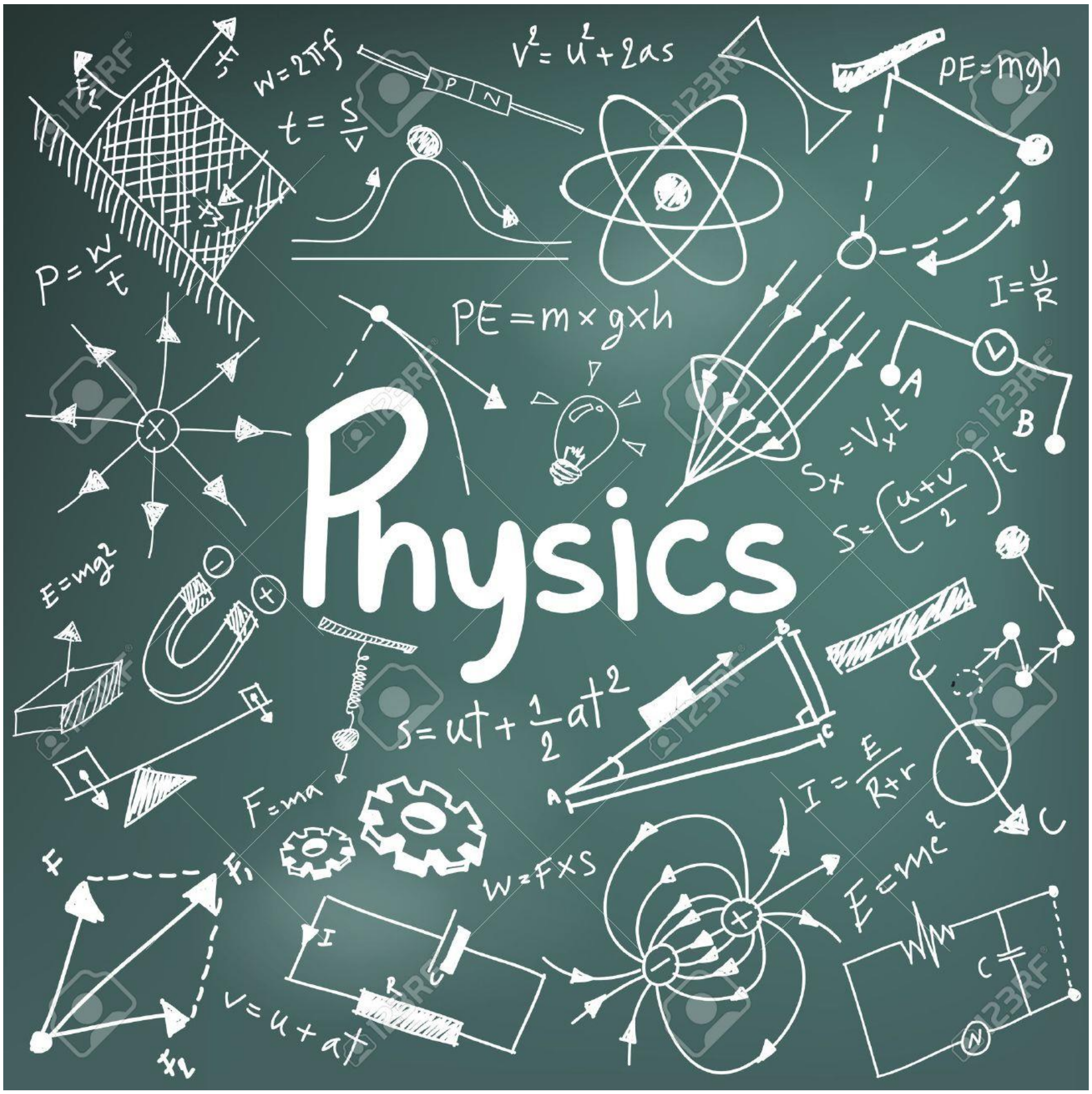
Grade 11 Physics

Outcomes, Concepts, and Problems



Mr. P. MacDonald

The reference digital text ***Openstax: Physics High School*** will be used throughout the course (grade 12 will also require the *College Physics* digital text for outcome 6.3). Readings can be found within the outcome description and the page reference is for the PDF file, not a printed version. Within the first few days of school, go through ***Chapter 1*** to add to your understanding of what physics is all about. Each unit breaks down into *Outcomes*. Each outcome is assessed and will be used to determine your grade for course.

1 Kinematic Motion in One Dimension: Kinematics is the study of *how* objects move.

- 1.1 Mathematics Review: Math is the foundation for solving problems in physics. Students will review order of operations, solving linear and quadratic equations, and trigonometric ratios.
- 1.2 Define and identify scalars and vectors. Classify each of the quantities listed in outcome 1.2 as a scalar or vector. ***Chapter 2, Pg. 67 – 80*** Recognize and define position, displacement, distance, velocity, speed, acceleration, force, and their units. ***Course Concept Guide***
- 1.3 Graphically analyze one-dimensional relationships among position, velocity, acceleration, and time. ***Chapter 2, Pg. 81 – 96***
- 1.4 Mathematically analyze the relationship among position, velocity, acceleration, and time in 1D. ***Chapter 3. Pg. 107 – 127***

2 Kinematic Motion in Two Dimensions: Much of physics relies on being able to solve problems in 1, 2 or 3 dimensions. In high school, we study 1 and 2 dimensional problems. Outcomes 2.1 – 2.3 are about the mathematics behind analyzing 2D problems. Outcomes 2.4 and 2.5 apply the 2D analysis to kinematic concepts from Unit 1. ***Chapter 5.1 & 5.2, Pg. 157-175***

- 2.1 Measuring vectors and the resultant position for 2D vectors using a scale diagram.
- 2.2 Calculating perpendicular components of vectors and calculating vectors (magnitude and direction) given the components.
- 2.3 Mathematically analyze the relationship among displacement (position), velocity, acceleration, and time in 2D.
- 2.4 Vector addition and solving for missing vectors.

3 Dynamics: The study of *why* objects move. This unit describes the types of forces and Newton's Laws of motion. ***Chapter 4, Pg. 129 – 156***

- 3.1 Explain, describe, and analyze the forces of gravity and friction. Including the types and causes of friction, the coefficient of friction, and determining normal force given the situation in one dimension.
- 3.2 Describe and apply Newton's 3 Laws of motion within inertial and non-inertial frames of reference in one dimension.
- 3.3 Qualitatively and quantitatively analyze the various types of forces including friction, normal, gravitational, applied, tension, and the concept of net force in two dimensions.

4 Conservation of Mechanical Energy: Understanding changes that take place in a system is often aided by considering energy exchanges. Students will learn the concept of work, relative to physics, identify forms of energy and that energy for a system is a constant. ***Chapter 9, Pg. 293-313***

- 4.1 Define and apply the concept of work incorporating the following: kinetic, gravitational potential, and elastic potential energy.
- 4.2 Qualitatively and quantitatively analyze the conservation of mechanical energy with conservative and non-conservative forces.

Final Assessment

- 2-hour final assessment during *Assessment Week* (January/June).
- Weighted the same as semester unit assessments.
- Clear outline of topics to review and build of the final assessment will be provided.

5 **Wave Phenomena and Applications**: In this unit students will come to understand waves and related phenomena. The two main topics are mechanical and electromagnetic waves. Students will learn about wave properties, the different parts and properties of the electromagnetic spectrum, standing waves, doppler shift, refraction, and their applications.
- 5.1 Mechanical waves: General properties, types of waves, wave speed, doppler shift, reflection, interference, and standing waves. ***Chapter 13, Pg. 403-427. Chapter 14, Pg. 444-448***
- 5.2 Electromagnetic waves and refraction: Parts of the EM spectrum and their properties. Refraction will focus on visible light as it enters and exits various media. ***Chapter 15, Pg. 470- 476, Chapter 16.2, Pg. 501-512.***
- 5.3 Spherical Lenses: A qualitative and quantitative analysis of using lenses to create images. Also, students will apply the lens maker's equation to problems. ***Chapter 16.3, Pg. 512 – 526***.

6 **Kinematics & Dynamics in 2D**: The study of why objects move in two-dimensional space. This unit applies the concept of perpendicular components to solve problems. Problems, or systems, involve:
- 6.1 In depth quantitative analysis of a projectile motion. ***Chapter 5.3, Pg. 176-185***
- 6.2 Applying Newton's Laws of motion for objects on an incline plane. ***Chapter 5.4, Pg. 185-192***
- 6.3 Learn and apply the concept of net torque to solve static equilibrium problems. ***OpenStax College Physics: Chapter 9, Pg. 329-354***
- 6.4 Collisions and explosions in one and two dimensions. ***Chapter 8, Pg. 267-287***

7 **Circular Motion & Universal Gravitation**: This explores how and why an object travels in a circular path (be it a ball on a string, an amusement park ride, or the Moon about the Earth, for example). Universal gravitation is a deeper analysis of the force of gravity between two masses and is further analyzed with Kepler's Laws of planetary motion.
- 7.1 Qualitatively and quantitatively analyze circular motion using vectors and Newton's laws. ***Chapter 6, Pg. 211-236***
- 7.2 Quantitatively apply Newton's law of gravitation and Kepler's third law of planetary motion to solve problems. Identify and explain Kepler's three laws of planetary motion. **Chapter 7, Pg. 243-260**

8 **Electric Circuits, Fields & Electromagnetic Forces**: Forces can affect objects over a distance and through space without physical contact. The analysis of that affect requires the physics concept of fields. When a current is made to pass through a wire, it creates a magnetic field, and vice-verse. Coulomb's Law allows for the mathematical analysis of the interaction of charged objects.
- 8.1 Qualitatively and quantitatively analyze electric charge and electric fields, including Coulomb's Law. ***Chapter 18, Pg. 563-606***
- 8.2 Quantitatively analyze electric circuits. Concepts include Ohm's Law, series and parallel circuits, and electrical power. ***Chapter 19, Pg. 617-653***
- 8.3 Qualitatively analyze magnetism including magnetic poles, fields, and moving charges. ***Chapter 20, Pg. 663-669***

Physics Investigations

- Physics investigations may take place throughout the course. Such investigations will link directly to learning a particular outcome. Often, questions relating to any investigations will appear on assessments, or they are assessed with a unique grade.

Final Assessment

- 2-hour final assessment during *Assessment Week* (January/June).
- Weighted the same as semester unit assessments.
- Clear outline of topics to review and build of the final assessment will be provided.

Classifying Mistakes

For any type of assessment, determine the type of mistake and the learning opportunity, that is, how to prevent the mistake from happening again. The purpose is to give us information on any learning barriers you are experiencing, and work towards removing them. Use the guide, below, to determine the type of mistake(s) and the learning opportunity.

Type	Type Definition and Examples of Learning Opportunities
Minor	An abnormal mistake, hard to catch, not made often, and not part of the problem solving. • Forgetting to divide by a number that is written down • A number or sign is not carried down to the next math step • Calculator buttons miss-hit • No units in final answers
Background	Usually in the form of mathematical mistakes. • Equation solving • Order of operations • Algebra rules (like and unlike terms)
Conceptual	These mistakes link to gaps in understanding the concepts of physics. • Vector signs • Incorrect formula • Incorrect setup or analysis • Concepts and definitions not fully understood

Strong Work Ethic and Skills for Success

- On task during class. This is the only time I can help you learn. Use it.
- Proper use of technology. Turn off your notifications. This is the main reason student work suffers.
- Time/task management.
- Problem solving skills. Not just math, but the approach to any problem.
- Reflection. No big write up necessary. “Did I work to your best today?”
- Take initiative with your learning. You have all the course materials for the entire semester. Use them.
- Personal workspace (outside of class).
- Goal setting.
- Ask questions during class lessons. Seek your own answers before asking the teacher during work time.
- Ask for feedback.
- Use of course resources. It is all there. Everything. Go forth, learn.
- Embrace mistakes. Then address them.

The Eight Science and Engineering Practices (NGSS)

Learning Target	Description
SEP1: Asking Questions and Defining Problems	A practice of science is to ask and refine questions that lead to descriptions and explanations of how the natural and designed world(s) works and which can be empirically tested. Engineering questions clarify problems to determine criteria for successful solutions and identify constraints to solve problems about the designed world. Both scientists and engineers also ask questions to clarify ideas.
SEP2: Developing and Using Models	A practice of both science and engineering is to use and construct models as helpful tools for representing ideas and explanations. These tools include diagrams, drawings, physical replicas, mathematical representations, analogies, and computer simulations. Modeling tools are used to develop questions, predictions, and explanations; analyze and identify flaws in systems; and communicate ideas. Models are used to build and revise scientific explanations and proposed engineered systems. Measurements and observations are used to revise models and designs.
SEP3: Planning and Carrying Out Investigations	Scientists and engineers plan and carry out investigations in the field or laboratory, working collaboratively as well as individually. Their investigations are systematic and require clarifying what counts as data and identifying variables or parameters. Engineering investigations identify the effectiveness, efficiency, and durability of designs under different conditions.
SEP4: Analyzing and Interpreting Data	Scientific investigations produce data that must be analyzed in order to derive meaning. Because data patterns and trends are not always obvious, scientists use a range of tools—including tabulation, graphical interpretation, visualization, and statistical analysis—to identify the significant features and patterns in the data. Scientists identify sources of error in the investigations and calculate the degree of certainty in the results. Modern technology makes the collection of large data sets much easier, providing secondary sources for analysis. Engineering investigations include analysis of data collected in the tests of designs. This allows comparison of different solutions and determines how well each meets specific design criteria—that is, which design best solves the problem within given constraints. Like scientists, engineers require a range of tools to identify patterns within data and interpret the results. Advances in science make analysis of proposed solutions more efficient and effective.
SEP5: Using Mathematics and Computational Thinking	In both science and engineering, mathematics and computation are fundamental tools for representing physical variables and their relationships. They are used for a range of tasks such as constructing simulations; solving equations exactly or approximately; and recognizing, expressing, and applying quantitative relationships. Mathematical and computational approaches enable scientists and engineers to predict the behavior of systems and test the validity of such predictions.
SEP6: Constructing Explanations and Designing Solutions	The end-products of science are explanations, and the end-products of engineering are solutions. The goal of science is the construction of theories that provide explanatory accounts of the world. A theory becomes accepted when it has multiple lines of empirical evidence and greater explanatory power of phenomena than previous theories. The goal of engineering design is to find a systematic solution to problems that is based on scientific knowledge and models of the material world. Each proposed solution results from a process of balancing competing criteria of desired functions, technical feasibility, cost, safety, aesthetics, and compliance with legal requirements. The optimal choice depends on how well the proposed solutions meet criteria and constraints.
SEP7: Engaging in Argument from Evidence	Argumentation is the process by which evidence-based conclusions and solutions are reached. In science and engineering, reasoning and argument based on evidence are essential to identifying the best explanation for a natural phenomenon or the best solution to a design problem. Scientists and engineers use argumentation to listen to, compare, and evaluate competing ideas and methods based on merits. Scientists and engineers engage in argumentation when investigating a phenomenon, testing a design solution, resolving questions about measurements, building data models, and using evidence to evaluate claims.
SEP8: Obtaining, Evaluating, and Communicating Information	Scientists and engineers must be able to communicate clearly and persuasively the ideas and methods they generate. Critiquing and communicating ideas individually and in groups is a critical professional activity. Communicating information and ideas can be done in multiple ways: using tables, diagrams, graphs, models, and equations as well as orally, in writing, and through extended discussions. Scientists and engineers employ multiple sources to obtain information that is used to evaluate the merit and validity of claims, methods, and designs.

Assessment and Evaluation

Outcomes will be graded from 1 to 6. That grade will be based on evidence from multiple sources including all or some of the following: observations, conversations, formative, and summative assessments.

<table>
<tr><td rowspan="2">Expert:
Demonstration of a deep/thorough understanding of the concept</td><td>6</td><td>• Chose an appropriate strategy.
• Successfully applied the necessary background skills and proper concepts to complete solutions.
• Solutions contained no minor mistakes, or a summative contains at most two.
• Clearly and concisely explained how to solve the problem using appropriate vocabulary, diagrams, and a coordinate system. “Did I show my work?”
• Evaluated the reasonableness of my answer. “Does this make sense for the situation?”
• Concept understood to a high degree to teach it to someone else.
• Concept can be applied to new or challenging problems.</td></tr>
<tr><td>5</td><td>• Chose an appropriate strategy.
• Solution(s) contained an error(s) related to a background skill.
• The concept can be explained using appropriate vocabulary.
• The concept can be applied successfully in known problems.</td></tr>
<tr><td rowspan="2">Apprentice:
Good/Satisfactory understanding of the concept</td><td>4</td><td>• Chose an appropriate strategy.
• A solution contained a concept error. A summative contained at most two such errors.
• Minor mistakes and background skill errors are common.
• Explanations of a problem contained mostly appropriate terminology.
• Mistakes were identified and corrected after referring to a key.
• More practice is needed solving this type of problem.</td></tr>
<tr><td>3</td><td>• Chose an appropriate strategy.
• Solution(s) contained a combination of concept errors, errors related to background skills and minor mistakes.
• A lack of necessary background skills to solve problems.
• Notes, examples, or help was needed to solve problems.
• Explanations did not contain proper terminology.
• Help from an expert is required solving this type of problem.</td></tr>
<tr><td rowspan="2">Novice:
Minimal-to-no understanding of the concept</td><td>2</td><td>• Incorrect strategy(ies) chosen for a problem(s).
• Step-by-step instructions are required to solve problems.
• Tasks could not be performed to an acceptable standard.
• Consistent extra help from an expert is required.</td></tr>
<tr><td>1</td><td>• Basics of what was needed to solve the problem was not known.
• Solution left blank; first step not known.
• Teaching by an expert is required.</td></tr>
</table>

<table>
<tr><th>Learning Category</th><th>Classification Level</th><th colspan="3">Only shortly before report cards will a percentage mark be determined</th></tr>
<tr><td rowspan="2">Expert</td><td>6</td><td colspan="3">95 – 100</td></tr>
<tr><td>5</td><td>86</td><td>90</td><td>94</td></tr>
<tr><td rowspan="2">Apprentice</td><td>4</td><td>73</td><td>80</td><td>85</td></tr>
<tr><td>3</td><td>60</td><td>66</td><td>72</td></tr>
<tr><td rowspan="2">Novice</td><td>2</td><td>50</td><td>56</td><td>59</td></tr>
<tr><td>1</td><td>0</td><td>25</td><td>49</td></tr>
</table>

Reassessing Unit Outcomes

This is not automatic. Available to students who are putting an effort in the unit. Students with a grade of 1 – 3 will be asked to explain or provide reasons why they deserve a reassessment and conference with the teacher (done through a form). Reassessing means to reassess the entire outcome. Not just a question you got wrong. A time will be arranged for these to occur.

Physics 112 – Course Outcome Tracking

Physics 112 Unit/Outcome	Description	Grade	Physics 112 Unit/Outcome	Description	Grade
1.1	Mathematics Review: Math is the foundation for solving problems in physics. Students will review order of operations, polynomials, solving linear and quadratic equations, and trigonometric ratios.		**2.4**	Vector addition and solving for missing vectors.	
1.2	Define and identify scalars and vectors. Classify each of quantity listed in outcome 1.2 as a scalar or vector. Recognize and define position, displacement, distance, velocity, speed, acceleration, force, and their units.		**3.1**	Explain, describe, and analyze the forces of gravity and friction. Including the types and causes of friction, the coefficient of friction, and determining normal force given the situation in one dimension.	
1.3	Graphically analyze one-dimensional relationships among position, velocity, acceleration, and time.		**3.2**	Describe and apply Newton's 3 Laws of motion within inertial and non-inertial frames of reference in one dimension.	
1.4	Mathematically analyze the relationship among position, velocity, acceleration, and time in 1D.		**3.3**	Qualitatively and quantitatively analyze the various types of forces including friction, normal, gravitational, applied, tension, and the concept of net force in two dimensions.	
2.1	Measuring vectors and the resultant position for 2D vectors using a scale diagram.		**4.1**	Define and apply the concept of work incorporating the following: kinetic, gravitational potential, and elastic potential energy.	
2.2	Calculating perpendicular components of vectors. Calculating vectors (magnitude and direction) given the components.		**4.2**	Qualitatively and quantitatively analyze the conservation of mechanical energy with conservative and non-conservative forces.	
2.3	Mathematically analyze the relationship among displacement (position), velocity, acceleration, and time in 2D.				

