Delta E_T$ | $P = \frac{W}{t}$ |
|--|-------------|-------------------------|-------------------------|------------------|------------------|------------------|------------------|-----------------------|-------------------|

**Current Electricity**

| Symbol           | Quantity (Unit)                                       | Symbol    | Quantity (Unit)                                     | Symbol | Quantity (Unit)                |
|------------------|---|-----------|---|--------|--------------------------------|
| $I$              | Current (A; amperes)                                  | $Q$       | Charge (C; coulombs)                                | $t$    | Time (s)                       |
| $N$              | Number of charges, resistors, etc                     | $e$       | Elementary Charge (C)                               | $R$    | Resistance ( $\Omega$ ; Ohm)   |
| $\rho$           | Resistivity ( $\Omega \cdot m$ )                      | $L$       | Length (m)  | $A$    | Cross-sectional area ( $m^2$ ) |
| $V, \varepsilon$ | Potential Difference (V; volts)                       | $V_T$     | Voltage of power source (V)                         | $I_T$  | Current from power source (A)  |
| $R_{eqs}$        | Equivalent Resistance-series ( $\Omega$ )             | $R_{eqp}$ | Equivalent Resistance-parallel ( $\Omega$ )         | $P$    | Power (W)                      |
| $C_{eqp}$        | Equivalent capacitance for capacitors in parallel (F) | $C_{eqs}$ | Equivalent capacitance for capacitors in series (F) | $\tau$ | Discharge rate (s)             |
| $emf$            | Electromotive force (V)                               | $r$       | Internal Resistance ( $\Omega$ )                    | $E$    | Electrical energy (J)          |

|                                  |                           |   |                                  |   |   |
|----------------------------------|---------------------------|---|----------------------------------|---|---|
| $I = \frac{\Delta q}{\Delta t}$  | $Q = Ne$                  | $R = \rho \frac{L}{A}$                                      | $V = IR$                         | $R_{eqs} = R_1 + R_2 + R_3 + \dots + R_N$ | $\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$ |
| $R_{eqp} = \frac{V_T}{I_T}$      | $P = IV$                  | $\tau = RC$   | $Q(t) = Q_0 e^{-\frac{t}{\tau}}$ | $C_{eqp} = C_1 + C_2 + C_3 + \dots + C_N$ | $\frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$ |
| $I(t) = I_0 e^{-\frac{t}{\tau}}$ | $V_{terminal} = emf - Ir$ | $Efficiency = \frac{E_{out}}{E_{in}} = \frac{Work}{E_{in}}$ |                                  |   |   |

**Gravitational & Electrical Fields**

| Symbol      | Quantity (Unit)                   | Symbol          | Quantity (Unit)   | Symbol    | Quantity (Unit)  |
|-------------|-----------------------------------|-----------------|---|-----------|--|
| $\vec{F}_q$ | Electrostatic Force (N)           | $q$             | Electric Charge (C)   | $k$       | $9.0 \times 10^9 \left(\frac{N \cdot m^2}{C^2}\right)$ , Coulomb's constant                      |
| $r$         | Distance from centres (m)         | $\vec{E}$       | Electric field intensity ( $\frac{N}{C}$ )                  | $\vec{g}$ | Gravitational Field Intensity ( $\frac{N}{kg}$ )   |
| $\vec{F}_g$ | Force of gravity (N)              | $E_g$           | Gravitational potential energy (J)                          | $E_q$     | Electric potential energy (J)  |
| $V$         | Electric potential difference (V) | $m$             | Mass of object (kg)   | $G$       | $6.672 \times 10^{-11} \left(\frac{N \cdot m^2}{kg^2}\right)$ , Universal gravitational constant |
| $W$         | Work (J)                          | $\varepsilon_0$ | $8.85 \times 10^{-12}$ (F/m)<br>Permittivity of free space. | $C$       | Capacitance (F, called farads)   |
| $Q$         | Charge stored in a capacitor (C)  | $\kappa$        | Dielectric constant (no unit)                               | $d$       | Plate separation (m)   |
| $A$         | Plate area ( $m^2$ )              | $E_c$           | Potential energy in a capacitor (J)                         |           |  |

