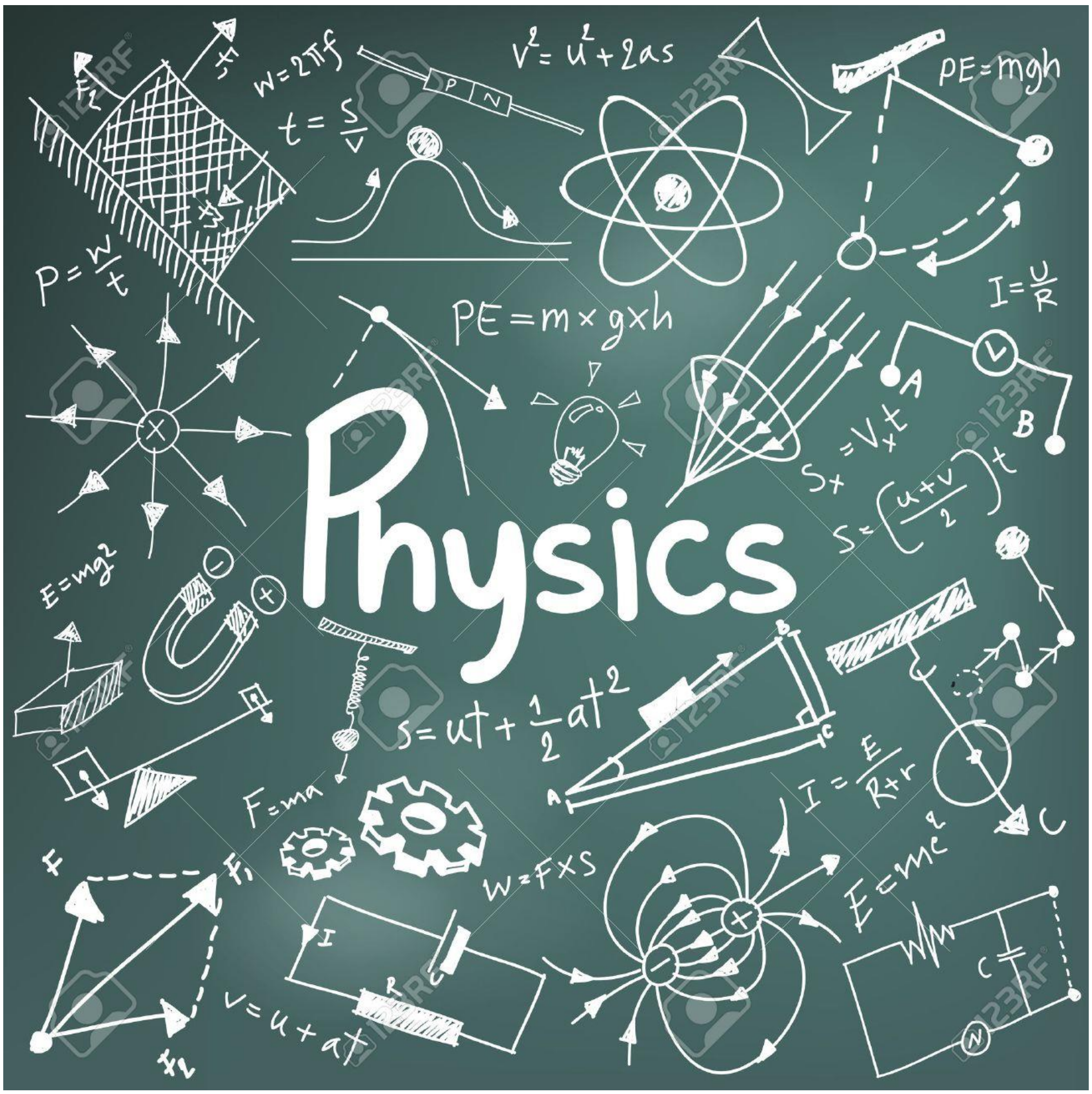
Grade 12 Physics

Outcomes, Concepts, and Problems



Mr. P. MacDonald

The reference digital text ***Openstax: Physics High School*** will be used throughout the course (grade 12 will also require the *College Physics* digital text for outcome 6.3). Readings can be found within the outcome description and the page reference is for the PDF file, not a printed version. Within the first few days of school, go through ***Chapter 1*** to add to your understanding of what physics is all about. Each unit breaks down into *Outcomes*. Each outcome is assessed and will be used to determine your grade for course.

1 Kinematic Motion in One Dimension: Kinematics is the study of *how* objects move.

- 1.1 Mathematics Review: Math is the foundation for solving problems in physics. Students will review order of operations, solving linear and quadratic equations, and trigonometric ratios.
- 1.2 Define and identify scalars and vectors. Classify each of the quantities listed in outcome 1.2 as a scalar or vector. ***Chapter 2, Pg. 67 – 80*** Recognize and define position, displacement, distance, velocity, speed, acceleration, force, and their units. ***Course Concept Guide***
- 1.3 Graphically analyze one-dimensional relationships among position, velocity, acceleration, and time. ***Chapter 2, Pg. 81 – 96***
- 1.4 Mathematically analyze the relationship among position, velocity, acceleration, and time in 1D. ***Chapter 3. Pg. 107 – 127***

2 Kinematic Motion in Two Dimensions: Much of physics relies on being able to solve problems in 1, 2 or 3 dimensions. In high school, we study 1 and 2 dimensional problems. Outcomes 2.1 – 2.3 are about the mathematics behind analyzing 2D problems. Outcomes 2.4 and 2.5 apply the 2D analysis to kinematic concepts from Unit 1. ***Chapter 5.1 & 5.2, Pg. 157-175***

- 2.1 Measuring vectors and the resultant position for 2D vectors using a scale diagram.
- 2.2 Calculating perpendicular components of vectors and calculating vectors (magnitude and direction) given the components.
- 2.3 Mathematically analyze the relationship among displacement (position), velocity, acceleration, and time in 2D.
- 2.4 Vector addition and solving for missing vectors.

3 Dynamics: The study of *why* objects move. This unit describes the types of forces and Newton's Laws of motion. ***Chapter 4, Pg. 129 – 156***

- 3.1 Explain, describe, and analyze the forces of gravity and friction. Including the types and causes of friction, the coefficient of friction, and determining normal force given the situation in one dimension.
- 3.2 Describe and apply Newton's 3 Laws of motion within inertial and non-inertial frames of reference in one dimension.
- 3.3 Qualitatively and quantitatively analyze the various types of forces including friction, normal, gravitational, applied, tension, and the concept of net force in two dimensions.

4 Conservation of Mechanical Energy: Understanding changes that take place in a system is often aided by considering energy exchanges. Students will learn the concept of work, relative to physics, identify forms of energy and that energy for a system is a constant. ***Chapter 9, Pg. 293-313***

- 4.1 Define and apply the concept of work incorporating the following: kinetic, gravitational potential, and elastic potential energy.
- 4.2 Qualitatively and quantitatively analyze the conservation of mechanical energy with conservative and non-conservative forces.