Overall Course Grade

- Calculate your *median* by arranging your grades from lowest to highest. The grade in the middle is *likely* your overall grade. If there is no exact middle number, average the two middle numbers.
- Calculate your mean by adding all the grades up and divide by how many there are.
- Use a pencil when writing your grades here because grades will fluctuate over the semester.

Smallest – to – Largest												

Median = _______ Mean = _______

Example Percent Determinations

Median	Mean	Percent	Reason
4	3.8 – 4.2	80 %	Median and mean match or are close
4	4.3 or higher	85 %	Mean is much higher than median
4	3.7 or lower	73 %	Mean is much lower than median

Physics 122 – Course Outcome Tracking

Physics 122 Unit/Outcome	Description	Grade	Physics 122 Unit/Outcome	Description	Grade
5.1	Mechanical waves: General properties, types of waves, speed, doppler shift, reflection, interference, and standing waves		6.4	Conservation of momentum: collisions and explosions in one and two dimensions.	
5.2	Electromagnetic waves: Types of EM radiation and refraction.		**7.1**	Qualitatively and quantitatively analyze circular motion using vectors and Newton's laws.	
5.3	Spherical lenses: A qualitative and quantitative analysis lenses to create images and the application of the lens maker's equation.		**7.2**	Quantitatively apply Newton's law of gravitation and Kepler's third law of planetary motion to solve problems. Identify and explain Kepler's three laws of planetary motion.	
6.1	In depth quantitative analysis of a projectile motion.		**8.1**	Qualitatively and quantitatively analyze electric charge and electric fields, including Coulomb's Law.	
6.2	Applying Newton's Laws of motion for objects on an incline plane.		**8.2**	Quantitatively analyze electric circuits. Concepts include Ohm's Law, series and parallel circuits and their components.	
6.3	Learn and apply the concept of net torque to solve static equilibrium problems.		**8.3**	Qualitatively analyze magnetism including magnetic poles, fields, and moving charges.	

Overall Course Grade

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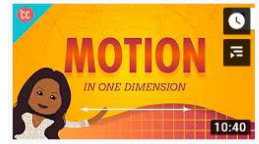
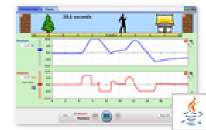
4	3.7 or lower	73 %	Mean is much lower than median

Crash Course Physics & Kahn Learning Academy: These are a series of YouTube videos produced by certified scientists and educators. They are suggested viewing material to complement many of the concepts introduced in Physics. The digital version of this document contains a link to the videos, otherwise a search of YouTube will find the video or related videos.



Unit 1: Kinematics in One Dimension

Kinematics: The study of *how* objects move. That is, analyzing motion relating to an object's current motion. The analysis is often a mathematical approach to solving for quantities such as position, final or initial velocity or acceleration at an instant in time. Investigate this with the PhET simulation *The Moving Man.*



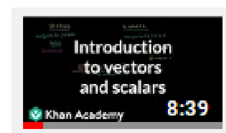
The Moving Man

Motion: An object is said to be in motion when its position changes.

Frame of Reference: Something not moving with respect to an observer that can be used to detect motion; it is the "point-of-view" of the observer. For example, suppose someone on the ground is watching another person walking on a train. The person on the train has a velocity *relative* to the ground or *relative* to the train. Both frames of reference are correct but will yield different numerical results when analyzed.

Coordinate System: The mathematical reference from which to measure quantities. It provides a reference for defining *direction* – up, down, left, right, north, west, east, south, etc. Solving problems in physics requires measurements to be positive or negative and it is the coordinate system that defines the positive direction.

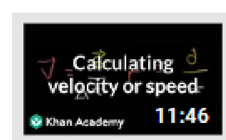
Scalar: Measurements that are independent of direction and always positive in value. It is not that you choose not to communicate a direction, but rather, stating a direction *does not make sense*. For example, you would not tell someone their body temperature is 38 °C East. Other scalar quantities include time, mass, distance, and speed.



Vector: Measurements that have a magnitude *and* a direction (magnitude is the numerical value of the measurement). All vectors must be used relative to the positive direction as determined in the coordinate system. Vector quantities can be positive or negative, depending on the coordinate system and are communicated in writing by being bold font or with an arrow above them. For example, the variable for force could be written as ***F*** or $\vec{F}$. Visually, in diagrams, vectors quantities are represented by arrows. The length represents magnitude and the way it points is the direction. Other examples of vectors include position, displacement, velocity, and acceleration.

Distance: The length of the route between two points. Measurements of distance can never decrease. That is, you cannot take away the fact an object moved a certain distance. The odometer of a car, for example, measures a car's total distance traveled and it is illegal to tamper with it.

Position: An object's distance and direction from a reference point (within a coordinate system) at an instant in time. The change in position is called *displacement*.

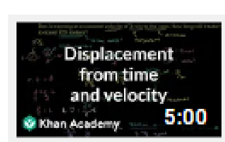


Average Speed: An object's *total distance* traveled per time interval.

Instantaneous Speed: The speed of an object at an instant in time. Determining this usually requires data to be analyzed. One example is the speedometer of a car.

Average Velocity: An object's *change in position* per time interval. Changes in direction average out so the path taken does not matter. It is possible to have a value of zero.

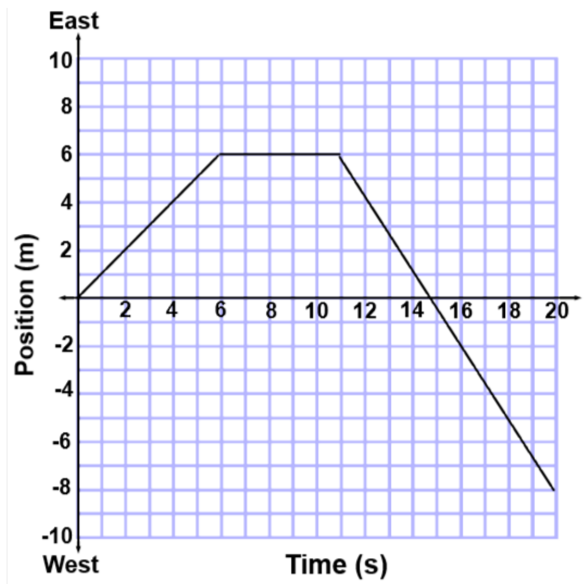
Instantaneous Velocity: The velocity of an object at an instant in time (usually requiring data analysis). It is the object's instantaneous speed *and* direction.



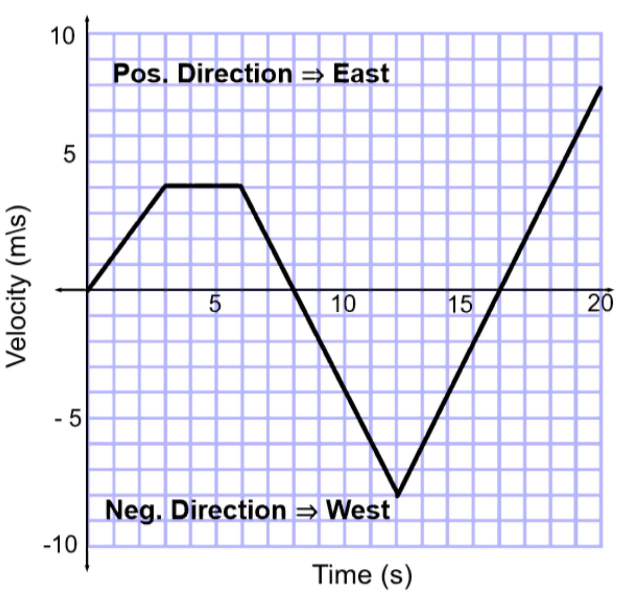
Acceleration: The change in an object's velocity per unit time. That means if an object's speed changes then it undergoes an acceleration. It also means that if only the object's direction changes then it accelerates! When an object changes direction it has an instantaneous velocity of zero, but its acceleration is not zero.

Analyzing Position–Time Graphs

- **Position**: Read from the position axis for the given time. It is how far and in what direction from a starting point.
- **Distance**: Add up all the motion using positive numbers. Follow the graph counting squares up and down until you get to the desired time. It is the total length an object traveled.
- **Direction Change**: When the velocity goes from positive to negative or vice versa.
- **Average Speed**: Distance (see above) divided by the time given in the question.
- **Instantaneous Speed**: Positive value of the slope of the line at the time given (find two points on the line and calculate rise/run).
- **Average Velocity**: Position (see above) divided by the time given in the question.
- **Instantaneous Velocity**: Slope of the line at the time given (find two points on the line and calculate rise/run). Can be positive or negative.



Analyzing Velocity–Time Graphs



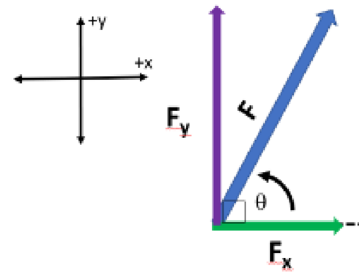
- **Velocity**: Read it from the velocity axis for the given time. It communicates the direction of travel, not the position.
- **Instantaneous Acceleration**: The slope of the line at the time given (find two points on the line and calculate rise/run). Can be positive or negative and the sign is the direction of the acceleration, not the object.
- **Distance**: For a certain time-interval, the distance is the area contained between the graph and the time axis (always use positive numbers when calculating).

- **Position**: For a certain time-interval, it is the area contained on the top minus the area contained on the bottom.
- **Direction Change**: When the velocity goes from positive to negative or vice versa.
- **Average Speed**: Total distance divided by the time.
- **Average Velocity**: Final position divided by the time.

Check out the PhET simulation again, but now make use of the graphical functions. Create a position-time and velocity-time graph to review the above concepts.

The Moving Man

Unit 2: Kinematics in Two Dimensions



Perpendicular Components: When a vector makes an angle with one of the axes of a coordinate system, that vector is acting partly in each direction. A physics problem must be solved by analyzing one dimension at a time because perpendicular vectors are *independent* of each other. Take the diagram to the right. The force, ***F***, makes an anlge to the coordinate system so part of the force acts in the x-direction, $\boldsymbol{F_x}$, and the other in the y-direction, $\boldsymbol{F_y}$.

- Mathematically, they are calculated using right triangle trigonometry: $\boldsymbol{F_x} = \boldsymbol{F}\cos\theta$, and $\boldsymbol{F_y} = \boldsymbol{F}\sin\theta$. The sign of the component matches the direction of the coordinate system.
- If you know the perpendicular components, the vector can be calculated with the Pythagorean Theorem and the angle from the horizontal using $\tan^{-1} F_y/F_x$ (use only positive component values to calculate the angle).

Kinematics in 2D: These types of problems will read like previous problems, but the vectors could be angled to the coordinate system. To solve, calculate the perpendicular components for all vectors (watch for negative directions) and apply the physics relationship(s) in each dimension separately. If you are calculating a vector, the final answer's magnitude is determined with the Pythagorean Theorem and use the inverse tangent of the components to find the direction, or the angle.

Relative Velocity: When multiple vectors of the same type (like velocity) act on an object, the *resultant vector* (or resulting velocity) is determined through *vector addition*. Essentially, each vector is broken down into perpendicular components, and components of the same dimension are added together (taking direction into consideration). The resultant is calculated using Pythagorean Theorem and the direction with inverse tangent of the resultant's components. Some problems will give you the resultant and you need to calculate a missing vector. The process is the same, add up the vectors and one of the components will be unknown.

Vector Addition

Unit 3: Dynamics in One and Two Dimensions

Dynamics: The study of *why* objects move. When all the forces acting on an object add to be greater than zero, then there is a *net force*. This will cause the object to accelerate, after which we apply kinematic concepts for an analysis.

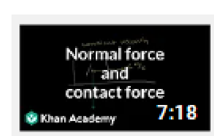
Force: ***F***, a vector quantity that is a push or a pull on and object. The unit of force is the Newton, N (pounds, or lb, in the United States). Many different forces can act on an object at the same time. The unit of a Newton breaks down into component units: $N = kg \cdot m/s^2$ (investigate forces using the PhET simulation to the right).

Forces and Motion: Basics

Types of Forces: Forces are grouped in to two categories – contact and noncontact forces. Contact forces require direct physical contact between objects (such as friction, applied and tension forces). Noncontact forces exert their forces on objects over a distance and create *fields* (such as gravity, magnetic and electric forces).

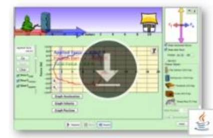
Applied Force: $\boldsymbol{F_a}$, this is usually a force created by a person or machine acting on an object.

Normal Force: $\boldsymbol{F_N}$, a force that acts perpendicular to a surface. For example, the stool you sit on provides a normal force upwards, equal to the magnitude of gravity's pull on you, your weight. The concept of normal force is prominent in our investigations and understanding of the force of friction.



Force of Friction: $\boldsymbol{F_f}$, this *electromagnetic* force acts between surface atoms of two objects in direct contact. These bonds must be broken to move the objects. The greater the normal force the more the objects are pushed together, so the stronger the bonds. The force of friction always acts opposite the direction of motion of the object or, if it is not moving, the direction of the net force acting on it. There are two types of frictional forces: static and kinetic.

Forces in 1 Dimension



Static Friction: A frictional force that must be overcome to get an object moving. Think of pushing a massive object, it does not move unless you apply a large force. That is because the force of friction is very strong due to a high normal force.

Kinetic Friction: A frictional force that must be overcome to keep an object moving. Kinetic friction is lower than static friction for any two surfaces. The electromagnetic bonds have less time to strengthen as they continually form and break due to the object's motion.

Coefficient of Friction: μ. Think of this as the "*electromagnetic stickiness*" between any two surfaces. It is unique for any two objects, and for our course, it is independent of surface area or location. All that matters are the two objects. Each type of friction has its own coefficient value. The symbol is the Greek letter mu, μ.

Forces and Motion

Force of Gravity: $\boldsymbol{F_g}$, Aa attractive force that acts over a distance between masses because of their warping of spacetime. For situations on Earth, it is the pull on objects towards the center of the Earth. It is often calculated by $\overrightarrow{F_g} = m\vec{g}$, where $\vec{g} = 9.81\, m/s^2$, the average acceleration due to gravity for the Earth. Another term for force of gravity is an object's *weight*.

Net Force: $\boldsymbol{F_{net}}$, is the vector sum of all the forces acting on an object. Vector sum means that only forces acting in the same *dimension* can be added together. For example, forces acting left, or right are not added with forces acting up or down – they are separate calculations. Objects accelerate in the direction of the net force.

Equilibrium: An object in equilibrium means that the net force acting on it is *zero*. This can happen if the object is at rest (a velocity of zero) or is moving with a constant velocity (an acceleration of zero).

Equilibrant: The vector, that when added, will result in the object achieving equilibrium.

Classical Mechanics: Developed in the late 1600s by Sir Isaac Newton. This system of physics treats matter and energy as separate entities, but it can predict the motion and interactions of objects. Such objects need to be much larger than an atom and traveling much slower than the speed of light.

Quantum Mechanics: Developed in the early 1900s, spearheaded by Albert Einstein, it treats matter and energy as one and the same through the famous equation $E = mc^2$. This system of physics can model the motion and energy of subatomic particles and objects traveling near the speed of light, $c \approx 300\,000\,000$ m/s as well as all classical physical systems.

Newton's 1st Law: An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by an external force. An external force brings an object out of equilibrium. For example, when traveling in a car and it takes a sharp turn you get squished into the door or another passenger. What is happening is that you are trying to continue uniform motion, but the car gets in your way.

Newton's 2nd Law: The net (average) force acting on an object is equal to the product of the object's mass and (average) acceleration, ***F*** = *ma*. This force has the same direction as the acceleration. This mathematically relates concepts of dynamics and kinematics.

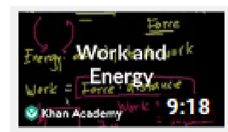
Newton's 3rd Law: For every action there is an opposite and equal reaction. Forces act in pairs, called action-reaction pairs. Thus, if you punch a wall the wall "punches" back with the same force, possibly breaking your hand. Walking on the floor is possible because the floor pushes you forward (if you were to push the floor, the floor would move).

Inertial Frame of Reference: One in which Newton's Laws of motion are valid and can be applied. Every force can be explained. This frame of reference must be in equilibrium.

Non-Inertial Frame of Reference: One in which Newton's Laws of motion are not valid and cannot be applied. The frame of reference is not in equilibrium. In such a frame of reference, some aspects of an object's motion cannot be explained without the use of *fictitious forces*. For example, if a car is the frame of reference and it suddenly slows down, the passengers lunge forward. Relative to the car there was no push or pull to create such motion.

Unit 4: Conservation of Mechanical Energy

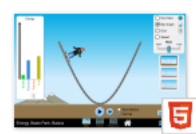
Work: In physics, work is done on an object when a force causes a displacement of the object. Work is a measure of energy and uses the unit called Joules, J. For reference, about 1 J of work is lifting an apple to the top of your head. The mathematical relationship is *W* = ***Fd***. Only a force parallel to the direction of motion does work on an object. Work is a scalar quantity, but it can be positive or negative. The sign of work is not determined by a coordinate system. If the force and displacement are in the same direction, that is positive work, otherwise, work is negative.



Conservative Force: Does work on an object in such a way that the amount of work done is independent of the path taken. For example, lifting an object 1 meter off the ground will be the same if you lift it straight up or use a ramp (in the absence of friction). A system of only conservative forces will have energy changes that are reversible.

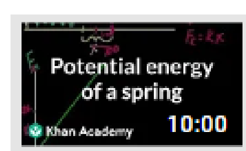
Nonconservative Force: The work done on an object is path dependent. Friction is such a force and removes energy from the system as heat. Such physics systems may not be reversible. In the natural world, most systems involve nonconservative forces. For example, when you bounce a ball it will not return to its original height because energy is lost from the collision with the floor in the form of deformations, heat, and sound.

Kinetic Energy: E_K, is the energy associated with motion. Represented by $E_K = \frac{1}{2}mv^2$, where *m* is the object's mass in *kg* and *v* is the instantaneous speed in *m/s*. Kinetic energy is measured in Joules, *J*.


Energy Skate Park: Basics

Potential Energy: Stored energy, the object has the *potential* to move. Such energy is associated with an object because of the position, shape, or condition of the object (e.g. pressing down on a spring or pulling a bow string stores energy because of a change of shape of the object).

Gravitational Potential Energy: E_g, is energy is stored in a gravitational field above a reference, or zero level of a gravitational source. (i.e. the surface of the Earth, but it can be set for each system). Mathematically, $E_g = mgh$, where *g* is the acceleration due to gravity and *h* is the height above a zero level (use only positive values).



Elastic Potential Energy: E_e, the energy stored that depends on the distance an object has been compressed or stretched. Elastic materials can restore its shape by applying a *restoring force*. That ability is summarized numerically by a what is called a spring constant, *k*. Highly flexible materials have a low k-value, whereas stiff materials have high k-values. Mathematically, the stored energy is $E_e = \frac{1}{2}kx^2$ where *x* is the compression or stretch length in meters. If a restoring force calculation is required, *Hooke's Law* is applied to the object: $F_s = -kx$. The expression is negative because the restoring force always acts opposite to the compression or stretch direction (this assumes stretch direction is positive). If maximum acceleration is at position *x*, then max velocity is at *x*/2. At maximum acceleration *v* = 0 m/s, and at max velocity, *a* = 0 m/s^2.