|                                     |                                 |                               |                       |  |                              |                                     |                                 |                               |
|-------------------------------------|---------------------------------|-------------------------------|-----------------------|--|------------------------------|-------------------------------------|---------------------------------|-------------------------------|
| $\vec{F}_g = G \frac{m_1 m_2}{r^2}$ | $\vec{g} = \frac{\vec{F}_g}{m}$ | $ \vec{g}  = G \frac{m}{r^2}$ | $G \frac{m}{r} = v^2$ | $\frac{Gm}{4\pi^2} = \frac{r^3}{T^2}$  | $E_g = -G \frac{m_1 m_2}{r}$ | $\vec{F}_q = k \frac{q_1 q_2}{r^2}$ | $\vec{E} = \frac{\vec{F}_q}{q}$ | $ \vec{E}  = k \frac{q}{r^2}$ |
| $V = \frac{E_q}{q}$                 | $V = k \frac{q}{r}$             | $\Delta E_q = q \Delta V$     | $Q = CV$              | $C = \frac{\kappa \varepsilon_0 A}{d}$ | $E_c = \frac{1}{2} CV^2$     | $\vec{E} = \frac{V}{d}$             | $E_q = k \frac{q_1 q_2}{r}$     |                               |

**General Waves & Sound Waves**

| Symbol    | Quantity (Unit)       | Symbol    | Quantity (Unit)                    | Symbol      | Quantity (Unit)         |
|-----------|-----------------------|-----------|------------------------------------|-------------|-------------------------|
| $T$       | Period (s)            | $f$       | Frequency (Hz)                     | $v$         | Wave speed (m/s)        |
| $\lambda$ | Wavelength (m)        | $T_{air}$ | Temp. of air ( $^{\circ}$ Celsius) | $v_{sound}$ | Sound speed (m/s)       |
| $v_{src}$ | Source speed (m/s)    | $v_{obs}$ | Observer's speed (m/s)             | $f_{obs}$   | Observed frequency (Hz) |
| $f_{src}$ | Source frequency (Hz) | $F_T$     | Force of Tension (N)               | $\mu$       | Mass/unit length (kg/m) |

|                                |                                |                   |                   |                |                              |                                 |  |
|--------------------------------|--------------------------------|-------------------|-------------------|----------------|------------------------------|---------------------------------|--|
| $T = \frac{\Delta t}{\#waves}$ | $f = \frac{\#waves}{\Delta t}$ | $T = \frac{1}{f}$ | $f = \frac{1}{T}$ | $v = f\lambda$ | $v = \sqrt{\frac{F_T}{\mu}}$ | $v_{sound} = 331 + 0.59T_{air}$ | $f_{obs} = f_{src} \left( \frac{v_{sound} \pm v_{obs}}{v_{sound} \mp v_{src}} \right)$ |
|--------------------------------|--------------------------------|-------------------|-------------------|----------------|------------------------------|---------------------------------|--|

**Refraction**

| Symbol     | Quantity (Unit)                                      | Symbol     | Quantity (Unit)                  | Symbol         | Quantity (Unit)                |
|------------|--|------------|----------------------------------|----------------|--------------------------------|
| $c$        | $3.00 \times 10^8$ (m/s), speed of light in a vacuum | $v$        | Speed of light in material (m/s) | $n$            | Index of refraction            |
| $n_i$      | Incident medium index                                | $n_R$      | Refractive medium index          | $\theta_i$     | Incident angle (degrees)       |
| $\theta_R$ | Refractive angle (degrees)                           | $\theta_c$ | Critical Angle (degrees)         | $\theta_{max}$ | Max angle refraction (degrees) |

|                   |   |                           |                               |
|-------------------|---|---------------------------|-------------------------------|
| $n = \frac{c}{v}$ | $n_i \sin \theta_i = n_R \sin \theta_R$ | $n_i \sin \theta_c = n_R$ | $n_i = n_R \sin \theta_{max}$ |
|-------------------|---|---------------------------|-------------------------------|