Final Assessment

- 2-hour final assessment during *Assessment Week* (January/June).
- Weighted the same as semester unit assessments.
- A clear outline of topics to review and build of the final assessment will be provided.

5 **Wave Phenomena and Applications**: In this unit students will come to understand waves and related phenomena. The two main topics are mechanical and electromagnetic waves. Students will learn about wave properties, the different parts and properties of the electromagnetic spectrum, standing waves, doppler shift, refraction, and their applications.

5.1 Mechanical waves: General properties, types of waves, wave speed, doppler shift, reflection, interference, and standing waves. ***Chapter 13, Pg. 403-427. Chapter 14, Pg. 444-448***

5.2 Electromagnetic waves and refraction: Parts of the EM spectrum and their properties. Refraction will focus on visible light as it enters and exits various media. ***Chapter 15, Pg. 470- 476, Chapter 16.2, Pg. 501-512.***

5.3 Spherical Lenses: A qualitative and quantitative analysis of using lenses to create images. Also, students will apply the lens maker's equation to problems. ***Chapter 16.3, Pg. 512 – 526***.

6 **Kinematics & Dynamics in 2D**: The study of why objects move in two-dimensional space. This unit applies the concept of perpendicular components to solve problems. Problems, or systems, involve:

6.1 In depth quantitative analysis of a projectile motion. ***Chapter 5.3, Pg. 176-185***

6.2 Applying Newton's Laws of motion for objects on an incline plane. ***Chapter 5.4, Pg. 185-192***

6.3 Learn and apply the concept of net torque to solve static equilibrium problems. ***OpenStax College Physics: Chapter 9, Pg. 329-354***

6.4 Collisions and explosions in one and two dimensions. ***Chapter 8, Pg. 267-287***

7 **Circular Motion & Universal Gravitation**: This explores how and why an object travels in a circular path (be it a ball on a string, an amusement park ride, or the Moon about the Earth, for example). Universal gravitation is a deeper analysis of the force of gravity between two masses and is further analyzed with Kepler's Laws of planetary motion.

7.1 Qualitatively and quantitatively analyze circular motion using vectors and Newton's laws. ***Chapter 6, Pg. 211-236***

7.2 Quantitatively apply Newton's law of gravitation and Kepler's third law of planetary motion to solve problems. Identify and explain Kepler's three laws of planetary motion. **Chapter 7, Pg. 243-260**

8 **Electric Circuits, Fields & Electromagnetic Forces**: Forces can affect objects over a distance and through space without physical contact. The analysis of that affect requires the physics concept of fields. When a current is made to pass through a wire, it creates a magnetic field, and vice-verse. Coulomb's Law allows for the mathematical analysis of the interaction of charged objects.

8.1 Qualitatively and quantitatively analyze electric charge and electric fields, including Coulomb's Law. ***Chapter 18, Pg. 563-606***

8.2 Quantitatively analyze electric circuits. Concepts include Ohm's Law, series and parallel circuits, and electrical power. ***Chapter 19, Pg. 617-653***

8.3 Qualitatively analyze magnetism including magnetic poles, fields, and moving charges. ***Chapter 20, Pg. 663-669***

Physics Investigations

- Physics investigations may take place throughout the course. Such investigations will link directly to learning a particular outcome. Often, questions relating to any investigations will appear on assessments, or they are assessed with a unique grade.

Final Assessment

- 2-hour final assessment during *Assessment Week* (January/June).
- Weighted the same as semester unit assessments.
- A clear outline of topics to review and build of the final assessment will be provided.

Classifying Mistakes

For any type of assessment, determine the type of mistake and the learning opportunity, that is, how to prevent the mistake from happening again. The purpose is to give us information on any learning barriers you are experiencing, and work towards removing them. Use the guide, below, to determine the type of mistake(s) and the learning opportunity.

Type	Type Definition and Examples of Learning Opportunities
Minor	An abnormal mistake, hard to catch, not made often, and not part of the problem solving. • Forgetting to divide by a number that is written down • A number or sign is not carried down to the next math step • Calculator buttons miss-hit • No units in final answers
Background	Usually in the form of mathematical mistakes. • Equation solving • Order of operations • Algebra rules (like and unlike terms)
Conceptual	These mistakes link to gaps in understanding the concepts of physics. • Vector signs • Incorrect formula • Incorrect setup or analysis • Concepts and definitions not fully understood

Strong Work Ethic and Skills for Success

- On task during class. This is the only time I can help you learn. Use it.
- Proper use of technology. Turn off your notifications. This is the main reason student work suffers.
- Time/task management.
- Problem solving skills. Not just math, but the approach to any problem.
- Reflection. No big write up necessary. "Did I work to your best today?"
- Take initiative with your learning. You have all the course materials for the entire semester. Use them.
- Personal workspace (outside of class).
- Goal setting.
- Ask questions during class lessons. Seek your own answers before asking the teacher during work time.
- Ask for feedback.
- Use of course resources. It is all there. Everything. Go forth, learn.
- Embrace mistakes. Then address them.

The Eight Science and Engineering Practices (NGSS)

Learning Target	Description
SEP1: Asking Questions and Defining Problems	A practice of science is to ask and refine questions that lead to descriptions and explanations of how the natural and designed world(s) works and which can be empirically tested. Engineering questions clarify problems to determine criteria for successful solutions and identify constraints to solve problems about the designed world. Both scientists and engineers also ask questions to clarify ideas.
SEP2: Developing and Using Models	A practice of both science and engineering is to use and construct models as helpful tools for representing ideas and explanations. These tools include diagrams, drawings, physical replicas, mathematical representations, analogies, and computer simulations. Modeling tools are used to develop questions, predictions, and explanations; analyze and identify flaws in systems; and communicate ideas. Models are used to build and revise scientific explanations and proposed engineered systems. Measurements and observations are used to revise models and designs.
SEP3: Planning and Carrying Out Investigations	Scientists and engineers plan and carry out investigations in the field or laboratory, working collaboratively as well as individually. Their investigations are systematic and require clarifying what counts as data and identifying variables or parameters. Engineering investigations identify the effectiveness, efficiency, and durability of designs under different conditions.
SEP4: Analyzing and Interpreting Data	Scientific investigations produce data that must be analyzed in order to derive meaning. Because data patterns and trends are not always obvious, scientists use a range of tools—including tabulation, graphical interpretation, visualization, and statistical analysis—to identify the significant features and patterns in the data. Scientists identify sources of error in the investigations and calculate the degree of certainty in the results. Modern technology makes the collection of large data sets much easier, providing secondary sources for analysis. Engineering investigations include analysis of data collected in the tests of designs. This allows comparison of different solutions and determines how well each meets specific design criteria—that is, which design best solves the problem within given constraints. Like scientists, engineers require a range of tools to identify patterns within data and interpret the results. Advances in science make analysis of proposed solutions more efficient and effective.
SEP5: Using Mathematics and Computational Thinking	In both science and engineering, mathematics and computation are fundamental tools for representing physical variables and their relationships. They are used for a range of tasks such as constructing simulations; solving equations exactly or approximately; and recognizing, expressing, and applying quantitative relationships. Mathematical and computational approaches enable scientists and engineers to predict the behavior of systems and test the validity of such predictions.
SEP6: Constructing Explanations and Designing Solutions	The end-products of science are explanations, and the end-products of engineering are solutions. The goal of science is the construction of theories that provide explanatory accounts of the world. A theory becomes accepted when it has multiple lines of empirical evidence and greater explanatory power of phenomena than previous theories. The goal of engineering design is to find a systematic solution to problems that is based on scientific knowledge and models of the material world. Each proposed solution results from a process of balancing competing criteria of desired functions, technical feasibility, cost, safety, aesthetics, and compliance with legal requirements. The optimal choice depends on how well the proposed solutions meet criteria and constraints.
SEP7: Engaging in Argument from Evidence	Argumentation is the process by which evidence-based conclusions and solutions are reached. In science and engineering, reasoning and argument based on evidence are essential to identifying the best explanation for a natural phenomenon or the best solution to a design problem. Scientists and engineers use argumentation to listen to, compare, and evaluate competing ideas and methods based on merits. Scientists and engineers engage in argumentation when investigating a phenomenon, testing a design solution, resolving questions about measurements, building data models, and using evidence to evaluate claims.
SEP8: Obtaining, Evaluating, and Communicating Information	Scientists and engineers must be able to communicate clearly and persuasively the ideas and methods they generate. Critiquing and communicating ideas individually and in groups is a critical professional activity. Communicating information and ideas can be done in multiple ways: using tables, diagrams, graphs, models, and equations as well as orally, in writing, and through extended discussions. Scientists and engineers employ multiple sources to obtain information that is used to evaluate the merit and validity of claims, methods, and designs.