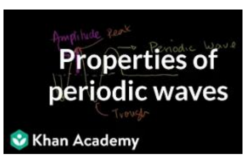
Work-Energy Theorem: Work must be done on an object to change its position. This theorem states that the work done on an object equals its energy gained or lost. Each form of energy applies its own version of the work-energy theorem: $W = \Delta E_K$, $W = \Delta E_g$, or $W = \Delta E_e$ and remember that $W = \boldsymbol{Fd}$, should force or displacement be part of the analysis. The symbol Δ (capital Greek letter delta) means "change in" so it will be an object's "final" – "initial" energy of that type.

Law of Conservation of Energy: Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant. Common examples include dropping an object – initially the object has a speed of zero but gravitational potential energy. Once released the gravitational potential energy decreases but its speed increases – gravitational potential energy is transformed into kinetic energy. In the absence of air resistance, the total energy remains constant. Use the PhET simulation on the right to explore energy transformations.

Energy Skate Park: Basics

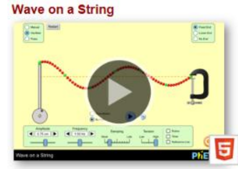
- If all the forces in the system are conservative, then mathematically $\Delta E_T = 0$ where E_T is the total of all types of energies and is a constant. That means the change in total energy is zero.
- If nonconservative forces are exist then, mathematically $\Delta E_T = W_{nc}$, where W_{nc} is the work done by the nonconservative forces (for example, friction acting over a distance) and will evaluate to a negative value since that energy is removed from the system.

Unit 5: Wave Phenomena and Applications



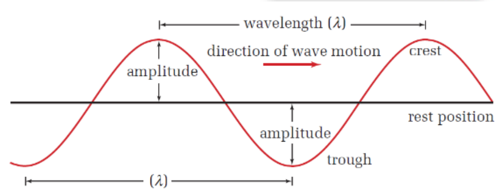
Wave: A disturbance that transfers energy through a medium. There are many types of waves and are generally classified as mechanical, electromagnetic, or gravity. The link left, *Properties of Waves*, is to a YouTube playlist by Kahn Academy. The playlist covers much of the topics that we will discuss in this unit.

Wave on a String

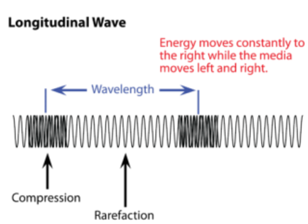


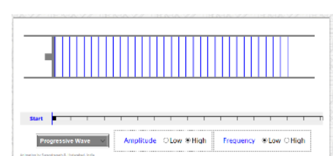
Mechanical Wave: Requires a physical medium to travel through. They are classified as either transversal or longitudinal.

- **Transverse wave**: One that vibrates particles of a medium perpendicular to the direction of wave travel.
- **Longitudinal wave**: One that vibrates particles of a medium parallel to the direction of wave travel.



Amplitude: The maximum displacement of a particle from its equilibrium, rest, position.





Wavelength, λ: The distance from a point on the wave to a corresponding point such that it includes one complete crest and trough.

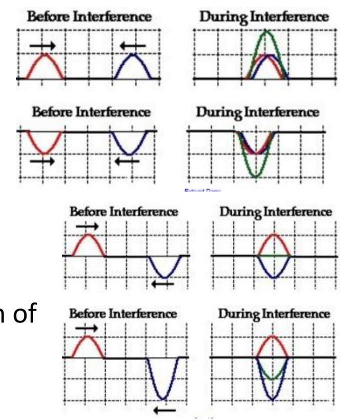
Frequency, *f*: The number of waves created per unit time. Measured in Hertz, Hz.

Period, *T*: The time to create one complete wave. Measured in seconds.

Medium: The material, or matter, a wave propagates though. Could be solid, liquid, gas or space.

Wave Speed, *v*: The speed of a wave is a constant for a particular medium. It is independent of the size or energy used to create it. It is a characteristic of that material, much like a boiling or freezing point. The speed is determined by the mass and forces between medium particles. Changing tension, for example, changes the force between particles, so it changes the medium – which changes wave properties. Those principles are prevalent in string instruments.

- **Effect of Force Between Particles (Tension)**: The greater the force between particles, the more rapidly the particles return to equilibrium position. This results in a faster wave. In other words, as the force of tension in a medium increases, wave speed increases and vice-versa.
- **Effect of Particle Mass**: The greater the mass of the particles, the slower they return to equilibrium position because of their inertia (resistance to change in motion). That results in a slower wave speed. So, as the mass of a medium particle increases, the wave speed decreases and vice-versa.
- **Effect of Friction**: Friction within a medium acts to dampen or reduce the amplitude of the wave but has no effect on wave speed (unlike sliding something on a table).

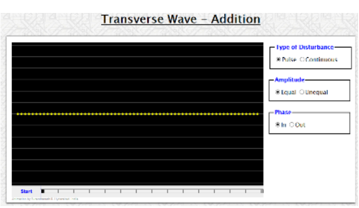


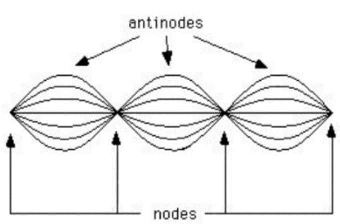
Wave Interference: When two or more waves act simultaneously on the same particles of a medium.

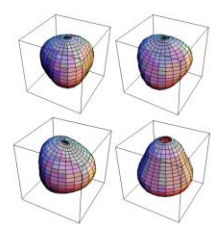
- **Constructive Interference**: Results when two or more waves interfere to produce a resultant displacement greater than the displacement caused by either wave itself.
- **Destructive Interference**: When the resultant displacement is smaller than the displacement that would be caused by one wave by itself.

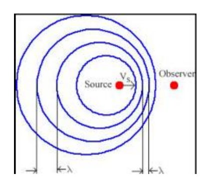
Principle of Superposition: The resultant displacement of a given particle is equal to the sum of the displacements that would have been produced by each wave acting independently.

Standing Waves: An interference pattern that occurs if interfering waves have the same amplitude, wavelength, frequency and are traveling in different directions. The pattern creates nodes and antinodes. Nodes (be it points or lines) are locations of total destructive interference. Can occur in one, two, or three dimensions.



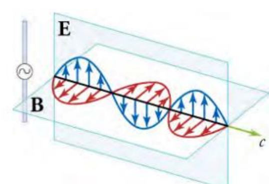




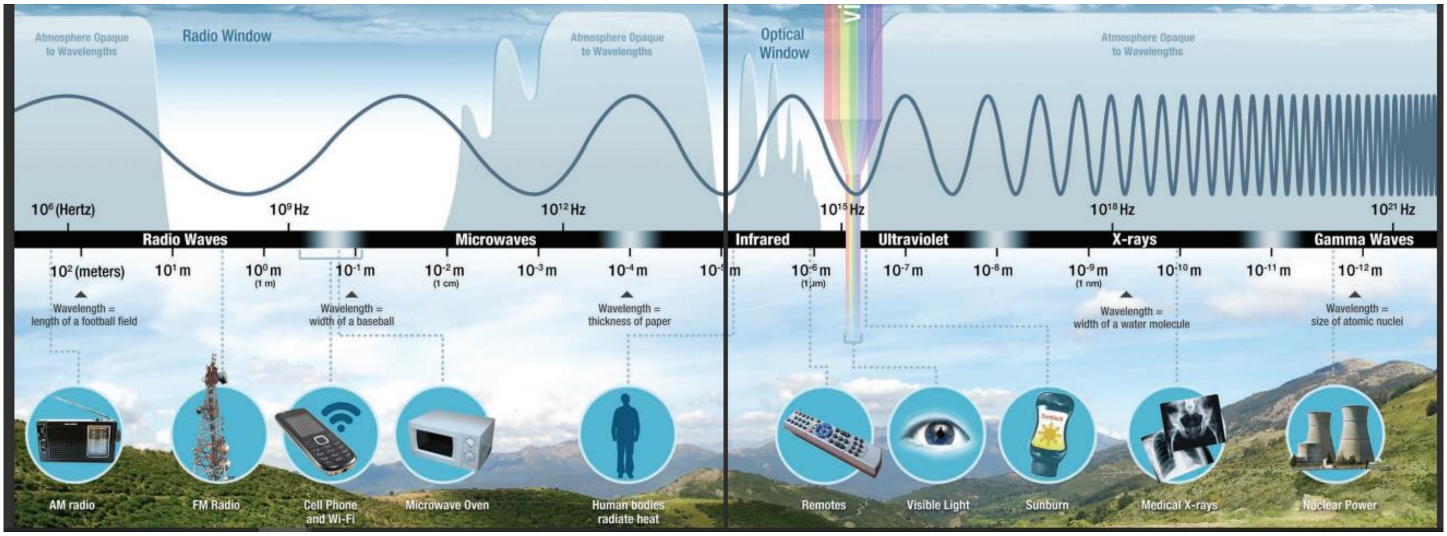


Doppler Shift: The apparent difference between the frequency at which sound (or electromagnetic radiation) waves leave a source and that at which they reach an observer, caused by the relative motion of the observer and the source.

Electromagnetic Radiation (wave): Energy waves generated by a moving electric charge, that is, by an electric current. An electric current generates both an electric field, ***E***, and a magnetic field, ***B***. These fields are perpendicular to each other. When the moving charge oscillates, as in an alternating current, an EM wave is propagated.

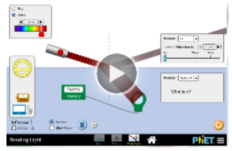


Electromagnetic Spectrum: Depending on the factors involved when an electric charge moves (acceleration), one of many different types of radiation can be produced. From lowest to highest energy, the EM spectrum encompasses radio, microwaves, infrared, visible light, ultraviolet, x-rays, and gamma radiation. All types of EM radiation travel the same speed, $c \approx 300\,000\,000$ m/s, in a perfect vacuum (outer space). Lowest energy EM waves have the largest wavelengths and lowest frequencies, whereas the highest energy EM waves have the smallest wavelengths and highest frequencies.



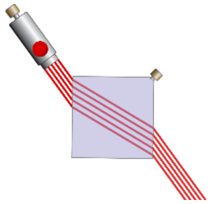
Refraction: The change in direction of a wave, in our case a light wave, as it enters a different medium.

Index of Refraction: The ratio of the speed of light in a perfect vacuum to that of the medium it is in. The number will always be greater or equal to 1 as it is a measure by what factor light slows down. It is given the variable, *n*.



Snell's Law: The mathematical relationship between two media's indices of refraction, incident angle and refracted angle. Light entering a medium where it slows down will bend toward the normal, and vice-versa.

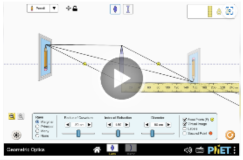
Critical Angle: For the case of light traveling into a medium where its speed increases. When the refracted angle calculates to 90°, that incident angle is called the critical angle. Any angle larger than the critical angle will result in *total internal reflection*. That is when the boundary acts like a mirror.



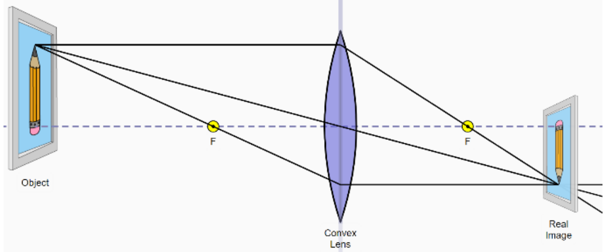
Lateral Displacement: The distance between a light ray's original path and its actual path when traveling through a refracting material.

Spherical Lense: A lens whose shape would be a section of a sphere. For this course, we assume the lenses are thin enough to ingnore the effects of lateral displacement. Lenses are used to focus, or redirect rays of light to create images. Images come in two formats, real or virtual.

- **Real Image**: Forms in space so it can be projected to a screen, film, or retina.
- **Virtual Image**: *Appears* to form in space, but in fact it cannot be captured on film (from the location it appears to be). Magnifying glasses make use of virtual images to enlarge objects.

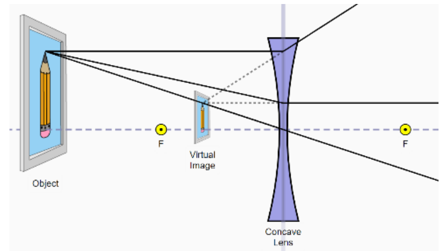


Convex Lens: This lens has been shaped so that all light rays that enter it parallel to its central axis cross one another at a single point on the opposite side of the lens. The central axis, or axis, is defined to be a line normal to the lens at its center. Such a lens is called a *converging lens* because of the converging effect it has on light rays.



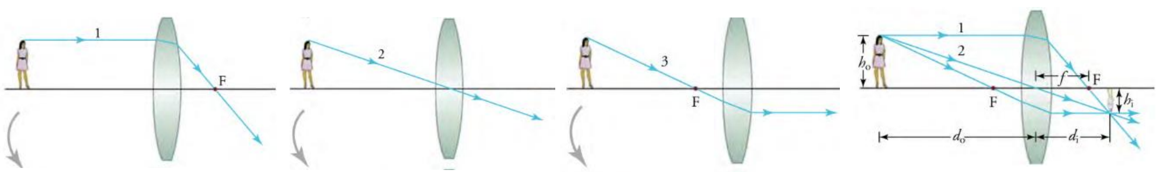
- The link to the left is a playlist from Kahn Academy that covers concepts and calculations involving thin convex and concave lenses and the thin lens equation. Note, we will not get into multiple lens systems, diopters or aberration.

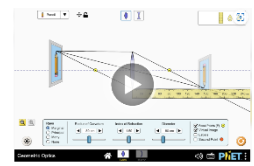
Concave Lens: This lens is a diverging lens because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length, or "f," of the lens. Note that the focal length of a diverging lens is defined to be negative.



Ray Tracing: This is used to determine where an image, either real or virtual, will form. An image will form, or appear to form, at the location where refracted light rays meet, or appear to meet. In diagrams, we use the following rules:

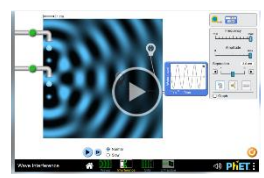
1. A ray entering a converging lens parallel to its optical axis passes through the focal point, F, of the lens on the other side.
2. A ray entering a diverging lens parallel to its optical axis seems to come from the focal point, F, on the side of the entering ray.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its optical axis.
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the optical axis.





Magnification: Determined by the ratio of image size to actual size of the object. It is negative for virtual images.

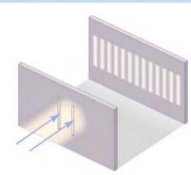
Diffraction: The bending of a wave around the edges of an opening or an obstacle. It is a wave characteristic that occurs for all types of waves. If diffraction is observed for a phenomenon, it is evidence that the phenomenon is produced by waves.



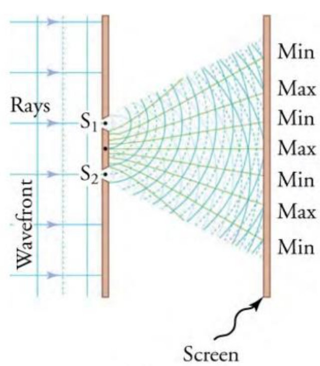
Wavefronts: The points on a wave's surface that share the same, constant phase (such as all the points that make up the crest of a water wave).

Huygens' Principle: "Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets."

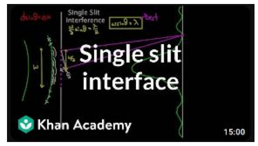
Young's Double Slit Experiment: Here, light of a single wavelength passes through a pair of vertical slits and produces a diffraction pattern on the screen—numerous vertical light and dark lines that are spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.



- When light passes through narrow slits, it is diffracted into semicircular waves. Pure constructive interference occurs where the waves line up crest to crest or trough to trough. Pure destructive interference occurs where they line up crest-to-trough. An analogous pattern for water waves is shown. Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. Those angles depend on wavelength and the distance between the slits.



- Check out a variety of videos from Kahn Academy to help with the concepts.



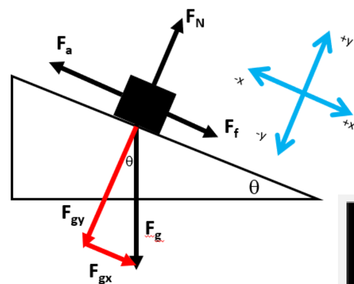
Unit 6: Extension of Kinematics & Dynamics in 2D

Projectile Motion: The only force acting on a launched object is gravity (we assume no air resistance).

- When launched at an angle, the projectile's velocity will have a horizontal, x, and vertical, y, component. Solving these problems requires the application of kinematic equations in each dimension separately.
- Since gravity only affects the vertical direction, there are no forces, hence no acceleration, in the horizontal direction.
- Horozintal and vertical motion occur within the same time interval, *t*, so time is the link between the motion in each dimension.
- **Range**: The horizontal position where the projectile lands, d_{fx}. The maximum range occurs for angles of 45°, and if landing at the same vertical position as it was launched, complementary angles (angles that add to 90°) have the same range.
- At maximum height the projectile is changing directions so the instantaneous velocity is zero.

Projectile Motion


Forces on an Incline: If the object is on an incline or ramp the coordinate system is rotated the same degree as the angle the ramp is to the ground, then it is the force of gravity that is broken into components. The trigonometry yields the following based on the plane of the ramp being the x-axis:

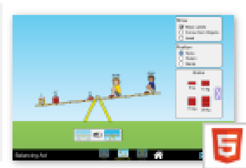


$F_{gx} = F_g sin\theta$
$F_{gy} = F_g cos\theta$

Connected Masses on a Ramp: Break the appropriate forces into components and then apply Newton's 2nd Law but now we include the sum of all the masses that accelerate. Picture the problem stretched out in one dimension to determine the direction of forces. Forces of tension are equal and opposite and cancel out when adding all forces. To find the force of tension, apply Newton's 2nd Law to one of the masses.


Ramp: Forces and Motion

Torque: Think of this as rotational power. When an object is made to rotate around a pivot point by a force acting on a lever arm (think prying something loose or a playground see-saw), that rotation is a result of a torque being applied. It is also the turning or twisting effectiveness of a force. The sign of a torque depends on which way a force would rotate an object. clockwise rotation is negative torque and counterclockwise is positive torque.

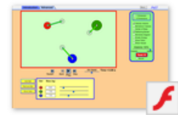

Balancing Act

Net Toque: The sum of all the torques acting on an object, incorporating all the rotational directions.

Static Equilibrium: If a system is said to be in static equilibrium, then the net force and net torque are zero. Often, solving such problems requires the use of net torque first because there are too many unknown force variables. By making the pivot point at the location of an unknown force, the net torque analysis eliminates that unknown force from the mathematical equation, allowing for another force to be calculated.

Center of Mass: If able to rotate freely, an object rotates about its center of mass. Objects (like a beam) can be analyzed as if all its mass is contained at one point.

Linear Momentum: An object's tendency to continue on-course, that is, to continue to move in the same direction. Momentum is directly proportional to the object's speed and mass, so the greater the mass and speed, the higher the momentum. The direction of an object's momentum is the same as its velocity, so objects traveling in different directions will have opposite signs.


Collision Lab

Elastic Collision: One in which objects separate after colliding. Kinetic energy is conserved.

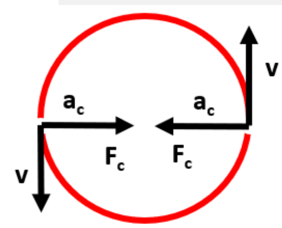
Inelastic Collision: One in which objects stick together after colliding. Kinetic energy is not conserved. Collision can be perfectly elastic, inelastic, or anything in-between.

Conservation of Momentum: When two, or more, objects interact (collide) the total momenta before and after the collision is conserved. That means momentum cannot be lost or gained. Momentum is conserved in all dimensions and velocities must be broken up into perpendicular components to allow for the mathematical analysis if each dimension. If an object explodes, the momentum of all the individual pieces must add to zero, assuming the object was at rest before in exploded.

Unit 7: Circular Motion & Universal Gravitation

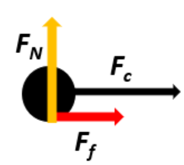
Uniform Circular Motion: Motion in a circular path at a constant speed, centered at one point. Note that it is not constant velocity because in a circle the direction continually changes. Because the direction continually changes, objects in circular motion are accelerating. The velocity as also called tangential velocity because it is always pointing at a tangent to the circular path.