**Table 9.2** Index of Refraction of Various Substances\*

| Substance   | Index of Refraction ( $n$ ) |
|---|-----------------------------|
| vacuum  | 1.00000                     |
| <b>gases at <math>0^{\circ}\text{C}</math>, <math>1.013 \times 10^5</math> Pa</b> |                             |
| hydrogen  | 1.00014                     |
| oxygen  | 1.00027                     |
| air   | 1.00029                     |
| carbon dioxide  | 1.00045                     |
| <b>liquids at <math>20^{\circ}\text{C}</math></b>                                 |                             |
| water   | 1.333                       |
| ethyl alcohol   | 1.362                       |
| glycerin  | 1.470                       |
| carbon disulfide  | 1.632                       |

| Substance  | Index of Refraction ( $n$ ) |
|--|-----------------------------|
| <b>solids at <math>20^{\circ}\text{C}</math></b> |                             |
| ice (at $0^{\circ}\text{C}$ )                    | 1.31                        |
| quartz (fused)                                   | 1.46                        |
| optical fibre (cladding)                         | 1.47                        |
| optical fibre (core)                             | 1.50                        |
| Plexiglas <sup>TM</sup> or Lucite <sup>TM</sup>  | 1.51                        |
| glass (crown)                                    | 1.52                        |
| sodium chloride                                  | 1.54                        |
| glass (crystal)                                  | 1.54                        |
| ruby   | 1.54                        |
| glass (flint)                                    | 1.65                        |
| zircon   | 1.92                        |
| diamond  | 2.42                        |

\* Measured using yellow light, with a wavelength of 589 nm in a vacuum.

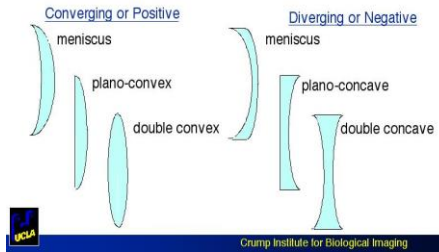
**Optics (Spherical Mirrors & Lenses)**

| Symbol     | Quantity (Unit)          | Symbol | Quantity (Unit)                     | Symbol | Quantity (Unit)                 |
|------------|--------------------------|--------|-------------------------------------|--------|---------------------------------|
| $f$        | Focal length (m, cm, mm) | $d_o$  | Object distance (m, cm, mm)         | $d_i$  | Image distance (m, cm, mm)      |
| $h_i$      | Image height (m, cm, mm) | $h_o$  | Object height (m, cm, mm)           | $M$    | Magnification                   |
| $n_{lens}$ | Lens refractive index    | $n_o$  | Surrounding medium refractive index | $R$    | Radius of curvature (m, cm, mm) |

|   |                       |                        |                                      |  |
|---|-----------------------|------------------------|--------------------------------------|--|
| $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ | $M = \frac{h_i}{h_o}$ | $M = -\frac{d_i}{d_o}$ | $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$ | $\frac{1}{f} = \left( \frac{n_{lens}}{n_o} - 1 \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$ |
|---|-----------------------|------------------------|--------------------------------------|--|

### Types of Lenses



### Diffraction

| Symbol | Quantity (Unit)                     | Symbol    | Quantity (Unit)          | Symbol   | Quantity (Unit)         |
|--------|-------------------------------------|-----------|--------------------------|----------|-------------------------|
| $m$    | Maximum # (integer)                 | $\lambda$ | wavelength (m)           | $d$      | separation of slits (m) |
| $W_m$  | Distance of maximum from centre (m) | $L$       | Distance from screen (m) | $\theta$ | angle (degrees)         |

|                             |                            |  |
|-----------------------------|----------------------------|--|
| $m\lambda = \frac{dW_m}{L}$ | $m\lambda = d \sin \theta$ | $m\lambda = 2nd$<br>Above is for thin film interference.<br>$n$ = refractive index of the film |
|-----------------------------|----------------------------|--|

### Solar System Data

| Quantity              | Sun                  | Mercury              | Venus                | Earth                | Mars                 | Jupiter              | Saturn               | Uranus               | Neptune              |
|-----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Distance from Sun (m) | N/A                  | $5.8 \times 10^{10}$ | $1.1 \times 10^{11}$ | $1.5 \times 10^{11}$ | $2.3 \times 10^{11}$ | $7.8 \times 10^{11}$ | $1.4 \times 10^{12}$ | $2.9 \times 10^{12}$ | $4.5 \times 10^{12}$ |
| Radius (m)            | $7.0 \times 10^8$    | $2.5 \times 10^6$    | $6.1 \times 10^6$    | $6.4 \times 10^6$    | $3.4 \times 10^6$    | $7.1 \times 10^7$    | $6.0 \times 10^7$    | $2.6 \times 10^7$    | $2.5 \times 10^7$    |
| Mass (kg)             | $2.0 \times 10^{30}$ | $3.3 \times 10^{23}$ | $4.9 \times 10^{24}$ | $6.0 \times 10^{24}$ | $6.4 \times 10^{23}$ | $2.0 \times 10^{27}$ | $5.7 \times 10^{26}$ | $8.7 \times 10^{25}$ | $1.0 \times 10^{26}$ |
| Revolution            | N/A                  | 88d                  | 225d                 | 365d                 | 1.88y                | 11.9y                | 29y                  | 84y                  | 164y                 |
| Rotation              | Varies               | 58d                  | 243d                 | 24h                  | 24h                  | 10h                  | 11h                  | 17h                  | 16h                  |