Assessment and Evaluation

Outcomes will be graded from 1 to 6. That grade will be based on evidence from multiple sources including all or some of the following: observations, conversations, formative, and summative assessments.

Expert: Demonstration of a deep/thorough understanding of the concept	**6**	• Chose an appropriate strategy. • Successfully applied the necessary background skills and proper concepts to complete solutions. • Solutions contained no minor mistakes, or a summative contains at most two. • Clearly and concisely explained how to solve the problem using appropriate vocabulary, diagrams, and a coordinate system. "Did I show my work?" • Evaluated the reasonableness of my answer. "Does this make sense for the situation?" • Concept understood to a high degree to teach it to someone else. • **Concept can be applied to new or challenging problems.**
	5	• Chose an appropriate strategy. • Solution(s) contained an error(s) related to a background skill. • The concept can be explained using appropriate vocabulary. • **The concept can be applied successfully in known problems.**
Apprentice: Good/Satisfactory understanding of the concept	**4**	• Chose an appropriate strategy. • A solution contained a concept error. A summative contained at most two such errors. • Minor mistakes and background skill errors are common. • Explanations of a problem contained *mostly* appropriate terminology. • Mistakes were identified and corrected after referring to a key. • **More practice is needed solving this type of problem**.
	3	• Chose an appropriate strategy. • Solution(s) contained a combination of concept errors, errors related to background skills and minor mistakes. • A lack of necessary background skills to solve problems. • Notes, examples, or help was needed to solve problems. • Explanations did not contain proper terminology. • **Help from an expert is required solving this type of problem**.
Novice: Minimal-to-no understanding of the concept	**2**	• Incorrect strategy(ies) chosen for a problem(s). • Step-by-step instructions are required to solve problems. • Tasks could not be performed to an acceptable standard. • **Consistent extra help from an expert is required**.
	1	• Basics of what was needed to solve the problem was not known. • Solution left blank; first step not known. • **Teaching by an expert is required**.

Learning Category	**Classification Level**	**Only shortly before report cards will a percentage mark be determined**		
Expert	6	95 – 100		
	5	86	90	94
Apprentice	4	73	80	85
	3	60	66	72
Novice	2	50	56	59
	1	0	25	49

Reassessing Units/Outcomes

One or two class days will be dedicated to for students to reassess a previous outcome. This will occur around mid-semester and towards the end of the semester. Note that a reassessment is the entire outcome, not just a question or two. Qualifying students (good work ethic, behaviour, attendance, etc.) can reassess any outcome to improve their grade and the highest grade is taken towards the mark calculation.

Physics 112 – Course Outcome Tracking

Physics 112 Unit/Outcome	Description	Grade	Physics 112 Unit/Outcome	Description	Grade
1.1	Mathematics Review: Math is the foundation for solving problems in physics. Students will review order of operations, polynomials, solving linear and quadratic equations, and trigonometric ratios.		**2.4**	Vector addition and solving for missing vectors.	
1.2	Define and identify scalars and vectors. Classify each of quantity listed in outcome 1.2 as a scalar or vector. Recognize and define position, displacement, distance, velocity, speed, acceleration, force, and their units.		**3.1**	Explain, describe, and analyze the forces of gravity and friction. Including the types and causes of friction, the coefficient of friction, and determining normal force given the situation in one dimension.	
1.3	Graphically analyze one-dimensional relationships among position, velocity, acceleration, and time.		**3.2**	Describe and apply Newton's 3 Laws of motion within inertial and non-inertial frames of reference in one dimension.	
1.4	Mathematically analyze the relationship among position, velocity, acceleration, and time in 1D.		**3.3**	Qualitatively and quantitatively analyze the various types of forces including friction, normal, gravitational, applied, tension, and the concept of net force in two dimensions.	
2.1	Measuring vectors and the resultant position for 2D vectors using a scale diagram.		**4.1**	Define and apply the concept of work incorporating the following: kinetic, gravitational potential, and elastic potential energy.	
2.2	Calculating perpendicular components of vectors. Calculating vectors (magnitude and direction) given the components.		**4.2**	Qualitatively and quantitatively analyze the conservation of mechanical energy with conservative and non-conservative forces.	
2.3	Mathematically analyze the relationship among displacement (position), velocity, acceleration, and time in 2D.				

Overall Course Grade

- Calculate your *median* by arranging your grades from lowest to highest. The grade in the middle is *likely* your overall grade. If there is no exact middle number, average the two middle numbers.
- Calculate your mean by adding all the grades up and divide by how many there are.
- Use a pencil when writing your grades here because grades will fluctuate over the semester.

Smallest – to – Largest												

Median = _______ Mean = _______

Example Percent Determinations

Median	Mean	Percent	Reason
4	3.8 – 4.2	80 %	Median and mean match or are close
4	4.3 or higher	85 %	Mean is much higher than median
4	3.7 or lower	73 %	Mean is much lower than median

Physics 122 – Course Outcome Tracking

Physics 122 Unit/Outcome	Description	Grade	Physics 122 Unit/Outcome	Description	Grade
5.1	Mechanical waves: General properties, types of waves, speed, doppler shift, reflection, interference, and standing waves		6.4	Conservation of momentum: collisions and explosions in one and two dimensions.	
5.2	Electromagnetic waves: Types of EM radiation and refraction.		**7.1**	Qualitatively and quantitatively analyze circular motion using vectors and Newton's laws.	
5.3	Spherical lenses: A qualitative and quantitative analysis lenses to create images and the application of the lens maker's equation.		**7.2**	Quantitatively apply Newton's law of gravitation and Kepler's third law of planetary motion to solve problems. Identify and explain Kepler's three laws of planetary motion.	
6.1	In depth quantitative analysis of a projectile motion.		**8.1**	Qualitatively and quantitatively analyze electric charge and electric fields, including Coulomb's Law.	
6.2	Applying Newton's Laws of motion for objects on an incline plane.		**8.2**	Quantitatively analyze electric circuits. Concepts include Ohm's Law, series and parallel circuits and their components.	
6.3	Learn and apply the concept of net torque to solve static equilibrium problems.		**8.3**	Qualitatively analyze magnetism including magnetic poles, fields, and moving charges.	