Radial Direction: When considering circular motion, any vector that is on a line that passes through the center of the circle. Vectors do not need to point towards the circle to be considered radial, just have to line up with the center.

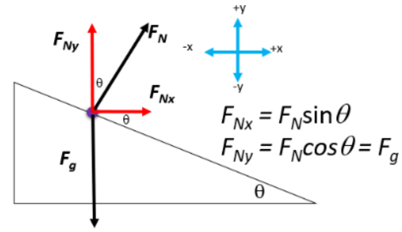


Centripetal Acceleration: The acceleration of an object moving in a circle. It always points towards the center of the circle (with is the direction of the change in velocity) and is perpendicular to the object's velocity.

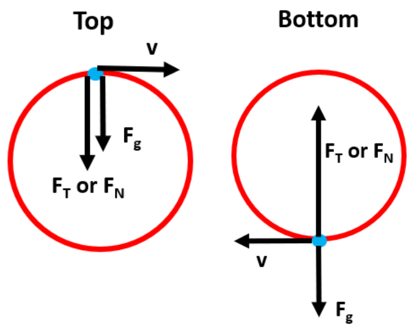
Centripetal Force: Any net force causing circular motion. It is directed towards the center of the circle. For an object to travel a circular path at a specific velocity, a certain amount of centripetal force is required. For example, the force of gravity provides the centripetal force for the Moon to travel about the Earth. Or swing a ball on a rope too fast and the rope could break because it was not designed to provide a centripetal force high enough. Travel around a corner too fast in a car and friction between the road and tires might not be high enough to maintain the circular motion and the car could leave the road.



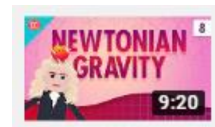
Unbanked Turn: When an object makes a circular turn on horizontal, level ground. The centripetal force for the turn comes from the force of static friction between the object and the ground. If it is a car, for example, it is the force of static friction between the tires and the road.



Banked Turn: When an object travels around a turn and that turn makes an angle with the horizontal, it is possible for the object to navigate the turn without relying on friction. Because it is banked, part of the normal force is directed towards the center of the circle, and this force provides the centripetal force necessary to safely make the turn. When friction is taken into consideration, it is possible to safely make the turn at a higher maximum speed. In that case, there is a component of friction that points towards the center of the circle and one that points down to the Earth (the magnitude of the force of friction points down the banked turn). This is why many highway exit ramps, and race track corners, are banked.

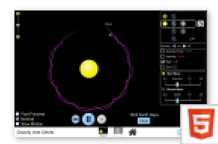


Vertical Circular Motion: This is considered when the object moves in a circle such that the force of gravity acts in the radial direction (so the object changes height above the Earth). The mathematics is similar, the difference being taking into account the force of gravity. The focus of interest will be the motion at the top and bottom of the circle. At the top, the object must have a minimum velocity to maintain the circular motion. At the bottom, the object would have a maximum velocity without breaking the string/rope (string tension is a maximum) or causing a person to suffer an injury (higher force on the body).



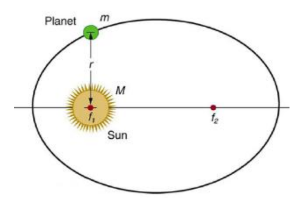
Universal Gravitation: Newton's work showed that the force of gravity is an attractive force proportional to the product of the two interacting masses and inversly proportional to the square of the distance between their centers. For this unit, assume all orbits are circular (the are actually ellipses) and the mass of the object in orbit is much, much lower compared to the mass of the other object (if the masses are comparable, then the object will orbit each other).

Gravity Force Lab: Basics

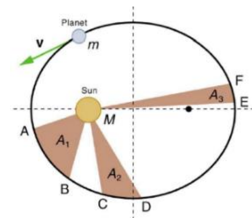


Gravity And Orbits

- **Orbital Velocity**: The tangential speed of the object about another object. It is perpendicular to the force of gravity.
- **Period**: The time to complete one full orbit.
- **Orbital Radius**: The distance between the centers of mass of each object.
- **Altitude**: The distance above the surface of the object in orbit.



Kepler's 1st Law: The orbit of each planet about the Sun is an ellipse with the Sun at one focus. The Earth, for example, is closest to the Sun in January and farthest away in June. The difference of those distances for the Earth are small compared to the average distance between the Earth and Sun. Some objects, like comets, have very elliptical orbits so their surface temperatures vary greatly.



Kepler's 2nd Law: Each planet moves to that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

Kepler's 3rd Law: For any system where objects orbit about another object that has a much greater mass, i.e., like the Moon or any satellites orbiting the Earth, the ratio of the squares of the periods for any two satellites is equal to the ratio of the cubes of their average distances from the central body. This comes out of applying the concepts and mathematics of circular motion and Newton's equations for the force of gravity.

Unit 8: Electromagnetic Forces & Fields

Electromagnetic Force: One of the four fundamental forces of nature. It consists of the static electricity, moving electricity, and magnetism. All atomic and molecular interactions (like friction, chemical reactions, physical properties like boiling points) are manifestations of the electromagnetic force.

Electric Charges: Protons are positively charged particles and electrons are negatively charged. Objects can become charged by gaining or losing electrons. Protons are fixed within the nucleus of atoms and the atoms, nor protons, are free to move in materials.


Balloons and Static Electricity

Law of Charges: There are two types of charges, positive and negative. Opposite charges attract, like charges repel, and both positively and negatively charged objects attract neutral objects by *polarizing* the charges in neutral objects.

Fundamental Charge, q: q = 1.60×10^{-19} C, measured in the SI unit of the coulomb, C. This is the charge of one electron and one proton. Any charged object is an integral multiple of the fundamental charge. Usually, this charge is thought of as an excess or deficit of electrons.

Quarks: While electrons do not have a substructure, high energy particle collision experiments have revealed that protons and neutrons are made of even smaller particles called quarks. There have been six types of quarks discovered, two of which, **Up** and **Down** quarks, make up protons and neutrons. Unexpectically, quarks have been shown to have a partial electrical charge. Up quarks have a charge of $+\frac{2}{3}q$, and down quarks a charge of $-\frac{1}{3}q$.

- Proton = 2 Up + 1 Down = $+\frac{2}{3}q+\frac{2}{3}q-\frac{1}{3}q=+1q$
- Neutron = 1 UP + 2 Down = $+\frac{2}{3}q-\frac{1}{3}q-\frac{1}{3}q=0q$

Law of Conservation of Charge: In any process, the total charge is constant. That means charge cannot be destroyed or created when objects interact.

Antimatter: A particle that has the same properties as a "normal" particle except it has an opposite charge. When a particle and its antimatter counterpart collide, they annihilate one another releasing a burst of energy. The anti-electron, called the positron, behaves like an electron but has a positive charge. The anti-proton is the counterpart to the proton and is made of 2 up antiquarks and 1 down antiquark. Interestingly, one of physics' biggest questions is, "*where is all the antimatter in the universe?*" as particles and anti-particles are created at the same time.

Electrical Conductor: A material that allows for the movement of valence electrons. Electrons collide with atoms or molecules in common conductors, resulting in some energy loss. The amount of energy loss varies by material. Metals and salt water are common examples.

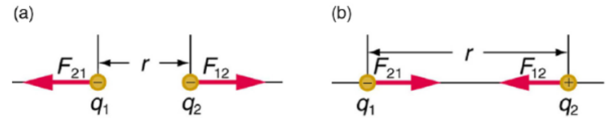
Superconductor: These allow for the movement of charge without any loss of electrical energy.

Electrical Insulator: A material that inhibits the movement of valence electrons. Some common examples are plastic, glass, rubber, and pure water.

Induction: This is a method of creating an electric charge on an object without direct contact. Devices that can charge batteries without contact were developed using this foundational concept (such devices also use *magnetic* induction).


Balloons and Static Electricity

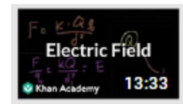
Electric Polarization: When a neutral object is made to have two regions of charge, one positive and one negative. This is usually the result of a charged object being placed near the neutral object. This is the reason charged object attract neutral objects.



Coulomb's Law: A mathematical foundation for calculating the force of electric attraction or repulsion. Named after Charles Coulomb who spearheaded much of the research of static electricity. The electric force is a vector, and each charged object applies a force on all other charges. Newton's 3rd Law still holds, and a reference charge experiences a force equal and opposite to the force it provides on another charge. (video is electrostatics).

Field: A field is a way of visualizing and tracking a force that acts over a distance (non-contact forces like gravity, electric and magnetic forces). Fields are represented by lines and the more lines per unit area the greater the force acting on a reference, or test, object.

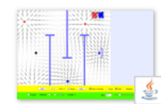
Electric Field: The region around a charged object, Q, that affects the motion of a positive test charge. Mathematically, its strength is the ratio of the Coulomb force to the positive test charge.



Electric Field Lines: Used to visually represent the electric field strength on a positive test charge from a charge, Q. The greater the field strength the more lines per unit area and arrows are used to indicate the direction of the electric force. Since the test charge is always positive, arrows always point toward negative Q-charges and away from positive Q-charges. The five electric field line properties (rules) are summarized below:


Charges and Fields

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.


Electric Field Hockey

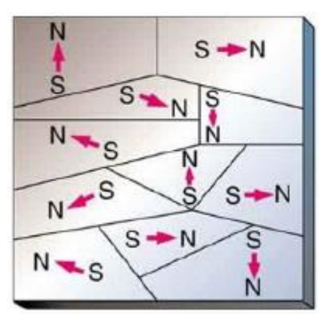
3. The strength of the field is proportional to the closeness of the field lines, i.e. it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is a tangent to the field line at any point in space.
5. Field lines can never cross. The field is unique at any point, so a test charge does not have a "choice" on how to move.

Electric Current: The rate at which a charge moves, measured in amperes (A, which is C/s).

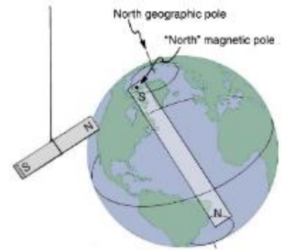
Conventional Current: The direction that a positive charge would flow. In wires, made of metal, positive charges do not move, and it is the negative electrons that flow. Lightning is another example of a discharge of electrons. However, positive ions can move in fluids and biological systems, so it is not only electrons that can create a current.

Magnetism: The result of moving charged particles. Within a solid magnet, this is the result of electron motions within the atom. When electrons are forced through a conducting wire, that motion creates a magnetic field. A magnet is created in a material when its magnetic domains all line up.

Magnetic Domains: Small regions within a materal where there has been a polarization of charge. The existance of these regions do not make a material a magnet, but if all of these regions allign, or are made to allign, then a magnet is created. The poles of a magnet line up with the poles of the magnetic domains and are named north and south poles.



Electromagnetic Force: The name given to all the forces that result from the motion of charged particles.



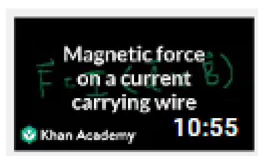
Magnetic Poles: The opposite ends of a magnet (or magnetic domains) and always occur in pairs. Slicing a magnet into smaller and smaller pieces creates smaller magnets with a north and south pole. Like poles repel and opposite poles attract. The Earth's core produces a large magnetic field with poles near the geographical north and south poles. Note that the geographical north pole and the magnetic south pole. The poles are defined based on what direction the north end of a compass points.

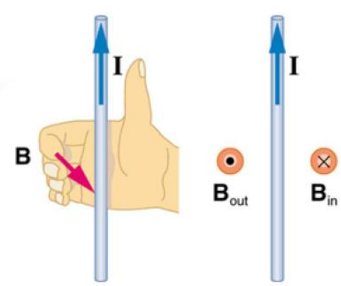
Magnets and Electromagnets

Electromagnet: Created by wrapping an electrically conductive wire around a metal (ferromagnetic) material. A current through the wire will produce a magnetic field and temporarily magnetize the metallic core. Within the core, there is an internal alignment of electron spins. The current creates a magnetic dipole.

Magnetic Field Lines: A visual representation of the magnetic field strength, the *B-field*. The direction of the field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field strength is represented by the variable ***B*** and is measure in units of Tesla, *T*, or Gauss, *G*. The four rules for magnetic fields are:

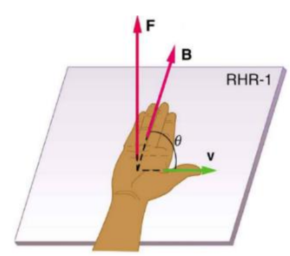
1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line (so to the south pole).
2. The strength of the field is proportional to the closeness of the line. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without a beginning or end. The go from north pole to the south pole. (Note this is different from electric field lines that begin at a positive charge and end at a negative charge.)





Right Hand Rule 2 (RHR-2): The direction of the magnetic field generated by a current is perpendicular to the direction of the current. The RHR-2 is a way to help remember and visualize the magnetic field (it is called RHR-2 because there is another right-hand-rule for a charge moving through a magnetic field). Picture the wire being held in your right-hand fingers with your thumb pointing in the direction of the current. Your fingers curl in the direction of the magnetic field, ***B***. Note the symbols used to represent the direction of the magnetic field for a current in a wire.

Lorentz Force: This is the name of the force applied on a moving charge from a magnetic field. The force is directly proportional to the charge on the moving object, its velocity and the magnetic field strength. The Lorentz force is perpendicular to the magnetic field and the force on a negative charge is in the exact opposite direction to that on a positive charge.



$F = qvB \sin\theta$

F $\perp$ plane of **v** and **B**

Right Hand Rule 1 (RHR-1): To determine the direction of the magnetic force on a positive moving charge, you point your thumb of the right hand in the direction of ***v***, the fingers in the direction of ***B***, and a perpendicular to the palm points in the direction of ***F***. There are no magnetic forces on a static (stationary) charge, only moving ones. Electric fields do not affect magnetic fields unless the charge is moving.

Outcome 1.1: Mathematics Review: Students will review order of operations, polynomials, solving linear and quadratic equations, and trigonometric ratios.

Evaluate using rules for order of operations.

1) $(6+3+2)\times 3$

2) $(5-1)^2+4$

3) $18\div(4-(1+2-2))$

4) $4+6-2+2\times 4$

5) $1+2+6\times 6+5+3$

6) $4\times 5+2\times 9\div 3\times 4$

Simplify each expression by collecting like terms.

7) $(4b+8b^3-8b^2)+(b^3+7b-5b^2)$

8) $(2a^3-4+a^2)-(2a^3+2+8a^2)$

9) $(-12n+2.1+11n^3)-(-8.3n^3+13.7n+9.7)$

10) $(7.1n^4+2.2n^5-2n^3)+(-12.2n^4-6n^5+7.92n^3)$

11) $(6.2b^3+6.5b^5+9.6b^2)-(13.7b^5-10.3b^2+12.49b^3)$

12) $(-2.8n^2-9.7n^5-9.06n^3)+(-2.7+10.5n^3+6.74n^5)$

Find each product by distribution.

13) $4(3n-5)$

14) $8(4n+8)$

15) $2.8(4.232n+1.03)$

16) $5.6(1.9n+4.4)$

17) $(2.21x+0.5)(7.87x-6.3)$

18) $(3.2n-4)(8n+4.6)$

19) $(7.8k+5.3)(6k-0.9)$

20) $(0.7x-1.2)(6.8x-7.628)$

Solve each linear equation.

21) $6+k-5=-4$

22) $-6n-1+4n=-9$

23) $-5x-3(2x+3)=-64$

24) $-93=3(5k-1)$

25) $-1.9r - 21.22 = 2.7 + 7(r + 6.5)$

26) $-4.1(x + 0.1) - 3.9 = -17.19 + 0.5x$

27) $3.8(n - 5) = 6.8(n + 3.7)$

28) $6a - 4.3 + 0.1a = 4.4(3.1a - 5.1) - 3.7(-6.5 + 2.2a)$

29) $4(1.5x + 6) - 5.2(x + 3.6) = -3.2x + 5.9 + 2.5$

30) $-4(1.6 - 3.8n) = -2n + 3(-2 + 5.8n)$

Solve each equation with the quadratic formula.

31) $3n^2 - 11n - 5 = 0$

32) $12k^2 - 4k - 11 = 0$

33) $5v^2 = 17 + 4v$

34) $4b^2 = 64$

35) $4a^2 - a = 95$

36) $-5 + 3x = -7x^2$

37) $4n^2 + 6n = 88$

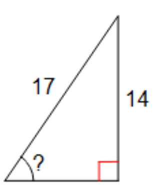
38) $-5x^2 - 2x = -39$

39) $-10x = -8x^2 + 2$

40) $17 = 4k^2 + 8k$

Find the measure of the indicated angle to the nearest degree.

41)



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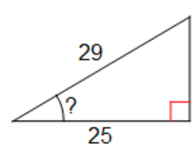

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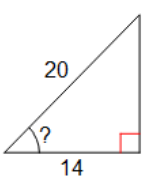

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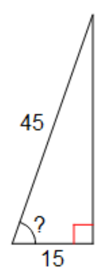

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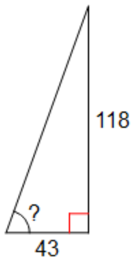

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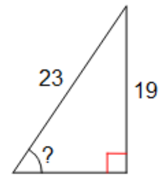

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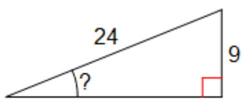

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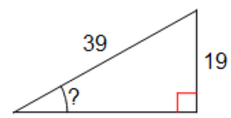

Find the missing side. Round to the nearest tenth.