**Table 4.4** Free-Fall Accelerations Due to Gravity in the Solar System

| Location | Acceleration due to gravity ( $\text{m/s}^2$ ) |
|----------|--|
| Earth    | 9.81   |
| Moon     | 1.64   |
| Mars     | 3.72   |
| Jupiter  | 25.9   |

**Table 4.3** Free-Fall Accelerations Due to Gravity on Earth

| Location                       | Acceleration due to gravity ( $\text{m/s}^2$ ) | Altitude (m)             | Distance from Earth's centre (km) |
|--------------------------------|--|--------------------------|-----------------------------------|
| North Pole                     | 9.8322   | 0 (sea level)            | 6357                              |
| equator                        | 9.7805   | 0 (sea level)            | 6378                              |
| Mt. Everest (peak)             | 9.7647   | 8850                     | 6387                              |
| Mariana Ocean Trench* (bottom) | 9.8331   | 11 034 (below sea level) | 6367                              |
| International Space Station*   | 9.0795   | 250 000                  | 6628                              |

\*These values are calculated.



## Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ , the roots of it can be found using

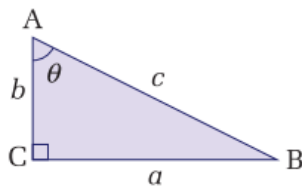
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Statistical Analysis

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are *mean*, *median*, and *mode*.

## Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{a}{b}$$

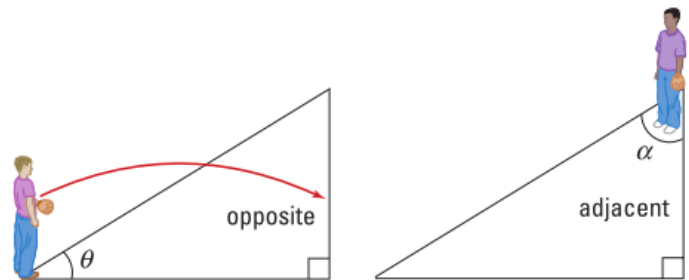
The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the  $90^\circ$  angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.

**Mean:** The sum of the numbers divided by the number of values. It is also called the “average.”

**Median:** When a set of numbers is organized in order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.

**Mode:** The number that occurs most often in a set of numbers. Some data sets will have more than one mode.

See examples of these on the following page.



A scientific calculator or trigonometry tables can be used to obtain an angle value from the ratio result. Your calculator performs a complex calculation (Maclaurin series summation) when the  $\sin^{-1}$ , or  $\cos^{-1}$ , or  $\tan^{-1}$  operation is used to determine the angle value.  $\sin^{-1}$  is not simply a  $1/\sin$  operation.

## Definition of the Pythagorean Theorem

The Pythagorean theorem is used to determine side lengths of a right-angle ( $90^\circ$ ) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$c^2 = a^2 + b^2$$

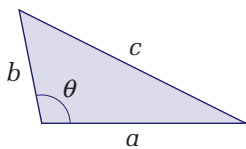
| Quantity   | Symbol | SI unit    |
|--|--------|------------|
| hypotenuse side is opposite the $90^\circ$ angle | $c$    | m (metres) |
| side $a$   | $a$    | m (metres) |
| side $b$   | $b$    | m (metres) |

**Note:** The hypotenuse is always the side across from the right ( $90^\circ$ ) angle. The Pythagorean theorem is a special case of a more general mathematical law called the “cosine law.” The cosine law works for all triangles.

## Definition of the Cosine Law

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths



Angle  $\theta$  is contained between sides  $a$  and  $b$ .

The cosine law states  $c^2 = a^2 + b^2 - 2ab \cos\theta$ .