Overall Course Grade

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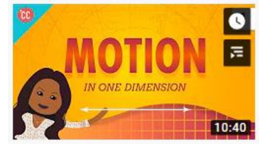
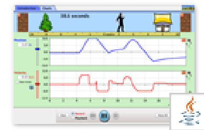


Crash Course Physics & Kahn Learning Academy: These are a series of YouTube videos produced by certified scientists and educators. They are suggested viewing material to complement many of the concepts introduced in Physics. The digital version of this document contains a link to the videos, otherwise a search of YouTube will find the video or related videos.

Unit 1: Kinematics in One Dimension



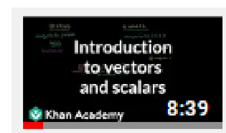
The Moving Man

Kinematics: The study of *how* objects move. That is, analyzing motion relating to an object's current motion. This analysis is often a mathematical approach to solving for quantities such as position, final or initial velocity or acceleration at an instant in time. Investigate this with the PhET simulation *The Moving Man.*

Motion: An object is said to be in motion when its position changes.

Frame of Reference: Something not moving with respect to an observer that can be used to detect motion; it is the "point-of-view" of the observer. For example, suppose someone on the ground is watching another person walking on a train. The person on the train has a velocity *relative* to the ground or *relative* to the train. Both frames of reference are correct but will yield different numerical results when analyzed.

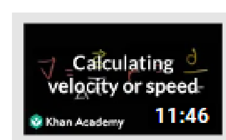
Coordinate System: The mathematical reference from which to measure quantities. It provides a reference for defining *direction* – up, down, left, right, north, west, east, south, etc. Solving problems in physics requires measurements to be positive or negative and it is the coordinate system that defines the positive direction.



Scalar: Measurements that are independent of direction and always positive in value. It is not that you choose not to communicate a direction, but rather, stating a direction *does not make sense*. For example, you would not tell someone their body temperature is 38 °C East. Other scalar quantities include time, mass, distance, and speed.

Vector: Measurements that have a magnitude *and* a direction (magnitude is the numerical value of the measurement). All vectors must be used relative to the positive direction as determined in the coordinate system. Vector quantities can be positive or negative, depending on the coordinate system and are communicated in writing by being bold font or with an arrow above them. For example, the variable for force could be written as ***F*** or $\vec{F}$. Visually, in diagrams, vectors quantities are represented by arrows. The length represents magnitude and the way it points is the direction. Other examples of vectors include position, displacement, velocity, and acceleration.

Distance: The length of the route between two points. Measurements of distance can never decrease. That is, you cannot take away the fact an object moved a certain distance. The odometer of a car, for example, measures a car's total distance traveled and it is illegal to tamper with it.



Position: An object's distance and direction from a reference point (within a coordinate system) at an instant in time. The change in position is called *displacement*.

Average Speed: An object's *total distance* traveled per time interval.

Instantaneous Speed: The speed of an object at an instant in time. Determining this usually requires data to be analyzed. One example is the speedometer of a car.

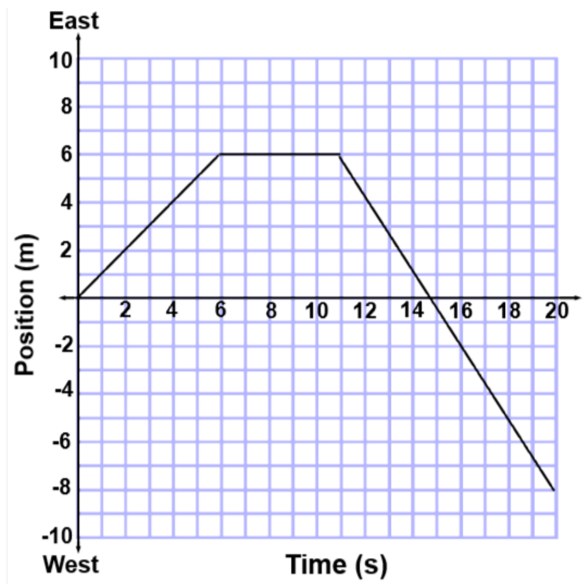
Average Velocity: An object's *change in position* per time interval. Changes in direction average out so the path taken does not matter. It is possible to have a value of zero.

Instantaneous Velocity: The velocity of an object at an instant in time (usually requiring data analysis). It is the object's instantaneous speed *and* direction.

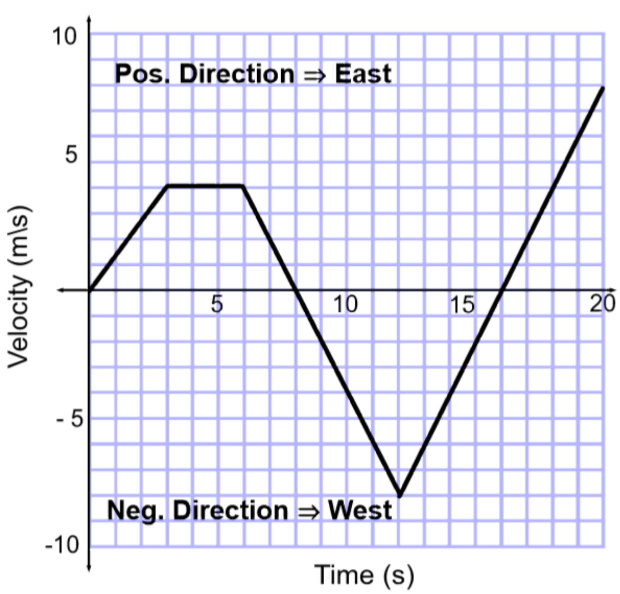
Acceleration: The change in an object's velocity per unit time. That means if an object's speed changes then it undergoes an acceleration. It also means that if only the object's direction changes then it accelerates! When an object changes direction it has an instantaneous velocity of zero, but its acceleration is not zero.

Analyzing Position–Time Graphs

- **Position**: Read from the position axis for the given time. It is how far and in what direction from a starting point.
- **Distance**: Add up all the motion using positive numbers. Follow the graph counting squares up and down until you get to the desired time. It is the total length an object traveled.
- **Direction Change**: When the velocity goes from positive to negative or vice versa.
- **Average Speed**: Distance (see above) divided by the time given in the question.
- **Instantaneous Speed**: Positive value of the slope of the line at the time given (find two points on the line and calculate rise/run).
- **Average Velocity**: Position (see above) divided by the time given in the question.
- **Instantaneous Velocity**: Slope of the line at the time given (find two points on the line and calculate rise/run). Can be positive or negative.