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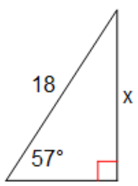

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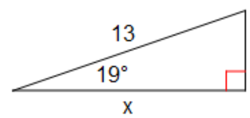

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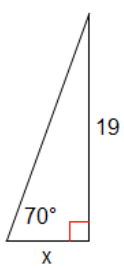

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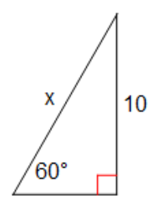

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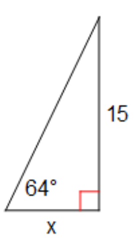

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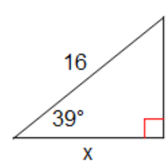

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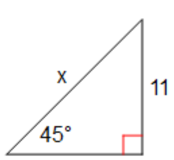

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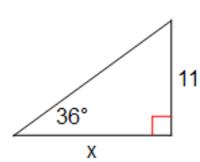


Answers to Outcome 1.1 Math Review

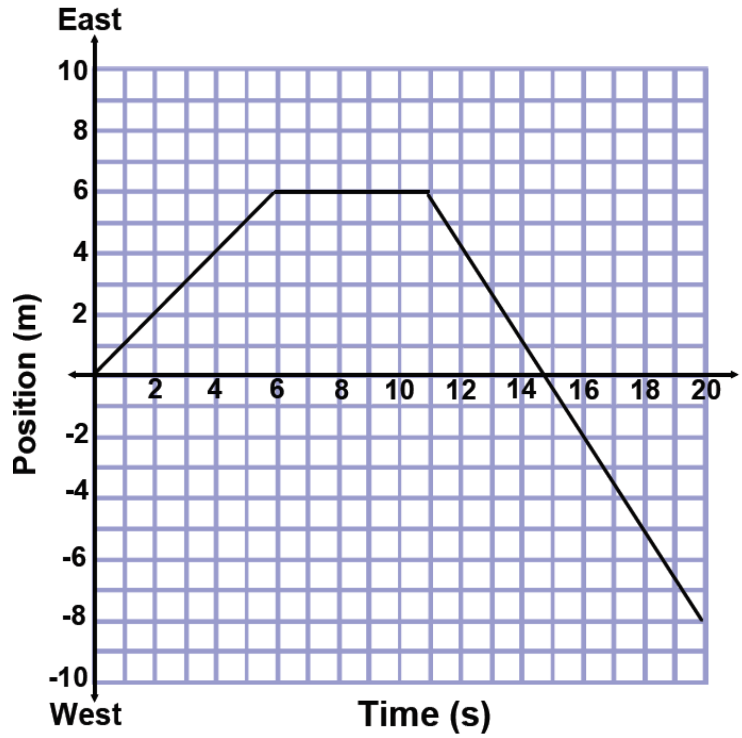
1) 33	2) 20	3) 6	4) 16
5) 47	6) 44	7) $9b^3 - 13b^2 + 11b$	8) $-7a^2 - 6$
9) $19.3n^3 - 25.7n - 7.6$	10) $-3.8n^5 - 5.1n^4 + 5.92n^3$	11) $-7.2b^5 - 6.29b^3 + 19.9b^2$	
12) $-2.96n^5 + 1.44n^3 - 2.8n^2 - 2.7$		13) $12n - 20$	14) $32n + 64$
15) $11.8496n + 2.884$	16) $10.64n + 24.64$	17) $17.3927x^2 - 9.988x - 3.15$	
18) $25.6n^2 - 17.28n - 18.4$	19) $46.8k^2 + 24.78k - 4.77$	20) $4.76x^2 - 13.4996x + 9.1536$	
21) $\{-5\}$	22) $\{4\}$	23) $\{5\}$	24) $\{-6\}$
25) $\{-7.8\}$	26) $\{2.8\}$	27) $\{-14.72\}$	28) $\{9.85\}$
29) $\{0.78\}$	30) $\{-2\}$	31) $\{4.076, -0.409\}$	32) $\{1.138, -0.805\}$
33) $\{2.287, -1.487\}$	34) $\{4, -4\}$	35) $\{5, -4.75\}$	36) $\{0.658, -1.086\}$
37) $\{4, -5.5\}$	38) $\{-3, 2.6\}$	39) $\{1.425, -0.175\}$	40) $\{-3.291, 1.291\}$
41) 55°	42) 30°	43) 46°	44) 71°
45) 70°	46) 56°	47) 22°	48) 29°
49) 15.1	50) 12.3	51) 6.9	52) 11.5
53) 7.3	54) 12.4	55) 15.6	56) 15.1

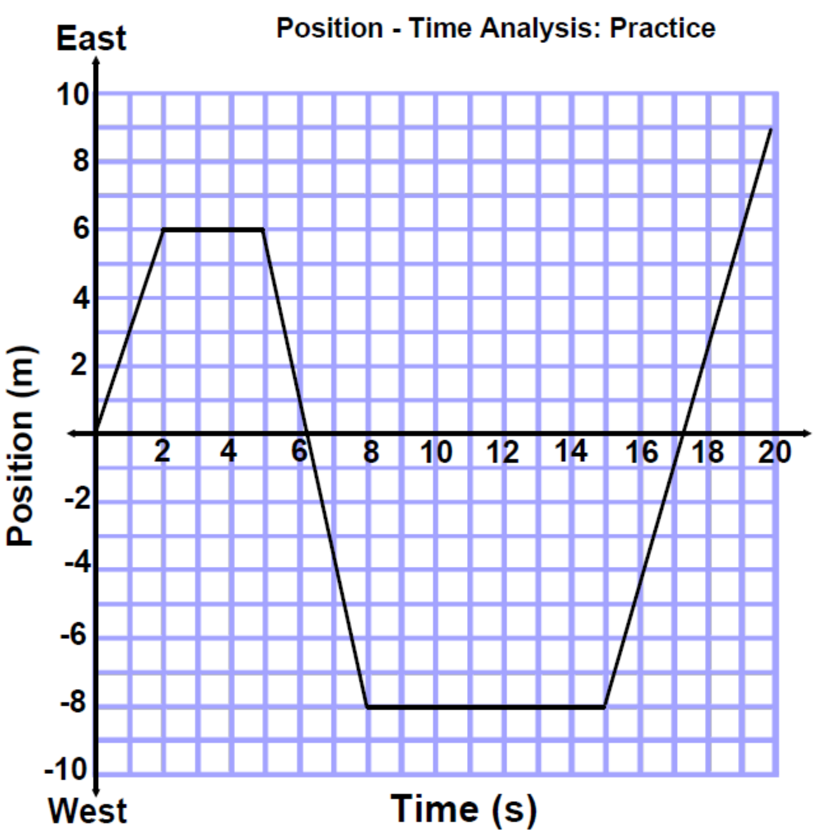
Outcome 1.2: Demonstrate an understanding that kinematic quantities are scalars or vectors and the importance that distinction plays when solving problems. Recognize and define kinematic quantities.

1. Describe why frame of reference is important and give an example of how a choice of frame of reference can give two different results for the same object in motion.
2. Define motion, speed, and velocity.
3. List four measurements that are scalars and four that are vectors.
4. Would a speeding ticket be given for average or instantaneous speed?
5. Define instantaneous and average velocity.
6. Identify each of the following as a vector or scalar: speed, force, acceleration, distance, time, velocity, weight, temperature, and mass.
7. In words, what is the definition of acceleration? How do the units reflect this definition?
8. A car goes from 30 km/h [E] to 50 km/h [E]. What is the direction of the acceleration?
9. A truck goes from 90 km/h [E] to 45 km/h [E]. What is the direction of the acceleration?
10. Give two examples where an object has a non-zero acceleration but an instantaneous velocity of zero.
11. Describe a situation where an object can have a constant speed and experience a non-zero acceleration.

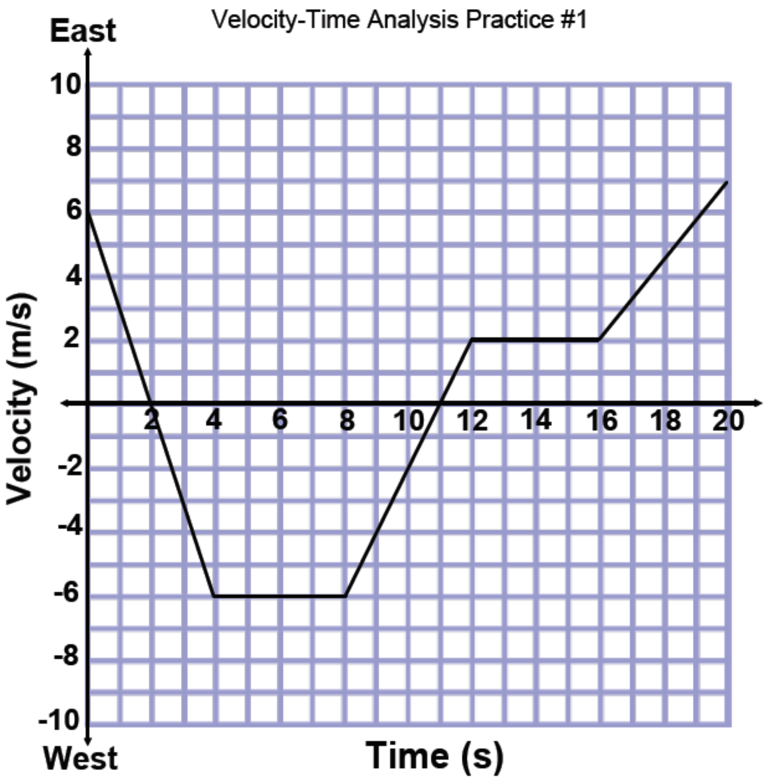
Outcome 1.3: Graphically analyze 1D relationships among position, velocity, acceleration, and time.

1. What was the object's position at the 4, 10 and 18 second marks? {relative to east: 4 m, 6 m, and -5 m}
2. Calculate the distance traveled during the first 14 seconds. {d = 11m}
3. Calculate the average speed during the first 14 seconds. {v_{sp} = 0.79 m/s}
4. Calculate the average velocity during the first 14 seconds. {$\mathbf{v_{avg}}$ = 0.071 m/s}
5. Calculate the instantaneous velocity at the 16-second mark. {**v** = –1.6 m/s}
6. Calculate the object's total distance traveled and final position. {d_{tot} = 20 m, $\mathbf{d_f}$ = –8 m}
7. Calculate the object's average speed and velocity for the full 20 seconds. {v_{sp} = 1.0 m/s, $\mathbf{v_{avg}}$ = 0.40 m/s}

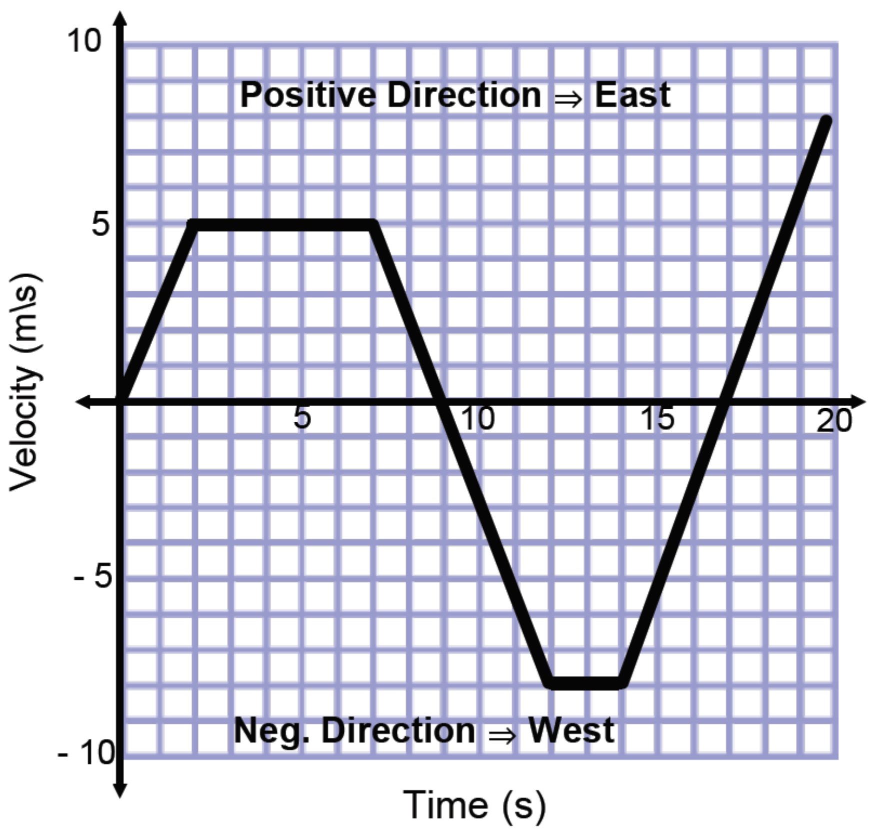




1. Calculate the instantaneous velocity at 1 and 6 seconds. {3 m/s, –5 m/s}
2. Calculate the distance covered during the first 8 seconds. {20 m}
3. At what times did the object return to the starting position? {around 6.2 and 17.2 seconds}
4. During what time interval{s} was the object traveling west? {5 – 8 seconds}
5. During what time interval{s} was the object's position west? {approx. 6 – 17 seconds}
6. How long was the object not moving? {10 seconds}
7. During what time interval{s} was the object east of the starting position, but traveling west? {5 – 6 s}
8. Calculate the average speed and velocity for the full 20 seconds. {v_{sp} = 1.85 m/s, $\mathbf{v_{avg}}$ = 0.45 m/s}
9. How long did it take the object to travel 11 m? {6 s}
10. At what time{s} was the object 3 m from the starting position? {1 s, 5.5 s, 6.8 s, 16.5 s, and 18 s}



1. What was the object's instantaneously velocity at 1, 6, and 18 seconds? {3 m/s, -6 m/s, 4.5 m/s}
2. For how many seconds was the object not accelerating? {8 seconds}
3. At what time{s} did the object change direction? {2 and 11 seconds}
4. Calculate the object's acceleration during the first 2 and last 4 seconds. {-3 m/s^2, 1.25 m/s^2}
5. Calculate the distance and displacement at the 4 second mark. {*d* = 12 m, ***d*** = 0 m}
6. Calculate the average speed and average velocity at the 4 s mark. {v_{sp} = 3.0 m/s, v_{avg} = 0 m/s}
 Calculate the total distance and displacement for the full 20 seconds. {*d* = 72 m, ***d*** = -6 m}
7. During what time interval{s} was the object traveling east but the acceleration was west? {0-2s}
8. During what time interval{s} was the object traveling west but the acceleration was east? {8-11s}
9. Calculate the average speed and average velocity for the 20 s. {v_{sp} = 3.6 m/s, v_{avg} = -0.3 m/s}



1. Calculate the acceleration from 0 to 2 seconds. {***a*** = 2.5 m/s^2}
2. Calculate the acceleration between 7 and 12 seconds {***a*** = -2.6 m/s^2}
3. At what times{s} did the object change direction? {9 & 17 s}
4. For how many seconds was the object not accelerating? {13s}
5. From the start, how many seconds did it take the object to travel 20 m? {5s}
6. Calculate the distance traveled east during the full twenty seconds. {*d* = 47 m}
7. Calculate the distance traveled west during the full twenty seconds. {*d* = 40 m}
8. Calculate the average speed and velocity during the full 20 s. {v_{sp} = 4.35 m/s; v_{avg} = 0.35 m/s}

Outcome 1.4: Mathematically analyze the relationship among position, velocity, acceleration, and time in one dimension.

Scalar Calculations – Speed, Distance, Time

1. Sam is driving along the highway towards Saint John. He travels 150 km in 3.00 h. What is his average speed for his trip? {50 km/h}

2. A vehicle travels 2345 m in 315 s. Calculate the average speed. {7.4 m/s}

3. What distance will a car, traveling 65 km/h, cover in 3.0 hrs? {195 km}

4. How long will it take to go 375 km [E] traveling at 62 km/hr [E]? {6.0 h}

5. A person travels once around a circular track in 75 seconds. The radius is 45 m.
   a. Calculate the average speed of the person. {3.8 m/s}
   b. Calculate the time to traveled around the circle 3.5 times. {262.5 s}

6. A soccer ball is kicked 25 m [E], then 15 m [E], 8 m [W], and finally 12 m [E]. All this takes place in 45 seconds. Calculate the average speed {v_{sp} = 1.3 m/s}

Vector Calculations – Velocity, Acceleration, Time

7. A soccer ball is kicked 25 m [E], then 15 m [E], 8 m [W], and finally 12 m [E]. All this takes place in 45 seconds. Calculate the average velocity of the ball. {v_{avg} = +0.98 m/s [E]}

8. What is the change in position of the Earth after one orbit about the Sun? What is the average velocity of the Earth after one orbit in m/s? {d_f = 0 m; v_{avg} = 0 m/s}

9. A car accelerates at a rate of 3.0 m/s^2. If the original velocity is 8.0 m /s, how many seconds, will it take the car to reach a final velocity of 25.0 m/s? {t = 5.7 s}

10. The final velocity of a car is 30m/s. The car is accelerating at a rate of 2.5m/s^2 over an 8-second period. What was the initial velocity of the car? {v_o = 10 m/s}

11. If a car, with an initial velocity of 10 m/s, accelerates at a rate of 50 m/s^2 for 3 seconds, Calculate its final velocity. {v_f = 160 m/s}

12. Calculate the time, undergoing an acceleration of 5.6 m/s^2 [W], to change velocity from 32 m/s [E] to 12 m/s [W]. {t = 7.9 s}

13. A baseball is thrown with an initially velocity of 46 m/s [E]. After leaving the bat, it is going 35 m/s [W]. Calculate the acceleration of the ball if it was in contact with the bat for 0.34 seconds. {a = -240 m/s^2}

14. Calculate the length of time a foot in contact with a soccer ball to change its velocity from 2.1 m/s [S] to 3.7 m/s [N] if the acceleration was 21.5 m/s^2 [N]. {t = 0.27 s}

Vector Calculations – Velocity, Acceleration, Position, Time

15. A car accelerates from 15 m/s [E] to 25 m/s [W] in 26 seconds.
    a. Calculate the acceleration of the car. { $\vec{a}$ = –1.54 m/s^2}
    b. Calculate the position of the car in that time. {$\vec{d}_f$ = –130 m}

16. A person is standing atop a cliff that is 250 m high cliff overlooking the water below. They mistakenly drop their phone to the water below.
    a. Calculate the time it takes for the phone to hit the water below. {t = 7.1 s}
    b. Calculate the velocity as it enters the water. {$\vec{v}_f$ = –70.0 m/s}
    c. Calculate the velocity of the phone 75 m above the water. {$\vec{v}_f$ = –58.6 m/s}
    d. Calculate the height of the phone when its velocity is 35 m/s downward. {$\vec{d}_f$ = 188 m}

17. Standing on the ground a person throws a rock (take initial position to be zero meters). It leaves his hand with an upward velocity of 21 m/s.
    a. Calculate the length of time for the rock to reach maximum height. {t = 2.1 s}
    b. Calculate the rock's maximum height. {$\vec{d}_f$ = 22.5 m}
    c. Calculate the velocity of the rock when it is 15 m above the ground. {$\vec{v}_f = \pm 12.1$ m/s}
    d. Calculate the time for the rock to be 18 m above the ground. {t = 1.2 s and t = 3.1 s}

18. A plane changes its velocity from 215 m/s [S] to 300 m/s [N]. The acceleration was 5.72 m/s^2 [N].
    a. Calculate the time it took the plane to change its velocity. {t = 90.0 s}
    b. Calculate the position of the plane after that time. {$\vec{d}_f$ = 3830 m [N]}
    c. Calculate the time for the place to have a location of 1500 m [S]. {t = 7.8 s and t = 67.4 s}

19. A ball is bounced such that it leaves the ground with an upward velocity of 25 m/s.
    a. Calculate the total time the ball is in the air. {t_{air} = 5.1 s}
    b. Calculate the velocity of the ball when it is 20 m above the ground. {$\vec{v}_f$ = ± 15 m/s}
    c. Calculate the time for the ball to be 16 m above the ground. {t = 0.75 s and 4.3 s}

Expert Level Questions

20. A car drives 12 m/s [S] for 5.0 seconds, then 18 m/s [N] for 9.0 seconds, and finally 15 m/s [S] for 11 seconds. Calculate the average speed and average velocity. {v_{sp} = 15 m/s; v_{avg} = -2.5 m/s or 2.5 m/s [S]}

21. Calculate the average speed of the Earth about the Sun in m/s. {29 885 m/s}

22. A plane changes its velocity from 215 m/s [S] to 300 m/s [N]. The acceleration was 5.72 m/s^2 [N]. Calculate the distance the plane traveled. {d = 11 900 m}

23. Derive the formula: $v_f^2 = v_o^2 + 2a(d_f – d_o)$

24. Show that when using the quadratic formula to solve $d_f = d_o + v_ot + ½at^2$ for time, t, the determinant (what is under the square-root symbol) is equal to $v_o^2 + 2a(d_f – d_o)$.

Outcome 2.1 Measuring vectors and calculating the resultant position for 2D vectors using a scale diagram.