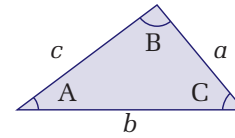
| Quantity                                 | Symbol   | SI unit    |
|--|----------|------------|
| unknown length side $c$                  | $c$      | m (metres) |
| opposite angle $\theta$                  | $c$      | m (metres) |
| length side $a$                          | $a$      | m (metres) |
| length side $b$                          | $b$      | m (metres) |
| angle $\theta$ opposite unknown side $c$ | $\theta$ | (radians)  |

**Note:** Applying the cosine law to a right angle triangle, setting  $\theta = 90^\circ$ , yields the special case of the Pythagorean theorem.

## Definition of the Sine Law

The sine law is useful when

- two angles and any one side length are known
- two side lengths and any one angle are known



Given any triangle ABC the sine law states

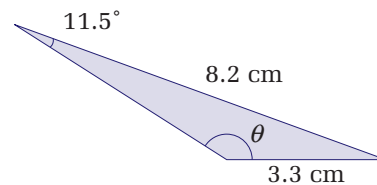
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

| Quantity                         | Symbol | SI unit    |
|----------------------------------|--------|------------|
| length side $a$ opposite angle A | $a$    | m (metres) |
| length side $b$ opposite angle B | $b$    | m (metres) |
| length side $c$ opposite angle C | $c$    | m (metres) |
| angle A opposite side $a$        | A      | (radians)  |
| angle B opposite side $b$        | B      | (radians)  |
| angle C opposite side $c$        | C      | (radians)  |

**Note:** The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

### Example

Use the sine law to solve for  $\theta$ .



Sine law:  
ambiguous case

$$\frac{\sin\theta}{3.3} = \frac{\sin 11.5^\circ}{8.2}$$

$$\sin\theta = 0.5$$

$$\theta = 30^\circ$$

Clearly, angle  $\theta$  is much greater than  $30^\circ$ . In this case, the supplementary angle is required ( $180^\circ - 30^\circ = 150^\circ$ ). It is important to recognize when dealing with obtuse angles ( $> 90^\circ$ ) that the supplementary angle might be required. Application of the cosine law in these situations will help reduce the potential for error.

## Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.



## TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} \\ \tan u &= \frac{1}{\cot u} & \cot u &= \frac{1}{\tan u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u}\end{aligned}$$

- **Pythagorean Identities**

$$\begin{aligned}\sin^2 u + \cos^2 u &= 1 \\ 1 + \tan^2 u &= \sec^2 u \\ 1 + \cot^2 u &= \csc^2 u\end{aligned}$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

- **Co-Function Identities**

$$\begin{aligned}\sin\left(\frac{\pi}{2} - u\right) &= \cos u & \cos\left(\frac{\pi}{2} - u\right) &= \sin u \\ \tan\left(\frac{\pi}{2} - u\right) &= \cot u & \cot\left(\frac{\pi}{2} - u\right) &= \tan u \\ \csc\left(\frac{\pi}{2} - u\right) &= \sec u & \sec\left(\frac{\pi}{2} - u\right) &= \csc u\end{aligned}$$

- **Parity Identities (Even & Odd)**

$$\begin{aligned}\sin(-u) &= -\sin u & \cos(-u) &= \cos u \\ \tan(-u) &= -\tan u & \cot(-u) &= -\cot u \\ \csc(-u) &= -\csc u & \sec(-u) &= \sec u\end{aligned}$$

- **Sum & Difference Formulas**

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

- **Double Angle Formulas**

$$\begin{aligned}\sin(2u) &= 2 \sin u \cos u \\ \cos(2u) &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u \\ \tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

- **Power-Reducing/Half Angle Formulas**

$$\begin{aligned}\sin^2 u &= \frac{1 - \cos(2u)}{2} \\ \cos^2 u &= \frac{1 + \cos(2u)}{2} \\ \tan^2 u &= \frac{1 - \cos(2u)}{1 + \cos(2u)}\end{aligned}$$

- **Sum-to-Product Formulas**

$$\begin{aligned}\sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)\end{aligned}$$

- **Product-to-Sum Formulas**

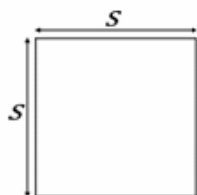
$$\begin{aligned}\sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)]\end{aligned}$$