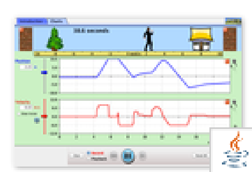
Analyzing Velocity–Time Graphs



- **Velocity**: Read it from the velocity axis for the given time. It communicates the direction of travel, not the position.
- **Instantaneous Acceleration**: The slope of the line at the time given (find two points on the line and calculate rise/run). Can be positive or negative and the sign is the direction of the acceleration, not the object.
- **Distance**: For a certain time-interval, the distance is the area contained between the graph and the time axis (always use positive numbers when calculating).

- **Position**: For a certain time-interval, it is the area contained on the top minus the area contained on the bottom.
- **Direction Change**: When the velocity goes from positive to negative or vice versa.
- **Average Speed**: Total distance divided by the time.
- **Average Velocity**: Final position divided by the time.

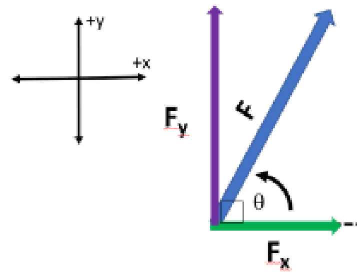
Check out the PhET simulation again, but now make use of the graphical functions. Create a position-time and velocity-time graph to review the above concepts.



The Moving Man

Unit 2: Kinematics in Two Dimensions

Perpendicular Components: When a vector makes an angle with one of the axes of a coordinate system, that vector is acting partly in each direction. A physics problem must be solved by analyzing one dimension at a time because perpendicular vectors are *independent* of each other. Take the diagram to the right. The force, ***F***, makes an anlge to the coordinate system so part of the force acts in the x-direction, $\boldsymbol{F_x}$, and the other in the y-direction, $\boldsymbol{F_y}$.



- Mathematically, they are calculated using right triangle trigonometry: $\boldsymbol{F_x} = \boldsymbol{F}\cos\theta$, and $\boldsymbol{F_y} = \boldsymbol{F}\sin\theta$. The sign of the component matches the direction of the coordinate system.
- If you know the perpendicular components, the vector can be calculated with the Pythagorean Theorem and the angle from the horizontal using $\tan^{-1} F_y/F_x$ (use only positive component values to calculate the angle).

Kinematics in 2D: These types of problems will read like previous problems, but the vectors could be angled to the coordinate system. To solve, calculate the perpendicular components for all vectors (watch for negative directions) and apply the physics relationship(s) in each dimension separately. If you are calculating a vector, the final answer's magnitude is determined with the Pythagorean Theorem and use the inverse tangent of the components to find the direction, or the angle.

Relative Velocity: When multiple vectors of the same type (like velocity) act on an object, the *resultant vector* (or resulting velocity) is determined through *vector addition*. Essentially, each vector is broken down into perpendicular components, and components of the same dimension are added together (taking direction into consideration). The resultant is calculated using Pythagorean Theorem and the direction with inverse tangent of the resultant's components. Some problems will give you the resultant and you need to calculate a missing vector. The process is the same, add up the vectors and one of the components will be unknown.

Vector Addition

Unit 3: Dynamics in One and Two Dimensions

Dynamics: The study of *why* objects move. When all the forces acting on an object add to be greater than zero, then there is a *net force*. This will cause the object to accelerate, after which we apply kinematic concepts for an analysis.

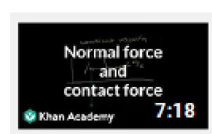
Force: ***F***, a vector quantity that is a push or a pull on an object. The unit of force is the Newton, N (pounds, or lb, in the United States). Many different forces can act on an object at the same time. The unit of a Newton breaks down into component units: $N = kg \cdot m/s^2$ (investigate forces using the PhET simulation to the right).

Forces and Motion: Basics

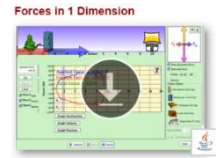
Types of Forces: Forces are grouped into two categories – contact and noncontact forces. Contact forces require direct physical contact between objects (such as friction, applied and tension forces). Noncontact forces exert their forces on objects over a distance and create *fields* (such as gravity, magnetic and electric forces).

Applied Force: $\boldsymbol{F_a}$, this is usually a force created by a person or machine acting on an object.

Normal Force: $\boldsymbol{F_N}$, a force that acts perpendicular to any surface. For example, the stool you sit on provides a normal force upwards, equal to the magnitude of gravity's pull on you, your weight. The concept of normal force is prominent in our investigations and understanding of the force of friction.



Force of Friction: $\boldsymbol{F_f}$, this *electromagnetic* force acts between surface atoms of two objects in direct contact. These bonds must be broken to move the objects. The greater the normal force the more the objects are pushed together, so the stronger the bonds. The force of friction always acts opposite the direction of motion of the object or, if it is not moving, the direction of the net force that acts on it. There are two types of frictional forces: static and kinetic.

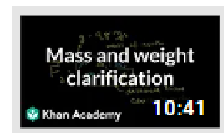


Static Friction: A frictional force that must be overcome to get an object moving. Think of pushing a massive object, it does not move unless you apply a large force. That is because the force of friction is very strong due to a high normal force.

Kinetic Friction: A frictional force that must be overcome to keep an object moving. Kinetic friction is lower than static friction for any two surfaces. The electromagnetic bonds have less time to strengthen as they continually form and break due to the object's motion.

Coefficient of Friction: μ. Think of this as the "*electromagnetic stickiness*" between any two surfaces. It is unique for any two objects, and for our course, it is independent of surface area or location. All that matters are the two objects. Each type of friction has its own coefficient value. The symbol is the Greek letter mu, μ.

Force of Gravity: $\boldsymbol{F_g}$, Aa attractive force that acts over a distance between masses because of their warping of spacetime. For situations on Earth, it is the pull-on objects towards the center of the Earth. It is often calculated by $\overrightarrow{F_g} = m\vec{g}$, where $\vec{g} = 9.81\, m/s^2$, the average acceleration due to gravity for the Earth. Another term for force of gravity is an object's *weight*.



Net Force: $\boldsymbol{F_{net}}$, is the vector sum of all the forces acting on an object. Vector sum means that only forces acting in the same *dimension* can be added together. For example, forces acting left, or right are not added with forces acting up or down – they are separate calculations. Objects accelerate in the direction of the net force.

Equilibrium: An object in equilibrium means that the net force acting on it is *zero*. This can happen if the object is at rest (a velocity of zero) or is moving with a constant velocity (an acceleration of zero).

Equilibrant: The vector, that when added, will result in the object achieving equilibrium.

Classical Mechanics: Developed in the late 1600s by Sir Isaac Newton. This system of physics treats matter and energy as separate entities, but it can predict the motion and interactions of objects. Such objects need to be much larger than an atom and travel much slower than the speed of light.

Quantum Mechanics: Developed in the early 1900s, spearheaded by Albert Einstein, it treats matter and energy as one and the same through the famous equation $E = mc^2$. This system of physics can model the motion and energy of subatomic particles and objects traveling near the speed of light, $c \approx 300\,000\,000$ m/s as well as all classical physical systems.