1. Use a protractor and ruler to measure all the following vectors. Measure all angles relative to the east-west line. The scale is 1 cm = 25 m.

a = ____

b = ____

c = ____

d = ____

e = ____

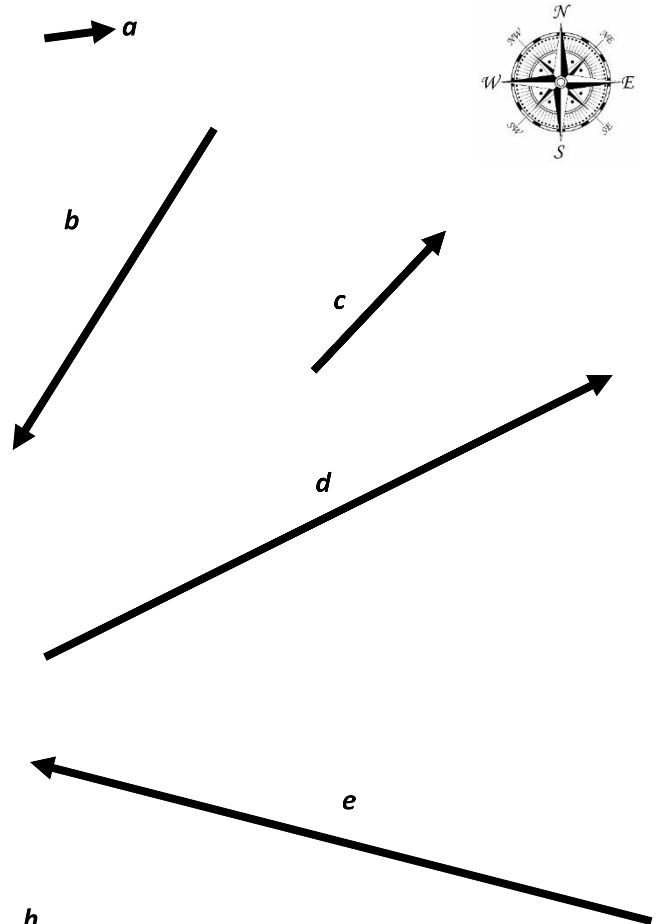
f = ____

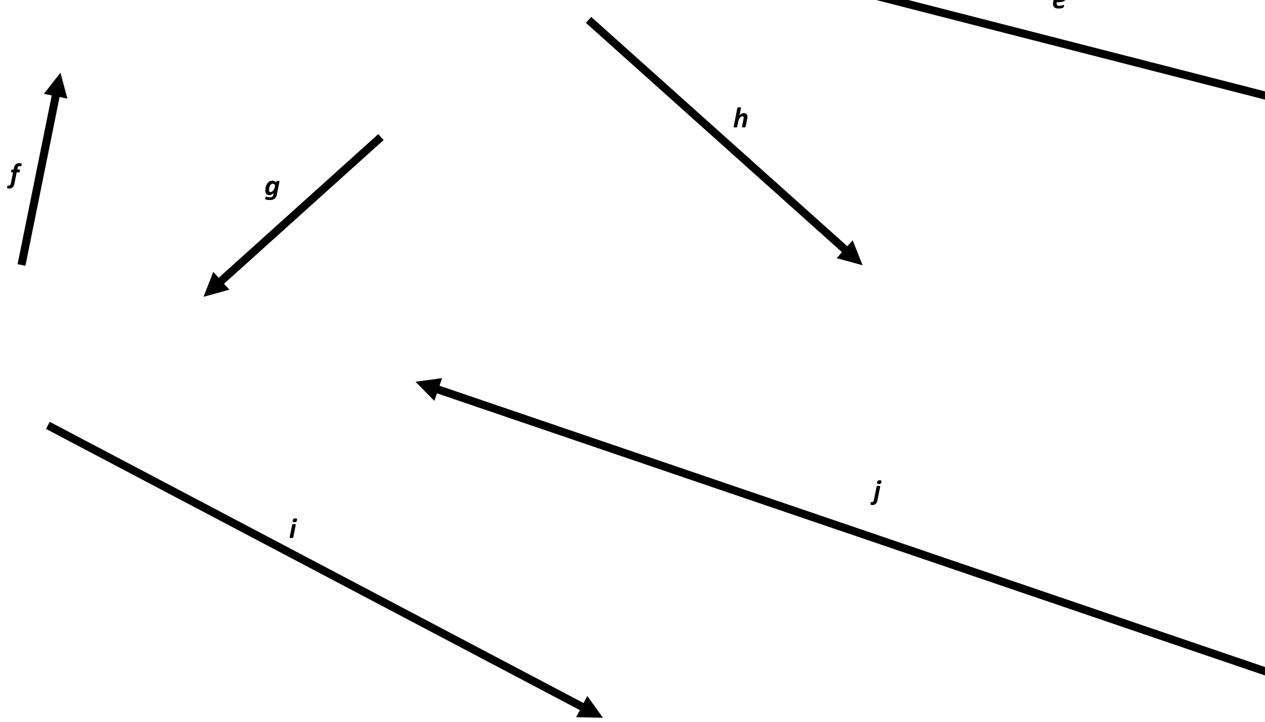
g = ____

h = ____

i = ____

j = ____





For questions 2 – 11: Using the scale given, draw the vectors and measure the resultant.

2. Scale: 1 cm = 10 m. ***a*** = 30 m [W], ***b*** = 45 m [N], ***c*** = 75 m [E] {***R*** = 64 m [E45N]}

3. Scale: 1 cm = 150 km. ***a*** = 630 km [S], ***b*** = 400 km [E], ***c*** = 275 km [S], ***d*** = 1050 km [W] {***R*** = 1114 km [W54S]}

4. Scale: 1 cm = 75 m. ***a*** = 620 m [E], ***b*** = 525 m [N], ***c*** = 900 m [W], ***d*** = 300 m [S] {***R*** = 359 m [W34N]}

5. Scale: 1 cm = 42 m. ***a*** = 228 m [E20S], ***b*** = 402 m [E70S], ***c*** = 519 m [W40N] {***R*** = 130 m [W70S]}

6. Scale: 1 cm = 10 m/s. ***a*** = 45 m/s [W60N], ***b*** = 70 m/s [W80S], ***c*** = 100 m/s [E45N] {***R*** = 54 m/s [E49N]}

7. 1 cm = 25 km. ***a*** = 208 km [E36S], ***b*** = 175 km [W72S], ***c*** = 133 km [W45N] {***R*** = 196 km [E84S]}

8. 1 cm = 65 m. ***a*** = 325 m [W60N], ***b*** = 429 m [E10N], ***c*** = 575 m [E33S] {***R*** = 743 m [E3S]}

9. 1 cm = 150 m. ***a*** = 982 m [E43S], ***b*** = 456 m [W15N], ***c*** = 621 m [E55N] {***R*** = 634 m [E4S]}

10. 1 cm = 38 km/h. ***a*** = 148 km/h [W23S], ***b*** = 275 km/h [E51S], ***c*** = 197 km/h [W65N] {***R*** = 104 km/h [W63S]}

11. 1 cm = 10 m/s^2. ***a*** = 75 m/s^2 [E], ***b*** = 32 m/s^2 [W29S], ***c*** = 105 m/s^2 [W75N] {***R*** = 88 m/s^2 [E77N]}

Outcome 2.2: Calculate the perpendicular components given the vector.

1. d_f = 652 m 20° up from positive x-axis. {d_{fx} = 613 m, d_{fy} = 223 m}

2. a = 7.8 m/s^2 80° down from negative x-axis. {a_x = -1.4 m/s^2, a_y = -7.7 m/s^2}

3. d_f = 7824 km 32° up from negative x-axis. {d_{fx} = -6635 km, d_{fy} = 4146 km}

4. v_o = 490 m/s [E25°N] {v_{oE} = 444 m/s, V_{oN} = 207 m/s}

5. d_o = 1200 m [E65°S] {d_{oE} = 507 m, d_{oN} = -1088 m}

6. a = 35 m/s^2 [W28°S] {a_E = -31 m/s^2, a_N = -16 m/s^2}

7. A ball is kicked from the ground with a velocity of 15.8 m/s at an angle of 30 degrees up from the horizontal. Calculate the initial vertical and horizontal velocity of the ball. {v_y = 7.9 m/s, v_x = 13.7 m/s}

8. A plane changes position by flying 623 km [W25N]. Calculate the East and North components of the change in position. {d_E = -565 km, d_N = 264 km}

9. From a location above the ground, a person tosses a coin to a person below. The coin has a velocity of 5.8 m/s at an angle of 56 degrees down from the positive horizontal axis. Calculate the components of this velocity. {v_x = 3.2 m/s, v_y = -4.8 m/s}

Outcome 2.2: Calculate the vector given its components.

10. a_x = 300 m/s^2, a_y = 195 m/s^2 {358 N, 33° up from the positive x-axis}

11. d_x = 437 km, d_y = -655 km {787 km, 56° down from positive x-axis}

12. v_E = -35 m/s, v_N = 50 m/s {61 m/s [W55°N]}

13. In calculating the acceleration on a car, the horizontal component is $\boldsymbol{a_E}$ = –5.1 m/s^2 and vertical $\boldsymbol{a_N}$ = 7.4 m/s^2. Calculate the acceleration of the car. {**a** = 9.0 m/s^2 [W55N]}

14. The horizontal component of a force is Fx = 46 N, the vertical component is Fy = – 21 N. Calculate the force vector. {**F**= 51 N 25° down from the positive x-axis}

15. A water current moves a sailboat 3.4 m/s [W]. The wind blows 2.6 m/s [N]. Calculate the velocity of the sailboat. {**v**=4.3 m/s [W37N]}

Outcome 2.3: Mathematically analyze kinematic problems in two dimensions.

1. Calculate the acceleration of an object that goes from 15.0 m/s [S] to 15 m/s [W] in 2.0 seconds. {***a*** = 10.6 m/s^2 [W45°N]}

2. A car is initially moving 7.5 m/s [N]. After 3.0 seconds the final velocity is 10.0 m/s [E40°N]. Calculate the acceleration. {***a*** = 2.57 m/s^2 [E8.1°S]}

3. A boat is sailing 6.5 m/s [E20°S]. A gust of wind provides an acceleration equal to 2.1 m/s^2 [E60°N] for 18 seconds.
   a. Calculate the velocity after the 18 seconds. {$\boldsymbol{v_f}$ = 39.4 m/s [E51°N]}
   b. Calculate the final position at that time. {***d*** = 378 m [E42°N]}

4. A glider is flying 9.2 m/s [E25°N]. A gust of wind changes the glider's trajectory to 11 m/s [E14°S] in 7.9 seconds.
   a. What was the acceleration of the glider? {***a*** = 0.88 m/s^2 [E70°S]}
   b. Calculate the final position of the glider at that time. {***d*** = 75 m [E3.7°N]}

5. An object accelerates at 18 m/s^2 [E60N] for 6.5 seconds. The final velocity is 39 m/s [E25N].
   a. Calculate the initial velocity. {$\mathbf{v_o}$ = 88 m/s[W74S]}
   b. Calculate the final velocity once a total of 25 seconds has passed. {$\mathbf{v_f}$ = 365 m/s [E57N]}

Expert Level Questions

6. An object, starting from position zero, is moving 35 m/s [E40°N] and undergoes an acceleration of 3.7 m/s^2 [W10°N]. How much time is required for the final position to be 609 m [W72°N]? {*t* = 20 s}

7. Show mathematically, or explain, why the direction of an object's final velocity will become the same direction as that object's acceleration.

8. An object is launched in the direction [E35°N], with initial speed v_o. There is no acceleration in the east direction; the acceleration north is –9.81 m/s^2. Calculate the initial speed necessary for the object to hit a target located 150 m [E] and 50 m [N] from the launch site. {v_o = 55 m/s}

Outcome 2.4: Vector addition and solving for missing vectors.

1. A car drives 55 km [W30S], 78 km [W65N] then 100 km [E]. Calculate the final position of the car. {47 km [E66°N]}

2. You are 37 km [W20°N] from Miramichi and must move to a position 15 km due West of the city. Calculate the displacement required. {***d*** = 24 km [E33°S]}

3. A coast guard boat is 75 km [E67°N] from port. A distress call comes in from a fishing vessel located 93km [E26°S] from port. Calculate the position of the fishing vessel from the coast guard boat? {***d*** = 122 km [E64°S]}

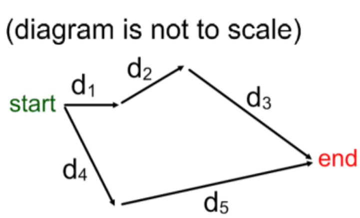
4. A boat's heading is directly across a river at 5.0 km/h [N]. The river is flowing east at 3.0 km/h.
   a. What is the velocity of the boat relative to someone standing on the dock where the boat departed? {***v*** = 5.8 km/h [E53°N]}
   b. How far down stream does it land if the trip takes 0.5 h? {$\boldsymbol{d_E}$ = 1.5 km}
   c. How wide is the river? {$\boldsymbol{d_N}$ = 2.5 km}

5. A river has a current of 6.0 m/s [E]. What speed must a boat be able to travel, relative to the water, to go straight across the river when it is aimed 75° upstream? {***v*** = 23.2 m/s}

6. It is 500 m straight east to get across a river. The river has a current of 3.7 m/s due south. You have a boat that can travel 10 m/s. Calculate the angle to aim the boat to get directly across the river? {E22°N}

Expert Level Questions

7. Given the information below, solve for the missing vector: {answer d_3 = 32 m [E23°S]}



d_1 = 7.5 m [E]
d_2 = 12 m [E25N]
d_3 = ?
d_4 = 24 m [E55S]
d_5 = 36 m [E20N]

8. A car is located 250 m [E40N] of a reference location. A truck is located 375 m [E55N] from the same reference location. Calculate the average velocity the car must drive to meet up with the truck in 30 seconds if the truck is driving 20 m/s [E30N]. {v_{avg} = 23.4 m/s [E39N]}

Outcome 3.1: Explain, describe, and analyze the forces of friction and gravity in one dimension.

Force of Gravity

1. Define an object's weight and mass.

2. How is the distance between objects considered in the equation for the force of gravity?

3. Explain the force of gravity, both its classical {Isaac Newton} and special relativity {Einstein} definitions. What observations supported Einstein's definition of gravity?

4. Suppose the weight of a person has a magnitude of 652.58 N on Earth's equator.
   a. Calculate the mass of the person. {m = 66.723 kg}
   b. Calculate the magnitude of that person's weight at the North Pole. {F_g = 656.03 N}
   c. Calculate the magnitude of that person's weight on the International Space Station. {F_g = 605.81}

5. A lunar rover has a mass of 209 kg. Calculate the magnitude of its weight on Earth and on the Moon. {F_{gE} = 2050 N, F_{gM} = 343 N}

6. On Earth, the magnitude of an object's weight is 451 N. Calculate the magnitude of the object's weight on the planet Mars. {F_{gMars} = 171 N}

7. On Earth, the magnitude of an object's weight is 725 N. On a different planet, the same object has a weight magnitude of 1062 N. Calculate the acceleration due to gravity on the other planet. {g_{planet} = 14.4 m/s^2}

Normal Force

8. What is the definition of a normal force?

9. A 33 kg mass, on Earth, rests on the floor. Calculate the normal force. {F_N = 324 N}

10. A 33 kg mass, on Mars, rests on the floor. Calculate the normal force. {F_N = 123 N}

11. A 21 kg mass, on Earth, rests on the floor. A person pushes down on the object with a force of 75 N. Calculate the normal force. {F_N = 281 N}

12. A 41 kg mass, on Earth, rests on the floor. A person pulls up on the object with 150 N of force. Calculate the normal force. {F_N = 252 N}

13. A 75 kg mass, on the Moon, rests on the floor. A person pulls up on the object with 61 N of force. Calculate the normal force. {F_N = 62 N}

14. Calculate the mass of an object if there is 126 N of normal force on the surface of Mars. {m = 34 kg}

15. A person is trying to lift a 24 kg box off the ground by applying a 75 N upward force. Calculate the normal force on the box. {F_N = 160 N [up]}

16. A person pushes down with 93 N of force on a 51 kg object. Calculate the normal force by the floor. {F_N = 593 N}

Static and Kinetic Friction

17. What causes friction?

18. What type of force is stronger, static, or kinetic friction and why?

19. What are the three conditions necessary to be able to apply the formula for calculating the force of friction?

20. Explain the coefficient of friction.

21. In what direction does kinetic friction always act?

22. A box is pushed along a table on Earth. The same box and table are used on the Moon. How do the forces and coefficients of friction compare for the object on Earth and the Moon?

23. The coefficient of kinetic friction between a book and a table is 0.21. The normal force provided by the table is 15 N. Calculate the force of kinetic friction. {F_{fk} = 3.2 N}

24. The force of static friction between a crate and the floor is 345 N. The normal force provided from the floor is 750 N. Calculate the coefficient of static friction. {μ_s = 0.46}

25. Calculate the normal force provided by a table if the force of kinetic friction is 65 N and the coefficient of kinetic friction is 0.18. {F_N = 361 N}

26. A 37 kg object is at rest on the floor. The coefficient of static friction is 0.35. Calculate the force of static friction that must be overcome to get the object moving. {F_{fs} = 127 N}

27. A 52 kg crate is moved across the floor. 78 N of kinetic friction act upon the crate. Calculate the coefficient of kinetic friction. {μ_k = 0.15}

28. A bag is dragged along the floor. The force of kinetic friction is 46 N, and the coefficient of kinetic friction is 0.23. Calculate the mass of the bag. {m = 20 kg}

29. An old, heavy TV sits on a table. The force of static friction is 91 N, and the coefficient of static friction is 0.39. Calculate the mass of the TV. {m = 24 kg}

30. A large box is pushed along the floor. The force of kinetic friction is 59 N and μ_k is 0.35. Calculate:
    a. The normal force acting on the box {F_N = 167 N}
    b. The mass of the box. {m = 17 kg}

31. A 75 kg object rests on the floor with μ_s = 0.67. Calculate the magnitude of the force of static friction if a person *pushes down* on the box with a force of 210 N. {F_f = 634 N}

32. A 75 kg object rests on the floor with μ_s = 0.67. Calculate the magnitude of the force of static friction if a person *pulls up* on the box with a force of 210 N. {F_f = 352 N}

33. A person pulls a 72 kg supply box across the surface of Mars. If μ_k = 0.41 calculate the magnitude of the force of friction. {F_f = 110 N}

34. An object is pressed up against a wall. In what direction do the normal and friction forces act?

35. Calculate the coefficient of static friction if a 12 kg box is held against the wall with a force of 325 N {μ_s = 0.36}

36. A 6.2 kg book is pressed against the wall. The coefficient of static friction between the book and wall is 0.16. Calculate the applied force necessary to keep the book from slipping down. {380 N}

37. A 14.7 kg box is pressed up against the wall using an applied force of 600 N. For the box not to fall, calculate the minimum coefficient of static friction necessary between the wall and the box. {0.24}

Expert Level Questions

38. When pushed along a certain material, the force of kinetic friction on the object is 47 N with μ_{k1} = 0.16. The object is then pushed along a second material with μ_{k2} = 0.25, calculate the force of kinetic friction required to be overcome on second material.

39. By what factor does the force of friction change for two objects taken from:
    a. The Earth to the Moon? {decrease by factor of 5.98}
    b. The Earth to Mars? {decrease by a factor of 2.64}
    c. The Moon to Mars? {Increase by a factor of 2.27}

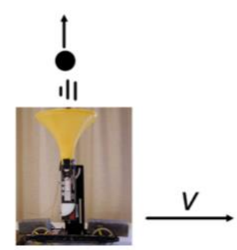
40. An object sits on a long plank that rests on the ground.
    a. How does the coefficient and force of static friction change as the plank is slowly lifted at one end to make an angle with the ground?
    b. The force of gravity down the ramp is $mg\sin\theta$ and the normal force perpendicular to the ramp is $mg\cos\theta$. Suppose an object with μ_k = 0.65 sits on the ramp. Calculate the angle of the ramp such that the force of gravity down the ramp equals the force of kinetic friction up the ramp. {θ = 33°}
    c. In a similar situation to (b), Calculate the μ_k if the object just begins to slide at an angle of 21°. {μ_k = 0.38}

Outcome 3.2: Describe and apply Newton's 3 Laws of motion within inertial and non-inertial frames of reference in one dimension.

1. Define and compare classical and quantum mechanics.

2. Why are Newton's Laws of motion applicable on the Earth even though the Earth is technically a non-inertial frame of reference?

3. Give two examples of objects that cannot be analyzed with Newtonian mechanics.



4. Explain if the ball in the image to the right likely to land in the funnel if the cart is maintaining a constant velocity? What about if the cart has a constant acceleration?