# GEOMETRY SHAPES AND SOLIDS

## SQUARE

$$P = 4s$$

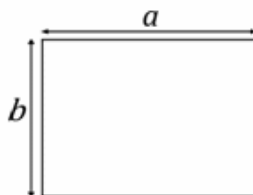
$$A = s^2$$



## RECTANGLE

$$P = 2a + 2b$$

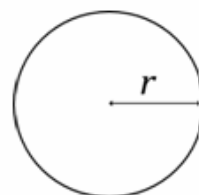
$$A = ab$$



## CIRCLE

$$P = 2\pi r$$

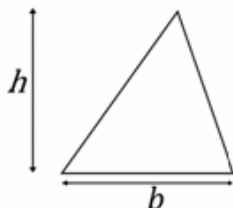
$$A = \pi r^2$$



## TRIANGLE

$$P = a + b + c$$

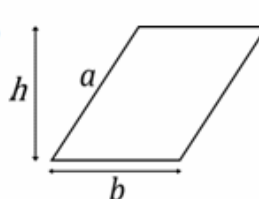
$$A = \frac{1}{2}bh$$



## PARALLELOGRAM

$$P = 2a + 2b$$

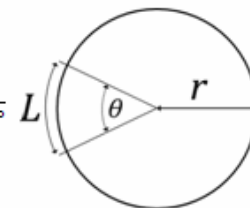
$$A = bh$$



## CIRCULAR SECTOR

$$L = \pi r \frac{\theta}{180^\circ}$$

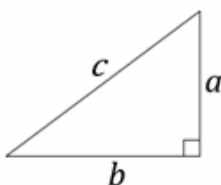
$$A = \pi r^2 \frac{\theta}{360^\circ}$$



## PYTHAGOREAN THEOREM

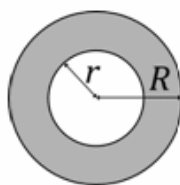
$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$



## CIRCULAR RING

$$A = \pi(R^2 - r^2)$$



## SPHERE

$$S = 4\pi r^2$$

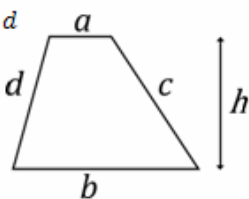
$$V = \frac{4\pi r^3}{3}$$



## TRAPEZOID

$$P = a + b + c + d$$

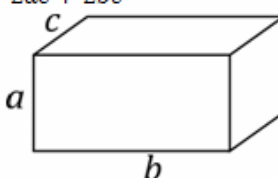
$$A = h \frac{a+b}{2}$$



## RECTANGULAR BOX

$$A = 2ab + 2ac + 2bc$$

$$V = abc$$

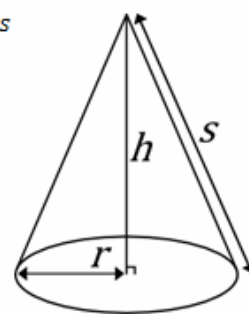


## RIGHT CIRCULAR CONE

$$A = \pi r^2 + \pi r s$$

$$s = \sqrt{r^2 + h^2}$$

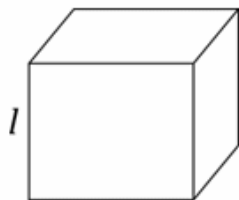
$$V = \frac{1}{3} \pi r^2 h$$



## CUBE

$$A = 6l^2$$

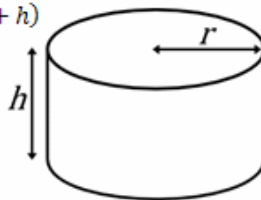
$$V = l^3$$



## CYLINDER

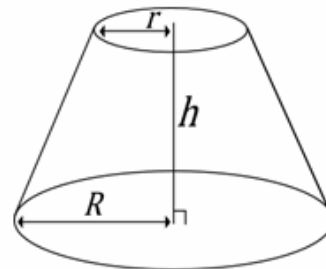
$$A = 2\pi r(r + h)$$

$$V = \pi r^2 h$$



## FRUSTUM OF A CONE

$$V = \frac{1}{3} \pi h (r^2 + rR + R^2)$$



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## The Metric System: Fundamental and Derived Units