Newton's 1st Law: An object at rest or in uniform motion will remain at rest or in uniform motion unless acted on by an external force. An external force brings an object out of equilibrium. For example, when traveling in a car and it takes a sharp turn you get squished into the door or another passenger. What is happening is that you are trying to continue uniform motion, but the car gets in your way.

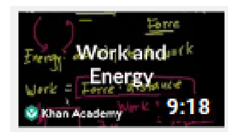
Newton's 2nd Law: The net (average) force acting on an object is equal to the product of the object's mass and (average) acceleration, ***F*** = *m**a***. This force has the same direction as the acceleration. This mathematically relates concepts of dynamics and kinematics.

Newton's 3rd Law: For every action there is an opposite and equal reaction. Forces act in pairs, called action-reaction pairs. Thus, if you punch a wall the wall "punches" back with the same force, possibly breaking your hand. Walking on the floor is possible because the floor pushes you forward (if you were to push the floor, the floor would move).

Inertial Frame of Reference: One in which Newton's Laws of motion are valid and can be applied. Every force can be explained. This frame of reference must be in equilibrium.

Non-Inertial Frame of Reference: One in which Newton's Laws of motion are not valid and cannot be applied. The frame of reference is not in equilibrium. In such a frame of reference, some aspects of an object's motion cannot be explained without the use of *fictitious forces*. For example, if a car is the frame of reference and it suddenly slows down, the passengers lunge forward. Relative to the car there was no push or pull to create such motion.

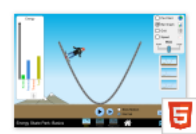
Unit 4: Conservation of Mechanical Energy



Work: In physics, work is done on an object when a force causes a displacement of the object. Work is a measure of energy and uses the unit called Joules, J. For reference, about 1 J of work is lifting an apple to the top of your head. The mathematical relationship is *W* = ***Fd***. Only a force parallel to the direction of motion does work on an object. Work is a scalar quantity, but it can be positive or negative. The sign of work is not determined by a coordinate system. If the force and displacement are in the same direction, that is positive work, otherwise, work is negative.

Conservative Force: Does work on an object in such a way that the amount of work done is independent of the path taken. For example, lifting an object 1 meter off the ground will be the same if you lift it straight up or use a ramp (in the absence of friction). A system of only conservative forces will have energy changes that are reversible.

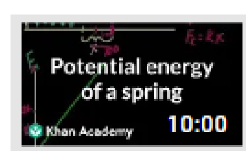
Nonconservative Force: The work done on an object is path dependent. Friction is such a force and removes energy from the system as heat. Such physics systems may not be reversible. In the natural world, most systems involve nonconservative forces. For example, when you bounce a ball, it will not return to its original height because energy is lost from the collision with the floor in the form of deformations, heat, and sound.


Energy Skate Park: Basics

Kinetic Energy: E_K, is the energy associated with motion. Represented by $E_K = \frac{1}{2}mv^2$, where *m* is the object's mass in *kg* and *v* is the instantaneous speed in *m/s*. Kinetic energy is measured in Joules, *J*.

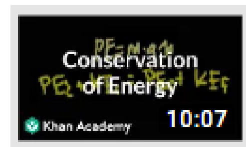
Potential Energy: Stored energy, the object has the *potential* to move. Such energy is associated with an object because of the position, shape, or condition of the object (e.g. pressing down on a spring or pulling a bow string stores energy because of a change of shape of the object).

Gravitational Potential Energy: E_g, is energy stored in a gravitational field above a reference, or zero level of a gravitational source. (i.e. the surface of the Earth, but it can be set for each system). Mathematically, $E_g = mgh$, where *g* is the acceleration due to gravity and *h* is the height above a zero level (use only positive values).



Elastic Potential Energy: E_e, the energy stored that depends on the distance an object has been compressed or stretched. Elastic materials can restore their shape by applying a *restoring force*. That ability is summarized numerically by a what is called a spring constant, *k*. Highly flexible materials have a low k-value, whereas stiff materials have high k-values. Mathematically, the stored energy is $E_e = \frac{1}{2}kx^2$ where *x* is the compression or stretch length in meters. If a restoring force calculation is required, *Hooke's Law* is applied to the object: $F_s = -kx$. The expression is negative because the restoring force always acts opposite to the compression or stretch direction (this assumes stretch direction is positive). If maximum acceleration is at position *x*, then max velocity is at *x*/2. At maximum acceleration *v* = 0 m/s, and at max velocity, *a* = 0 m/s^2.

Work-Energy Theorem: Work must be done on an object to change its position. This theorem states that the work done on an object equals its energy gained or lost. Each form of energy applies its own version of the work-energy theorem: $W = \Delta E_K$, $W = \Delta E_g$, or $W = \Delta E_e$ and remember that $W = \boldsymbol{Fd}$, should force or displacement be part of the analysis. The symbol Δ (capital Greek letter delta) means "change in" so it will be an object's "final" – "initial" energy of that type.

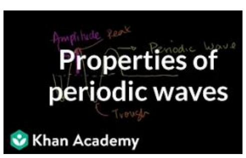


Law of Conservation of Energy: Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant. Common examples include dropping an object – initially the object has a speed of zero but gravitational potential energy. Once released the gravitational potential energy decreases but its speed increases – gravitational potential energy is transformed into kinetic energy. In the absence of air resistance, the total energy remains constant. Use the PhET simulation on the right to explore energy transformations.

Energy Skate Park: Basics

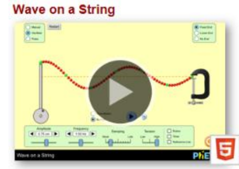
- If all the forces in the system are conservative, then mathematically $\Delta E_T = 0$ where E_T is the total of all types of energies and is a constant. That means the change in total energy is zero.
- If nonconservative forces exist then, mathematically $\Delta E_T = W_{nc}$, where W_{nc} is the work done by the nonconservative forces (for example, friction acting over a distance) and will evaluate to a negative value since that energy is removed from the system.

Unit 5: Wave Phenomena and Applications



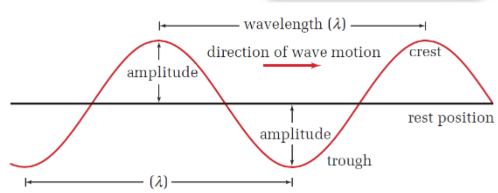
Wave: A disturbance that transfers energy through a medium. There are many types of waves and are generally classified as mechanical, electromagnetic, or gravity. The link left, *Properties of Waves*, is to a YouTube playlist by Kahn Academy. The playlist covers much of the topics that we will discuss in this unit.

Wave on a String

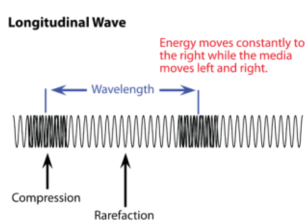


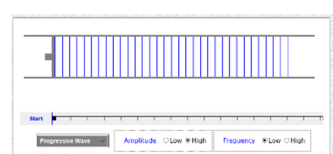
Mechanical Wave: Requires a physical medium to travel through. They are classified as either transversal or longitudinal.