5. Describe how the floor pushes you forward and that you do not push the floor.

6. Describe how Newton's 3rd Law applies to rocket launches.

7. Define net force.

8. Calculate the net force if these act on an object at the same time: F_1 = 42 N [E], F_2 = 70 N [W]. {F_{net} = 28 N [W]}

9. A box is pushed along the floor with an applied force of 93 N [E]. If 58 N of friction act on the box, calculate the net force. {F_{net} = 35 N [E]}

10. Calculate the necessary applied force to overcome 125 N of friction and have a net force of 182 N [E]. {F_a = 307 N [E]}

11. Describe how Newton's 2^{nd} Law applies to rockets.

12. A towrope is used to pull a 1750 kg car across a flat surface, giving it an acceleration of 1.35 m/s^2. Calculate the net force acting on the car. {F_{net} = 2360 N}

13. A 5.2 kg bowling ball is accelerated from rest to a velocity of 12 m/s as the bowler covers 5.0 m of approach before releasing the ball. Calculate the net force on the ball during this time. {F_{net} = 75 N}

14. A high jumper falling at a 4.0 m/s lands on foam pit and comes to rest compressing the pit 0.40 m. If the pit can exert a net force of 1200 N on the high jumper breaking the fall, calculate the jumper's mass. {m = 60 kg}

15. In bench pressing 100 kg, a weightlifter applies an upward force of 1040 N. How large is the upward acceleration of the weights during the lift? {a = 0.59 m/s^2}

16. An elevator that weighs 3000 N is accelerated upward at 1.5 m/s^2. What applied force does the cable apply to give this acceleration? {F_a = 3460 N}

17. A car has a mass of 710 Kg. It starts from rest and travels 40 m in 3.0 s. Calculate the net force acting on the car assuming a uniform acceleration. {F_{net} = 6300 N}

18. A force is applied to a 50 kg object and results in a constant acceleration of 3.6 m/s^2 on a flat floor. The coefficient of kinetic friction is 0.15.
    a. Calculate the magnitude of the force of friction. {$|F_f|$ = 73.6 N}
    b. Calculate the net force. {F_{net} = 180 N}
    c. Calculate the applied force. {F_a = 253 N}

19. The net force on a box is 125 N and results in an acceleration of 10.5 m/s^2 along the ground. However, to keep that box moving an applied force of 175 N is required.
    a. Calculate the mass of the box. {12 kg}
    b. Calculate the normal force on the box. {117 N}
    c. Calculate the force of friction acting on the box. {–50 N}
    d. Calculate the coefficient of kinetic friction between the box and the ground. {0.43}

20. A 13 kg mass starts from 3.1 m/s and is accelerated to 18.6 m/s in 4.5 seconds from an unknown applied force. The coefficient of kinetic friction is 0.24.
    a. Calculate the acceleration of the mass. {a = 3.4 m/s^2}
    b. Calculate the net force on the mass. {F_{net} = 45 N}
    c. Calculate the magnitude of the force of kinetic friction on the mass. {F_{fk} = 31 N}
    d. Calculate the applied force. {F_a = 76 N}

Expert Level Questions

21. The net force on an object is 231 N. The object's acceleration is 4.9 m/s^2 along the ground. To overcome friction, an applied force of 292 N is required. Calculate the coefficient of kinetic friction. {μ_k = 0.13}

22. An object sliding along the floor goes from 1.5 m/s to 12 m/s in 2.6 seconds. Calculate the applied force if the object has a mass of 15 kg and $\mu_k = 0.28$. {$F_a = 102$ N}

23. A 45 kg person stands at the end of a diving board that is 12 m above the water. Taking the initial velocity off the diving board to be zero, calculate the minimum water depth to safely stop the diver with an acceleration of 25 m/s^2. {depth = 4.7 m}

24. To avoid an accident, a car slams on the breaks to a stop (so the only horizontal force on the car is kinetic friction). If the car's initial velocity is 15 m/s [E], calculate the stopping distance if $\mu_k = 0.25$. {$d_f = 46$ m}

25. In a similar situation to question 24, calculate the coefficient of kinetic friction between the tires and the road to stop a vehicle traveling 30 m/s in 55 m. {$\mu_k = 0.83$}

Outcome 3.3: Qualitatively and quantitatively analyze the various types of forces including friction, normal, gravitational, applied, tension, and the concept of net force in two dimensions.

Net Force

1. Three forces act simultaneously on an object. One force is 10.0 N [N], the second is 15 N [W], and the third is 15.0 N [E60N]. Calculate the net force. {$F_{net} = 24.2$ N [W72°N]}

2. Two forces are acting on an object: $F_1 = 345$ N [E71°S] and $F_2 = 415$ N [W19°S]. Calculate a third force to create a net force of 378 N [E30°N]. {$F_3 = 889$ N [E47°N]}

Pulling or Pushing Objects at an Angle

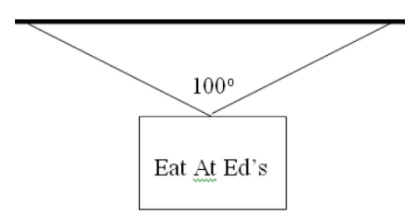
3. A student pushes a 25 kg lawn mower with a force of 150 N. The handle makes an angle of 35° to the horizontal.
    a. Calculate the vertical and horizontal components of the applied force. {$F_{ay} = -86$ N, $F_{ax} = 123$ N}
    b. Calculate the normal force. {$F_N = 330$ N}
    c. Calculate the net force if 85 N of friction exists. {$F_{netx} = 38$ N}
    d. Calculate the acceleration of the lawn mower. {$a_x = 1.5$ m/s^2}

4. Calculate the acceleration of a 15 kg toboggan that is pulled with an applied force of 45 N at an angle of 40° to the horizontal. The opposing force of friction is 28 N. {$a_x = 0.43$ m/s^2}

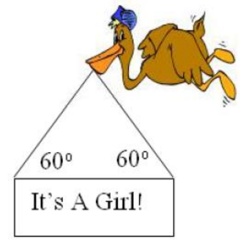
5. A 45 kg box is pulled with a force of 205 N by a rope held at an angle of 47° to the horizontal. The horizontal acceleration of the box is 0.20 m/s^2. Calculate the coefficient of kinetic friction. {$\mu_k = 0.45$}

Tension and Hanging Objects

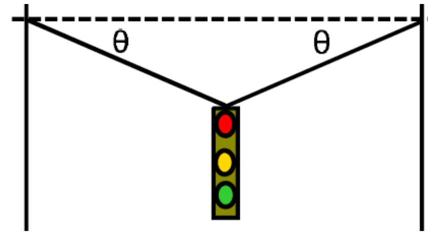
6. The sign has a mass of 5.0 kg. Calculate the tension in the cables. {$F_T = 38$ N}



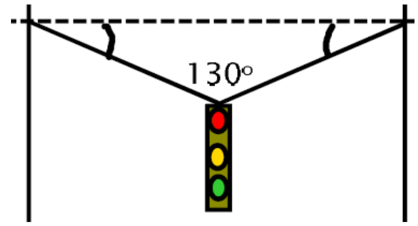
7. Calculate the force of tension in the cables if the sign has a mass of 10 kg. {F_T = 57 N}



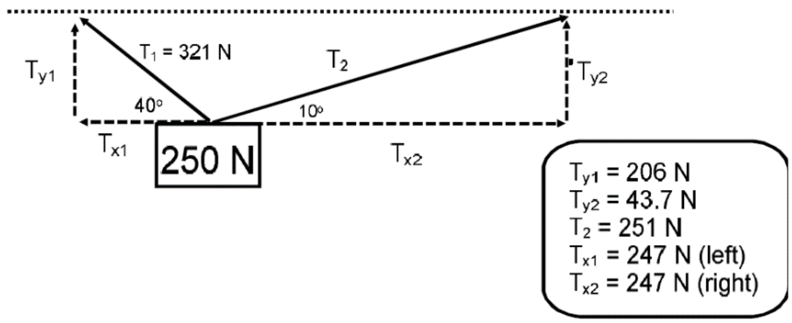
8. The cable being used to support a traffic light will break if the tension reaches 2100 N. Calculate the smallest angle possible if the traffic light has a mass of 110 kg. {θ = 15°}



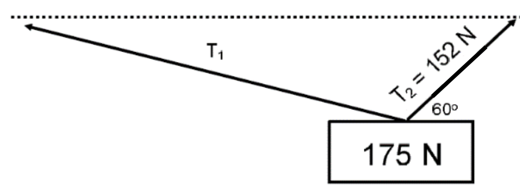
9. The cable being used to support a traffic light will break if the tension reaches 1750 N. Calculate the largest mass that can be hung on the wire. {m = 151 kg}



10. Calculate all the unknown variables labeled in the following sketch {the answers are in the block to the bottom right of the image}.



11. Calculate T_1 given the following information in the image on the right. {T_1 = 87 N}



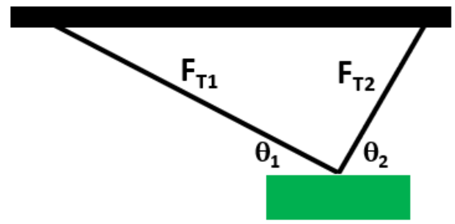
Expert Level Questions

12. A 29 kg object is pulled, to move horizontally along the ground, with an applied force of 75 N, 25° up from the horizontal. The coefficient of kinetic friction is 0.23 and the object starts from rest.
    a. Calculate the time for the object to travel 125 m along the ground. {t = 27 s}
    b. Using the information from the question, calculate how far away the object is when its instantaneous speed is 11 m/s. {d_f = 178 m}

13. A rope is attached to a 25 kg box to drag it horizontally across the floor with an acceleration of 0.25 m/s^2. The coefficient of kinetic friction is 0.35 and the rope makes an angle of 30° with the horizontal. Calculate the applied force in the rope. {F_a = 89 N}

14. A 45 kg sled is pulled across the snow with an acceleration of 0.15 m/s^2. The sled experiences 47 N of friction and the coefficient of kinetic friction is 0.20. Calculate the applied force acting on the sled. {F_a = 137, 58° up from the horizontal}

15. Calculate the force of tension in each wire given θ_1 = 32°, θ_2 = 48° and the mass of the sign is 125 kg. {F_{T1} = 836 N, F_{T2} = 1054 N}



Outcome: 4.1 Define and apply the concept of work incorporating the following: kinetic, gravitational potential, elastic potential energy, and energy loss due to friction.

Work

1. A piano is moved 12.7 m across a room. Calculate the force exerted if the work done is 2750 J. {***F*** = 217 N}

2. A person holds a briefcase at a constant height above the floor by exerting an upwards force of 30.0 N This person walks 20.0 m down the hall to her office. Calculate the work done on the briefcase, by the upwards force, during the walk. {W = 0 J}

3. Calculate the work done by friction and the applied force on a box. The applied force is 212 N moving the box across a 9.5 m long floor. The force of friction between the box and the floor is 87 N. {W_f = -826 J, W_{Fa} = 2014 J}

4. To move an object along the floor, an applied force of 291 N does 1750 J of work. The force of friction is 105 N. Calculate the work done by the force of friction. {W_f = -631 J}

5. Starting from rest, a car drives straight east from an applied force. It accelerates to a desired velocity, maintains that velocity, and then accelerates to a stop.
   a. In what direction is the applied force during the initial acceleration? Is the work done on the car, by this force positive or negative?
   b. In what direction is the applied force during the constant velocity? Is the work done on the car, by this force positive or negative?
   c. In what direction is the applied breaking force during the final acceleration? Is the work done on the car, by this force positive or negative?

6. To drive a car in a circle, the force of friction acts towards the center, at 90° to the velocity. Suppose a car drives at a constant speed in a circular path, explain why the work done by friction on the car is zero.

7. A 75 kg boulder rolled off a cliff and fell to the ground. If the force of gravity did 60 000 J of work on the boulder, how far did it fall? {d = 81.5m}

8. A person pushes a box down a hall by exerting 300 N of force. If the work done by the person is 1900 J, calculate the length of the hallway. {d = 6.33 m}

9. A 25 kg rock is rolled up a hill to a height of 12 m above its previous location. Calculate the work done by gravity. {W = -2943 J}

Types of Energy

10. A spring is used to stop a sliding object by applying an average force of 40 N. If the stopping distance was 0.73 m, calculate the work done by the spring. {W = -29.2 J}

11. A bowling ball moving at 0.95 m/s has 4.5 J of kinetic energy. Calculate the mass of the bowling ball. {m = 10 kg}

12. A 56 kg skier reaches the bottom of a ski hill with a velocity of 7.25 m/s. Calculate their kinetic energy. {E_k = 1472 J}

13. A slingshot has an elastic cord. The cord has a spring constant of 1100 N/m.
    a. Using Hooke's Law, calculate the stretch length of the cord if a force of 455 N is applied. {x = 0.41 m}
    b. Calculate the elastic potential energy in the cord. {E_e = 92 J}

14. An object is hung from a vertical spring, extending it by 0.24 m. If the spring constant is 35 N/m, calculate the potential energy in the spring. {E_e = 1.0 J}

15. A force of 18 N compresses a spring by 0.15 m. Calculate the spring's potential energy. {E_e = 1.4 J}

16. A 4.45 kg picture is hung on the wall. Not happy with its position, the owner moves the picture to a location 2.5 m higher. Calculate how much gravitational potential energy the picture has gained. {E_g = 109 J}

17. A roller-coaster train lifts its passengers up vertically through a height of 39.4 m from its starting position. Calculate the gravitational potential energy if the entire mass is 3900 kg. {E_g = 1.51 x 10^6 J}

Expert Level Questions

18. A 12 kg box has a speed of 5.5 m/s. In 8.1 m the force of friction has brought it to a stop. Calculate the work done by friction. {W_f = –182 J}

19. A 17 kg object, moving along the floor, has an acceleration of 0.65 m/s^2 under an applied force of 315 N. Friction does –1525 J of work on the object. Calculate the work done by the applied force. {W_a = 1580 J}

20. A 41 kg object is pulled along the ground with an applied force of 450 N, 32° up from the horizontal. The object is moved 37 m under an acceleration of 0.96 m/s^2. Calculate the work done by friction. {W_f = – 12664 J}

Outcome 4.2: Qualitatively and quantitatively analyze the conservation of mechanical energy with conservative and non-conservative forces.

1. The length of a rollercoaster track is 1425 m. The cart starts from rest and comes to a stop at ground level. Calculate the average force of friction if all the cart's energy was lost to thermal energy. The initial height of the roller coaster and mass are 25.9 m and 341 kg, respectively. {$\boldsymbol{F_f}$ = 61 N}

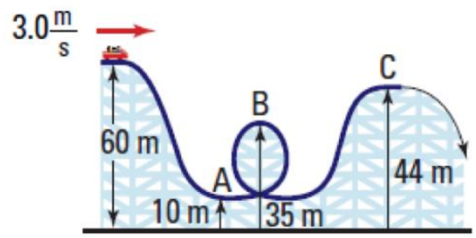
2. A 1.2 kg cart is moving along a table at 3.6 m/s when it collides with a spring bumper. Answer the following if it has a spring constant of 200 N/m, assuming no frictional forces.
   a. Calculate the maximum compression of the spring. {x = 0.29 m}
   b. Calculate the speed of the cart the instant the spring has been compressed 0.11 m. {v_f = 3.3 m/s}

3. A clown car has a total mass of 150 kg and is moving at 6.0 m/s when it hits a large spring. The cart is brought to a stop in 2.0 m. Calculate the spring constant. {k = 1350 N/m}

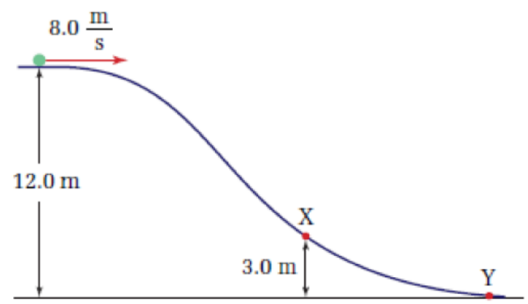
4. An archery bow has a spring constant of 485 N/m. If the bow is stretched 0.45 m and the arrow has a mass of 0.031 kg, calculate the speed of the arrow as it leaves the bow. {v_f = 56.3 m/s}

5. A 70.0 kg stunt person falls 8.25 m to a suspended net. Calculate the spring constant of the net if it stretches 1.40 m to stop the person's fall. {k = 6762 N/m}

6. A 250 kg roller coaster cart loaded with people has an initial velocity of 3.0 m/s. Calculate the velocity of the cart at A, B, and C. Assume the roller coaster is frictionless. {v_A = 31 m/s; v_B = 22.3 m/s; v_C = 18 m/s}

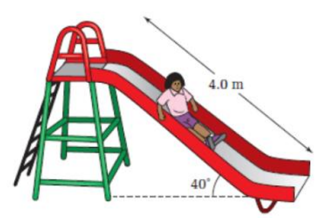


7. A sled at the top of a snowy hill is moving forward at 8.0 m/s. The total mass of the sled and rider is 70.0 kg. Calculate the speed of the sled at point X, which is 3.0 m above the base of the hill, if the snow does 1220 J of work on the sled on the way to point X. {v_f = 14.3 m/s}

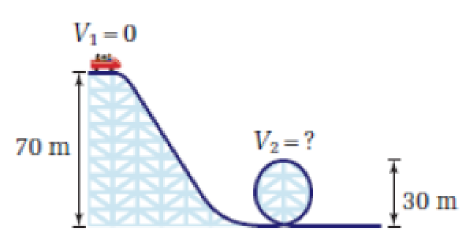


8. The tallest point of a roller-coaster is 94.5 m, take v_o = 0 m/s.
   a. Calculate the speed of a cart at ground level. {v_f = 43.1 m/s}
   b. The actual ground level speed is 41.1 m/s, calculate the percentage of total mechanical energy lost to thermal energy due to friction. {E_{loss} = 9.1%}

9. A 15 kg child, at rest, slides down a 4.0 m long slide that makes a 40° angle with the ground. The child's speed at the bottom is 3.2 m/s. Calculate the force of friction exerted on the child from the slide. {$\boldsymbol{F_f}$ = 75 N [up the slide]}

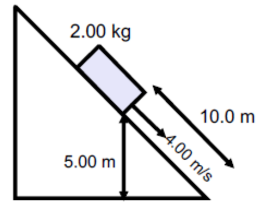


10. A roller coaster is constructed with a circular vertical loop. Assuming no friction calculate the speed at the top of the loop. {v_f = 28 m/s}



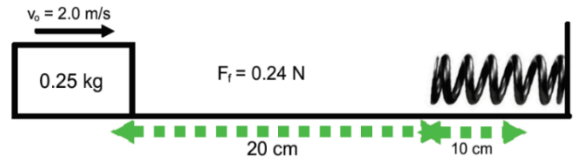
11. A 45 kg cyclist travelling 7.6 m/s on a 7.0 kg bike brakes suddenly and slides to a stop in 3.2 m. Calculate the work done by friction to stop the cyclist. {W_{nc} = -1502 J}

12. Calculate the velocity at the bottom of the ramp if the force of friction acting on the mass is 3.3 N {refer to the image to the right for more details}. {v_f = 9.0 m/s}

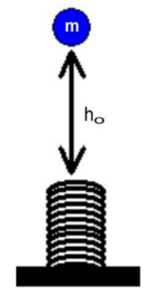


13. A 2.75 kg object is dropped 1.85 m on to a platform which is supported by a strong spring. The platform compresses 0.75 m to bring the object to a stop. Calculate the k-value of the spring. {k = 249 N/m}

14. Given the information in the diagram below and that the spring is compressed 10 cm to bring the moving block to a stop, calculate (remember to use distances in meters):
    a. The spring constant, k. {k = 85.6 N/m}
    b. The speed of the mass as it leaves the spring. {v_f = 1.8 m/s}



15. A 5.0 kg mass is dropped from 2.0 m on to a large spring. The k-value of the spring is 1406 N/m. Calculate how much the spring compresses to bring the mass to a stop. {x = 0.41 m}



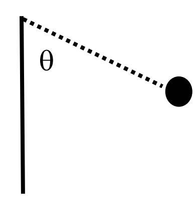
16. An 8.9 kg mass is dropped from a height of 7.6 meters above a large spring in neutral position. The spring constant is 175 N/m. Calculate the compression distance of the spring when the object has a speed of 5.3 m/s. {x = 3.0 m}