### Metric System Prefixes

| Prefix | Symbol | Factor                                 |
|--------|--------|--|
| tera   | T      | 1 000 000 000 000 = $10^{12}$          |
| giga   | G      | 1 000 000 000 = $10^9$                 |
| mega   | M      | 1 000 000 = $10^6$                     |
| kilo   | k      | 1000 = $10^3$                          |
| hecto  | h      | 100 = $10^2$                           |
| deca   | da     | 10 = $10^1$                            |
|        |        | 1 = $10^0$                             |
| deci   | d      | 0.1 = $10^{-1}$                        |
| centi  | c      | 0.01 = $10^{-2}$                       |
| milli  | m      | 0.001 = $10^{-3}$                      |
| micro  | $\mu$  | 0.000 001 = $10^{-6}$                  |
| nano   | n      | 0.000 000 001 = $10^{-9}$              |
| pico   | p      | 0.000 000 000 001 = $10^{-12}$         |
| femto  | f      | 0.000 000 000 000 001 = $10^{-15}$     |
| atto   | a      | 0.000 000 000 000 000 001 = $10^{-18}$ |

### Fundamental Physical Quantities and Their SI Units

| Quantity             | Symbol | Unit            | Symbol |
|----------------------|--------|-----------------|--------|
| length               | $l$    | metre           | m      |
| mass                 | $m$    | kilogram        | kg     |
| time                 | $t$    | second          | s      |
| absolute temperature | $T$    | Kelvin          | K      |
| electric current     | $I$    | ampère<br>(amp) | A      |
| amount of substance  | mol    | mole            | mol    |

### Derived SI Units

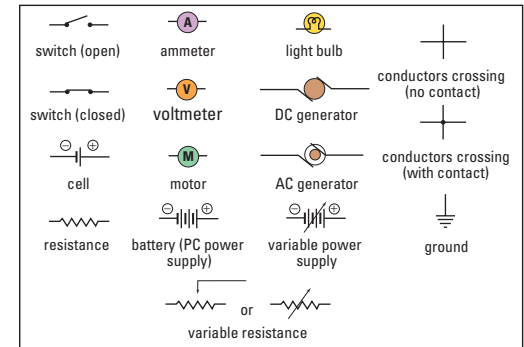
| Quantity                 | Quantity symbol     | Unit                           | Unit symbol | Equivalent unit(s)                                       |
|--------------------------|---------------------|--------------------------------|-------------|--|
| area                     | $A$                 | square metre                   | $m^2$       |  |
| volume                   | $V$                 | cubic metre                    | $m^3$       |  |
| velocity                 | $v$                 | metre per second               | m/s         |  |
| acceleration             | $a$                 | metre per second<br>per second | $m/s^2$     |  |
| force                    | $F$                 | newton                         | N           | $kg \cdot m/s^2$   |
| work                     | $W$                 | joule                          | J           | $N \cdot m$ , $kg \cdot m^2/s^2$                         |
| energy                   | $E$                 | joule                          | J           | $N \cdot m$ , $kg \cdot m^2/s^2$                         |
| power                    | $P$                 | watt                           | W           | J/s, $kg \cdot m^2/s^3$                                  |
| density                  | $\rho$              | kilogram per cubic metre       | $kg/m^3$    |  |
| pressure                 | $p$                 | pascal                         | Pa          | $N/m^2$ , $kg/(m \cdot s^2)$                             |
| frequency                | $f$                 | hertz                          | Hz          | $s^{-1}$   |
| period                   | $T$                 | second                         | s           |  |
| wavelength               | $\lambda$           | metre                          | m           |  |
| electric charge          | $Q$                 | coulomb                        | C           | $A \cdot s$  |
| electric potential       | $V$                 | volt                           | V           | W/A, J/C,<br>$kg \cdot m^2/(C \cdot s^2)$                |
| resistance               | $R$                 | ohm                            | $\Omega$    | V/A,<br>$kg \cdot m^2/(C^2 \cdot s)$                     |
| magnetic field intensity | $B$                 | tesla                          | T           | $N \cdot s/(C \cdot m)$ , $N/(A \cdot m)$                |
| magnetic flux            | $\Phi$              | weber                          | Wb          | $V \cdot s$ , $T \cdot m^2$ , $m^2 \cdot kg/(C \cdot s)$ |
| radioactivity            | $\Delta N/\Delta t$ | becquerel                      | Bq          | $s^{-1}$   |
| radiation dose           |                     | gray                           | Gy          | $J/kg \cdot m^2/s^2$                                     |
| temperature (Celsius)    | $T$                 | degree Celsius                 | $^{\circ}C$ | $T^{\circ}C = (T + 273.15) K$                            |
|                          |                     | atomic mass unit               | u           | $1u = 1.660\ 566 \times 10^{-27} kg$                     |
|                          |                     | electron volt                  | eV          | $1 eV = 1.602 \times 10^{-19} J$                         |