- **Transverse wave**: One that vibrates particles of a medium perpendicular to the direction of wave travel.
- **Longitudinal wave**: One that vibrates particles of a medium parallel to the direction of wave travel.



Amplitude: The maximum displacement of a particle from its equilibrium, rest, position.





Wavelength, λ: The distance from a point on the wave to a corresponding point such that it includes one complete crest and trough.

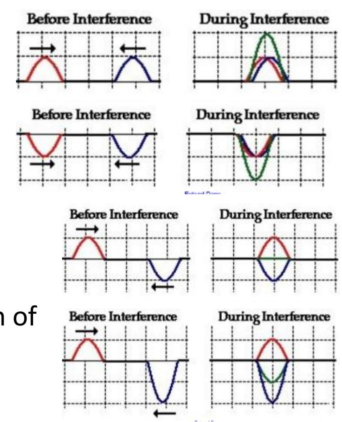
Frequency, *f*: The number of waves created per unit time. Measured in Hertz, Hz.

Period, *T*: The time to create one complete wave. Measured in seconds.

Medium: The material, or matter, a wave propagates though. Could be solid, liquid, gas or space.

Wave Speed, *v*: The speed of a wave is a constant for a particular medium. It is independent of the size or energy used to create it. It is a characteristic of that material, much like a boiling or freezing point. The speed is determined by the mass and forces between medium particles. Changing tension, for example, changes the force between particles, so it changes the medium – which changes wave properties. Those principles are prevalent in string instruments.

- **Effect of Force Between Particles (Tension)**: The greater the force between particles, the more rapidly the particles return to equilibrium position. This results in a faster wave. In other words, as the force of tension in a medium increase, wave speed increases and vice versa.
- **Effect of Particle Mass**: The greater the mass of the particles, the slower they return to equilibrium position because of their inertia (resistance to change in motion). That results in a slower wave speed. So, as the mass of a medium particle increases, the wave speed decreases and vice versa.
- **Effect of Friction**: Frictional forces within a medium act to dampen or reduce the amplitude of the wave but have no effect on wave speed (unlike sliding something on a table).

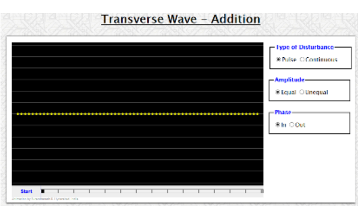


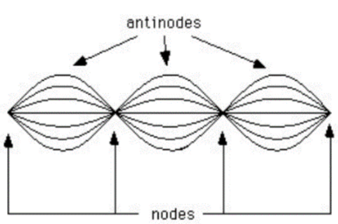
Wave Interference: When two or more waves act simultaneously on the same particles of a medium.

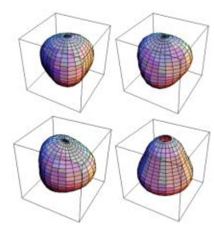
- **Constructive Interference**: Results when two or more waves interfere to produce a resultant displacement greater than the displacement caused by either wave itself.
- **Destructive Interference**: When the resultant displacement is smaller than the displacement that would be caused by one wave by itself.

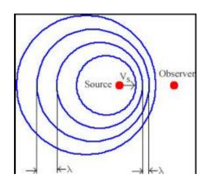
Principle of Superposition: The resultant displacement of a given particle is equal to the sum of the displacements that would have been produced by each wave acting independently.

Standing Waves: An interference pattern that occurs if interfering waves have the same amplitude, wavelength, frequency and are traveling in different directions. The pattern creates nodes and antinodes. Nodes (be it points or lines) are locations of total destructive interference. Can occur in one, two, or three dimensions.



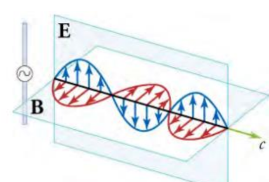


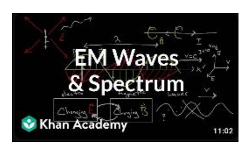




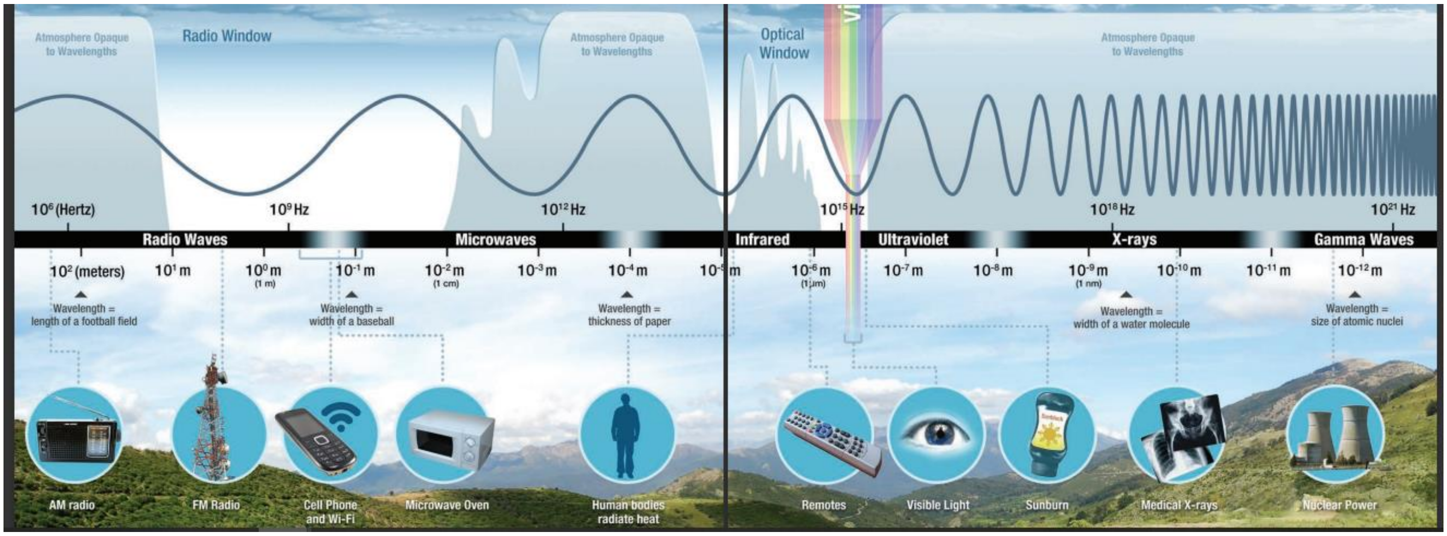
Doppler Shift: The apparent difference between the frequency at which sound (or electromagnetic radiation) waves leave a source and that at which they reach an observer, caused by the relative motion of the observer and the source.

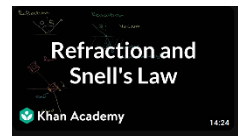
Electromagnetic Radiation (wave): Energy waves generated by a moving electric charge, that is, by an electric current. An electric current generates both an electric field, ***E***, and a magnetic field, ***B***. These fields are perpendicular to each other. When the moving charge oscillates, as in an alternating current, an EM wave is propagated.