17. A 5.7 kg block is located 3.1 m from a large spring. At that point, the block is moving at 6.5 m/s. The force of kinetic friction between the block and the floor is 8.5 N. The spring constant is 80 N/m. Calculate the compression length of the spring to stop the block from moving. {x = 1.4 m}

18. A 5.7 kg block is located 7.4 m from a large spring. At that point, the block is moving at 12 m/s. The force of kinetic friction between the block and the floor is 25 N. The spring constant is 40 N/m. Calculate the compression length of the spring to stop the block from moving. {x = 2.8 m}

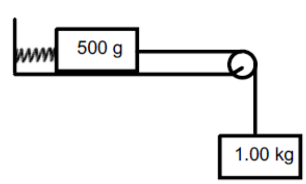
Expert Level Questions

19. A mass is attached to a 3.0 m long string and raised at an angle of 65° relative to the rest position and released. Calculate the velocity of the mass when it makes an angle of 25° on the other side of the rest position. {v_f = 5.3 m/s}



20. A toy foam ball, m = 0.0019 kg is launched directly upwards by compressing a spring. That spring has a k-value of 285 N/m. The spring is compressed 0.05 m and air resistance (friction) keeps the ball's maximum height to 12 m. Calculate the maximum height if the spring constant is now 450 N/m while keeping the same amount of compression. {h_f = 19 m}

21. In the diagram, the force of friction between the 500 g mass (or 0.5 kg) and the table is 4.8 N. The spring constant is 50 N/m. The system starts at rest with the spring at its natural length. The 500 g mass will not leave the table. Calculate the maximum speed the 1.0 kg mass will reach. {v_{max} = 0.58 m/s}



Unit 1 – Kinematics Equations

$$v = \frac{d}{t}$$

$$\vec{v}_{avg} = \frac{\vec{d}_f - \vec{d}_o}{t}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{t}$$

$$\vec{d}_f = \vec{d}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f = \vec{v}_o + 2a\left(\vec{d}_f - \vec{d}_o\right)$$

Variable	Quantity	Unit
v	Average/Instantaneous speed	m/s
$\vec{v}_{avg}$	Average velocity	m/s
$\vec{v}_f$	Final velocity	m/s
$\vec{v}_o$	Initial velocity	m/s
$\vec{d}_f$	Final position	m
$\vec{d}_f$	Initial position	m
d	Distance	m
$\vec{a}$	Acceleration	m/s^2
t	Time	s

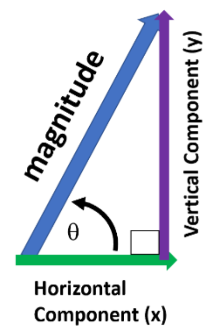
Unit 2 – Vector Components

Vertical component = Magnitude $\times$ $\sin\theta$

Horizontal component = Magnitude $\times$ $\cos\theta$

Magnitude = $\sqrt{(horizontal\ component)^2 + (vertical\ component)^2}$

Angle, $\theta = \tan^{-1}\left(\frac{vertical\ component}{horizontal\ component}\right)$



Unit 3 – Dynamics

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_{net} = \sum Forces$$

$$|\vec{F}_f| = \mu|\vec{F}_N|$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = \sum \text{connected masses} \times \vec{a}$$

Variable	Quantity	Unit
m	Mass	kg
$\vec{F}_g$	Force of gravity	N
$\vec{g}$	Acceleration due to gravity 9.81 for objects on Earth	m/s^2
$\vec{F}_{net}$	Net force	N
$\vec{F}_f$	Force of friction	N
$\vec{F}_N$	Normal force	N
μ	Coefficient of friction	none

Location	$\lvert\vec{g}\rvert$
Earth	9.81
Moon	1.64
Mars	3.72
Jupiter	25.9
North Pole	9.8322
South Pole	9.7805
Mt. Everest	9.7647
Mariana Trench	9.8331
ISS	9.0795

Unit 4 – Mechanical Energy

$$W = F_{\parallel}\left(\vec{d}_f - \vec{d}_o\right)$$

$$E_g = mgh$$

$$E_k = \frac{1}{2}mv^2$$

$$E_e = \frac{1}{2}kx^2$$

$$\vec{F}_s = -kx$$

$$\Delta E_T = 0$$

$$W_{nc} = \Delta E_T$$

Variable	Quantity	Unit
W	Work	J
$F_{\parallel}$	Force parallel to motion	N
E_g	Gravitational potential energy	J
g	Acceleration due to gravity	m/s^2
h	Change in height	m
E_k	Kinetic energy	J
E_e	Elastic potential energy	J
$\vec{F}_s$	Restoring force	N
k	Spring constant	N/m
x	Stretch or compression length	m
ΔE_T	Change in total mechanical energy	J
W_{nc}	Work done by non-conservative forces	J

Unit 5 – Wave Phenomena

$$T = \frac{1}{f}$$

$$v = f\lambda$$

$$f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$$

Refraction

$$n = \frac{c}{v}$$

$$n_i \sin\theta_i = n_R \sin\theta_R$$

Substance	Index of Refraction (n)
vacuum	1.00000
gases at 0°C, 1.013×10^5 Pa	
hydrogen	1.00014
oxygen	1.00027
air	1.00029
carbon dioxide	1.00045
liquids at 20°C	
water	1.333
ethyl alcohol	1.362
glycerin	1.470
carbon disulfide	1.632

Substance	Index of Refraction (n)
solids at 20°C	
ice (at 0°C)	1.31
quartz (fused)	1.46
optical fibre (cladding)	1.47
optical fibre (core)	1.50
Plexiglas™ or Lucite™	1.51
glass (crown)	1.52
sodium chloride	1.54
glass (crystal)	1.54
ruby	1.54
glass (flint)	1.65
zircon	1.92
diamond	2.42

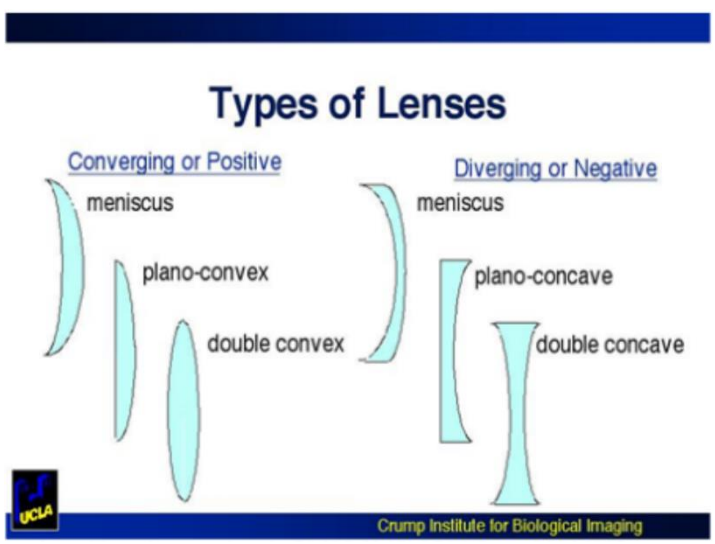
Spherical Lenses

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Variable	Quantity	Unit
Waves		
T	Period	s
f	Frequency	Hz
v	Wave speed	m/s
λ	Wavelength	m
f_{obs}	Observed frequency	Hz
f_s	Source frequency	Hz
v_w	Speed of sound	m/s
v_s	Speed of the source	m/s
Refraction		
n	Index of refraction	none
c	3.00×10^8; Speed of light in a vacuum	m/s
n_i	Incident-ray medium index of refraction	none
θ_i	Angle of incidence	degrees
n_R	Refracted-ray medium index	none
θ_R	Angle of refraction	degrees
Spherical Lenses		
f	Focal length	cm, m
d_o	Object distance	cm, m
d_i	Image distance	cm, m
m	magnification	none
h_i	Image height	cm, m
h_o	Object height	cm, m
n_{lens}	Lens refractive index	none
n_o	Surrounding medium refractive index	none
R_1	Radius of curvature of first side of lens	cm, m
R_2	Radius of curvature of second side of lens	cm, m



Unit 6 – Kinematics & Dynamics in 2D

$\vec{\tau} = rF_{\perp}$

$\vec{\tau}_{net} = \sum \text{Torques}$

$\vec{p} = m\vec{v}$

$\vec{p}_{oT} = \vec{p}_{fT}$

Variable	Quantity	Unit
$\vec{\tau}$	Torque	Nm
r	Distance from pivot (lever arm)	m
$F_{\perp}$	Perpendicular force component	N
$\vec{\tau}_{net}$	Net torque	Nm
$\vec{p}$	momentum	$\frac{kg \cdot m}{s}$
m	mass	kg
$\vec{v}$	velocity	m/s
$\vec{p}_{oT}$	Total initial momenta	$\frac{kg \cdot m}{s}$
$\vec{p}_{fT}$	Total final momenta	$\frac{kg \cdot m}{s}$

Unit 7 – Circular Motion and Universal Gravitation

Circular Motion

$v = \frac{2\pi r}{T}$

$a_c = \frac{v^2}{r}$

$F_c = \frac{mv^2}{r}$

$T = \frac{1}{f}$

$v = \sqrt{rg\mu_s}$

$v = \sqrt{rg \tan\theta}$

Universal Gravitation

$\vec{F}_g = G\frac{m_1 m_2}{r^2}$

$|\vec{g}| = G\frac{m}{r^2}$

$v^2 = G\frac{m}{r}$

$\frac{Gm}{4\pi^2} = \frac{r^3}{T^2}$

Variable	Quantity	Unit
v	Circular or orbital speed	m/s
r	Circular or orbital radius; distance between centers of mass	m
T	Period	s
a_c	Centripetal acceleration	m/s^2
F_c	Centripetal force	N
f	Frequency	Hz
m	Mass or planet mass	kg
g	Acceleration due to gravity	m/s^2
μ_s	Coefficient of static friction	none
θ	Bank angle	degrees
$\vec{F}_g$	Force of gravity	N
G	6.673×10^{-11}, Universal gravitational constant	$N\frac{m^2}{kg^2}$

Inner Solar System Data					
Quantity	**Sun**	**Mercury**	**Venus**	**Earth**	**Moon**
Mass (kg)	1.99×10^{30}	3.3×10^{23}	4.9×10^{24}	5.98×10^{24}	7.36×10^{22}
Radius (m)	6.96×10^{8}	2.5×10^{6}	6.1×10^{6}	6.38×10^{6}	1.74×10^{6}
Orbital Radius (m)	N/A	5.8×10^{10}	1.1×10^{11}	1.49×10^{11}	3.84×10^{8}
Period of Orbit	N/A	88 Earth days	225 Earth days	365.25 Earth days	27.3 Earth days
Rotation	Varies	58 Earth days	243 Earth days	24 hours	27.3 Earth days

Outer Solar System Data					
Quantity	**Mars**	**Jupiter**	**Saturn**	**Uranus**	**Neptune**
Mass (kg)	6.4×10^{23}	2.0×10^{27}	5.7×10^{26}	8.7×10^{25}	1.0×10^{26}
Radius (m)	3.4×10^{6}	7.1×10^{7}	6.0×10^{7}	2.6×10^{7}	2.5×10^{7}
Orbital Radius (m)	2.3×10^{11}	7.8×10^{11}	1.4×10^{12}	2.9×10^{12}	4.5×10^{12}
Period	1.88 Earth years	11.9 Earth years	29 Earth years	84 Earth years	164 Earth years
Rotation	24 hours	10 hours	11 hours	17 hours	16 hours

Unit 8 – Electric Circuits, Fields, and Electromagnetic Forces

$q = Ne$

$\vec{F}_q = k\frac{q_1 q_2}{r^2}$

$\vec{E} = \frac{\vec{F}_q}{q}$

$\left|\vec{E}\right| = k\frac{q}{r^2}$

$\vec{E}_q = k\frac{q_1 q_2}{r}$

$V = \frac{E_q}{q}$

$V = k\frac{q}{r}$

$I = \frac{\Delta q}{\Delta t}$

$R = \rho\frac{L}{A}$

$V = IR$

$P = IV$

$R_{eqs} = R_1 + R_2 + \cdots + R_n$

$C_{eqp} = C_1 + C_2 + \cdots + C_n$

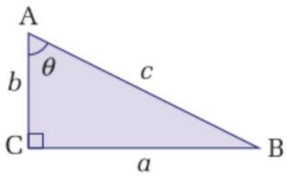
$\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

Variable	Quantity	Unit
q	Electric charge	C
N	Number of charges	none
e	1.602×10^{-19}, Elementary charge	C
$\vec{F}_q$	Electrostatic Force	N
k	8.988×10^{9}, Coulomb's constant	$N\frac{m^2}{C^2}$
r	Charge separation	m
$\vec{E}_q$	Electric potential energy	J
V	Electric potential difference	V
I	Electric current	A
R	Electrical Resistance	Ω
ρ	Resistivity	Ω·m
L	Length of wire	m
A	Cross sectional area of wire	m²
P	Electrical power	W
R_{eqs}	Equivalent resistance (series)	Ω
C_{eqp}	Equivalent capacitance (parallel)	F
R_{eqp}	Equivalent resistance (parallel)	Ω
C_{eqs}	Equivalent capacitance (series)	F

Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{a}{b}$$

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the 90° angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.

TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u}$$

- **Pythagorean Identities**

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

The Metric System: Fundamental and Derived Units

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1000 = 10^{3}
hecto	h	100 = 10^{2}
deca	da	10 = 10^{1}
		1 = 10^{0}
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Fundamental Physical Quantities and Their SI Units

Quantity	Symbol	Unit	Symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
absolute temperature	T	Kelvin	K
electric current	I	ampère (amp)	A
amount of substance	mol	mole	mol

Derived SI Units

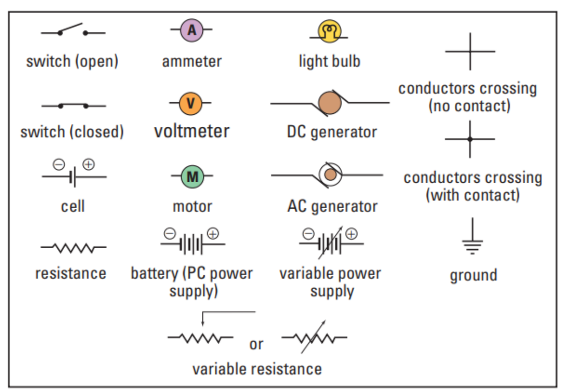
Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m,\ kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m,\ kg \cdot m^2/s^2$
power	P	watt	W	$J/s,\ kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	$N/m^2,\ kg/(m \cdot s^2)$
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field intensity	B	tesla	T	$N \cdot s/(C \cdot m),\ N/(A \cdot m)$
magnetic flux	Φ	weber	Wb	$V \cdot s,\ T \cdot m^2, m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	T°C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\ 566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Physical Constants and Data

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^8 m/s
gravitational constant	G	$6.673 \times 10^{-11}\ N \cdot m^2/kg^2$
Coulomb's constant	k	$8.988 \times 10^9\ N \cdot m^2/C^2$
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
Planck's constant	h	$6.626 \times 10^{-34}\ J \cdot s$

Electric Circuit Symbols

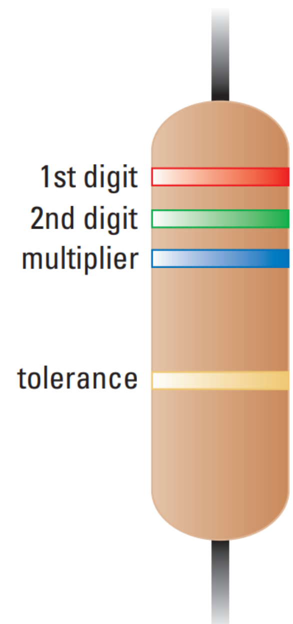


Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^5 Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^3 kg/m^3
latent heat of fusion		3.34×10^5 J/kg
latent heat of vaporization		2.26×10^6 J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^6 J
acceleration due to Earth's gravity	g	9.81 m/s^2 (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^6 m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^7 s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^6 m
mean radius of Moon's orbit	R_M	3.84×10^8 m
period of Moon's orbit	T_M	27.3 days or 2.36×10^6 s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^8 m

Resistor Colour Codes

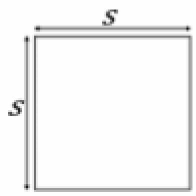
Colour	Digit represented	Multiplier	Tolerance
black	0	$\times 1$	
brown	1	$\times 1.0 \times 10^1$	
red	2	$\times 1.0 \times 10^2$	
orange	3	$\times 1.0 \times 10^3$	
yellow	4	$\times 1.0 \times 10^4$	
green	5	$\times 1.0 \times 10^5$	
blue	6	$\times 1.0 \times 10^6$	
violet	7	$\times 1.0 \times 10^7$	
gray	8	$\times 1.0 \times 10^8$	
white	9	$\times 1.0 \times 10^9$	
gold		$\times 1.0 \times 10^{-1}$	5%
silver		$\times 1.0 \times 10^{-2}$	10%
no colour			20%



GEOMETRY SHAPES AND SOLIDS

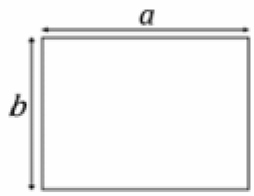
SQUARE

$P = 4s$
$A = s^2$



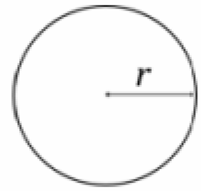
RECTANGLE

$P = 2a + 2b$
$A = ab$



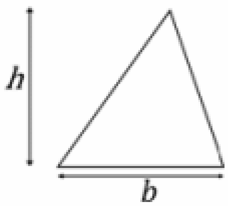
CIRCLE

$P = 2\pi r$
$A = \pi r^2$



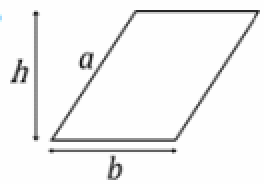
TRIANGLE

$P = a + b + c$
$A = \frac{1}{2}bh$



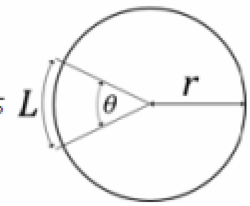
PARALLELOGRAM

$P = 2a + 2b$
$A = bh$



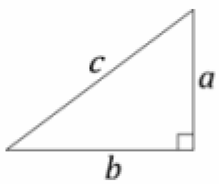
CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$



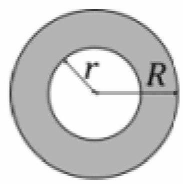
PYTHAGOREAN THEOREM

$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$



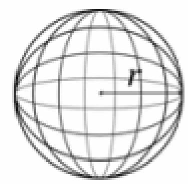
CIRCULAR RING

$A = \pi(R^2 - r^2)$



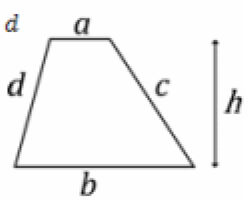
SPHERE

$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$



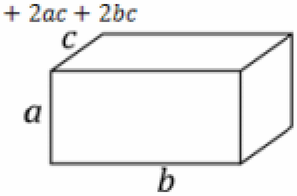
TRAPEZOID

$P = a + b + c + d$
$A = h\frac{a+b}{2}$



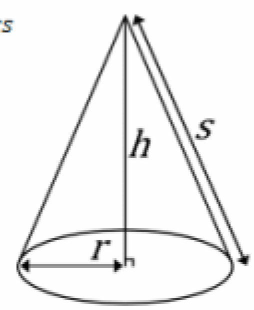
RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$
$V = abc$



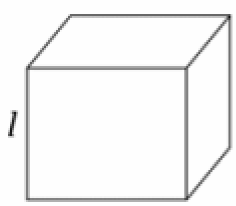
RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



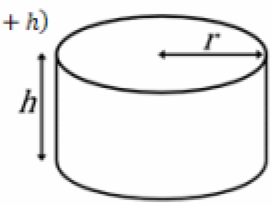
CUBE

$A = 6l^2$
$V = l^3$



CYLINDER

$A = 2\pi r(r + h)$
$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$