## Physical Constants and Data

### Fundamental Physical Constants

| Quantity                   | Symbol | Accepted value  |
|----------------------------|--------|---|
| speed of light in a vacuum | $c$    | $2.998 \times 10^8$ m/s                                     |
| gravitational constant     | $G$    | $6.673 \times 10^{-11}$ N · m <sup>2</sup> /kg <sup>2</sup> |
| Coulomb's constant         | $k$    | $8.988 \times 10^9$ N · m <sup>2</sup> /C <sup>2</sup>      |
| charge on an electron      | $e$    | $1.602 \times 10^{-19}$ C                                   |
| rest mass of an electron   | $m_e$  | $9.109 \times 10^{-31}$ kg                                  |
| rest mass of a proton      | $m_p$  | $1.673 \times 10^{-27}$ kg                                  |
| rest mass of a neutron     | $m_n$  | $1.675 \times 10^{-27}$ kg                                  |
| Planck's constant          | $h$    | $6.626 \times 10^{-34}$ J · s                               |

### Electric Circuit Symbols



### Other Physical Data

| Quantity                            | Symbol | Accepted value                                       |
|-------------------------------------|--------|--|
| standard atmospheric pressure       | $P$    | $1.013 \times 10^5$ Pa                               |
| speed of sound in air               |        | 343 m/s (at 20°C)                                    |
| water: density (4°C)                |        | $1.000 \times 10^3$ kg/m <sup>3</sup>                |
| latent heat of fusion               |        | $3.34 \times 10^5$ J/kg                              |
| latent heat of vaporization         |        | $2.26 \times 10^6$ J/kg                              |
| specific heat capacity (15°C)       |        | 4186 J/(kg°C)  |
| kilowatt hour                       | $E$    | $3.6 \times 10^6$ J                                  |
| acceleration due to Earth's gravity | $g$    | 9.81 m/s <sup>2</sup> (standard value; at sea level) |
| mass of Earth                       | $m_E$  | $5.98 \times 10^{24}$ kg                             |
| mean radius of Earth                | $r_E$  | $6.38 \times 10^6$ m                                 |
| mean radius of Earth's orbit        | $R_E$  | $1.49 \times 10^{11}$ m                              |
| period of Earth's orbit             | $T_E$  | 365.25 days or $3.16 \times 10^7$ s                  |
| mass of Moon                        | $m_M$  | $7.36 \times 10^{22}$ kg                             |
| mean radius of Moon                 | $r_M$  | $1.74 \times 10^6$ m                                 |
| mean radius of Moon's orbit         | $R_M$  | $3.84 \times 10^8$ m                                 |
| period of Moon's orbit              | $T_M$  | 27.3 days or $2.36 \times 10^6$ s                    |
| mass of Sun                         | $m_s$  | $1.99 \times 10^{30}$ kg                             |
| radius of Sun                       | $r_s$  | $6.96 \times 10^8$ m                                 |

### Resistor Colour Codes

| Colour    | Digit represented | Multiplier                  | Tolerance |
|-----------|-------------------|-----------------------------|-----------|
| black     | 0                 | $\times 1$                  |           |
| brown     | 1                 | $\times 1.0 \times 10^1$    |           |
| red       | 2                 | $\times 1.0 \times 10^2$    |           |
| orange    | 3                 | $\times 1.0 \times 10^3$    |           |
| yellow    | 4                 | $\times 1.0 \times 10^4$    |           |
| green     | 5                 | $\times 1.0 \times 10^5$    |           |
| blue      | 6                 | $\times 1.0 \times 10^6$    |           |
| violet    | 7                 | $\times 1.0 \times 10^7$    |           |
| gray      | 8                 | $\times 1.0 \times 10^8$    |           |
| white     | 9                 | $\times 1.0 \times 10^9$    |           |
| gold      |                   | $\times 1.0 \times 10^{-1}$ | 5%        |
| silver    |                   | $\times 1.0 \times 10^{-2}$ | 10%       |
| no colour |                   |                             | 20%       |