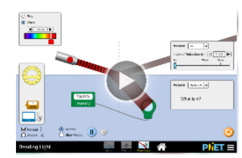
Electromagnetic Spectrum: Depending on the factors involved when an electric charge moves (acceleration), one of many different types of radiation can be produced. From lowest to highest energy, the EM spectrum encompasses radio, microwaves, infrared, visible light, ultraviolet, x-rays, and gamma radiation. All types of EM radiation travel the same speed, $c \approx 300\ 000\ 000$ m/s, in a perfect vacuum (outer space). Lowest energy EM waves have the largest wavelengths and lowest frequencies, whereas the highest energy EM waves have the smallest wavelengths and highest frequencies.





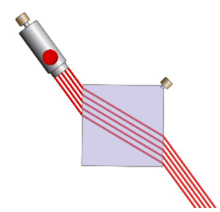
Refraction: The change in direction of a wave, in our case a light wave, as it enters a different medium.

Index of Refraction: The ratio of the speed of light in a perfect vacuum to that of the medium it is in. The number will always be greater or equal to 1 as it is a measure by what factor light slows down. It is given the variable, *n*.



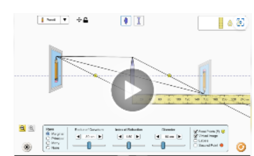
Snell's Law: The mathematical relationship between two media's indices of refraction, incident angle and refracted angle. Light entering a medium where it slows down will bend toward the normal, and vice-versa.

Critical Angle: For the case of light traveling into a medium where its speed increases. When the refracted angle calculates to 90°, that incident angle is called the critical angle. Any angle larger than the critical angle will result in *total internal reflection*. That is when the boundary acts like a mirror.

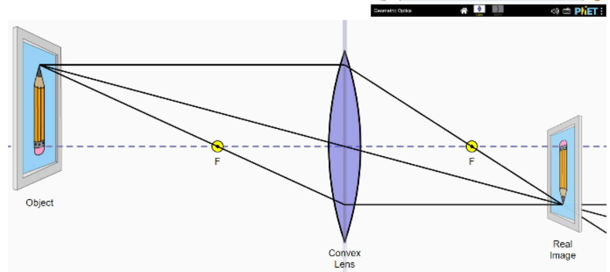


Lateral Displacement: The distance between a light ray's original path and its actual path when traveling through a refracting material.

Spherical Lense: A lens whose shape would be a section of a sphere. For this course, we assume the lenses are thin enough to ingnore the effects of lateral displacement. Lenses are used to focus, or redirect rays of light to create images. Images come in two formats, real or virtual.



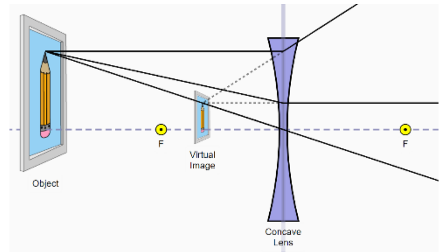
- **Real Image**: Forms in space so it can be projected to a screen, film, or retina.
- **Virtual Image**: *Appears* to form in space, but in fact it cannot be captured on film (from the location it appears to be). Magnifying glasses make use of virtual images to enlarge objects.



Convex Lens: This lens has been shaped so that all light rays that enter it parallel to its central axis cross one another at a single point on the opposite side of the lens. The central axis, or axis, is defined to be a line normal to the lens at its center. Such a lens is called a *converging lens* because of the converging effect it has on light rays.

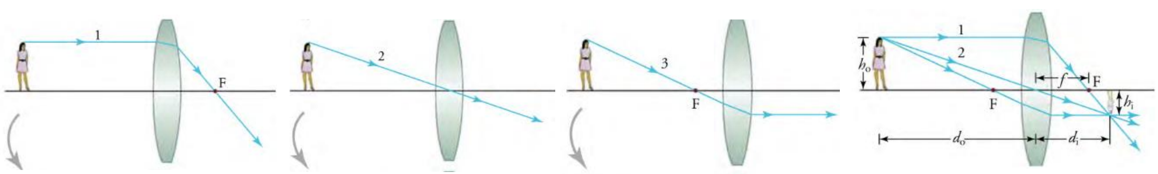
- The link to the right is a playlist from Kahn Academy that covers concepts and calculations involving thin convex and concave lenses and the thin lens equation. Note, we will not get into multiple lens systems, diopters, or aberration.

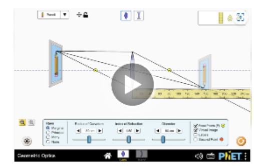
Concave Lens: This lens is a diverging lens because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length, or "ƒ," of the lens. Note that the focal length of a diverging lens is defined to be negative.



Ray Tracing: This is used to determine where an image, either real or virtual, will form. An image will form, or appear to form, at the location where refracted light rays meet, or appear to meet. In diagrams, we use the following rules:

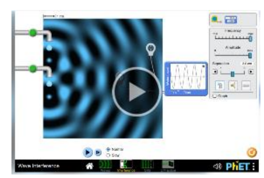
1. A ray entering a converging lens parallel to its optical axis passes through the focal point, F, of the lens on the other side.
2. A ray entering a diverging lens parallel to its optical axis seems to come from the focal point, F, on the side of the entering ray.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its optical axis.
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the optical axis.





Magnification: Determined by the ratio of image size to actual size of the object. It is negative for inverted images.

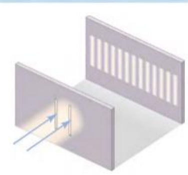
Diffraction: The bending of a wave around the edges of an opening or an obstacle. It is a wave characteristic that occurs for all types of waves. If diffraction is observed for a phenomenon, it is evidence that the phenomenon is produced by waves.



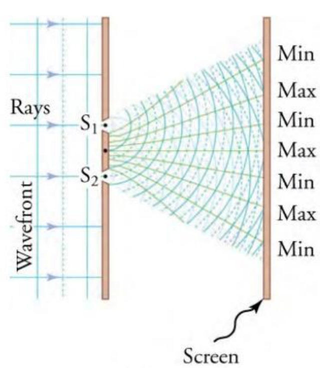
Wavefronts: The points on a wave's surface that share the same, constant phase (such as all the points that make up the crest of a water wave).

Huygens' Principle: "Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to each of the wavelets."

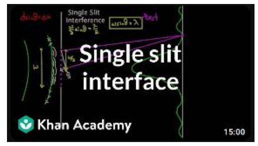
Young's Double Slit Experiment: Here, light of a single wavelength passes through a pair of vertical slits and produces a diffraction pattern on the screen—numerous vertical light and dark lines that are spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.



- When light passes through narrow slits, it is diffracted into semicircular waves. Pure constructive interference occurs where the waves line up crest to crest or trough to trough. Pure destructive interference occurs where they line up crest-to-trough. An analogous pattern for water waves is shown. Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. Those angles depend on wavelength and the distance between the slits.



- Check out a variety of videos from Kahn Academy to help with the concepts.

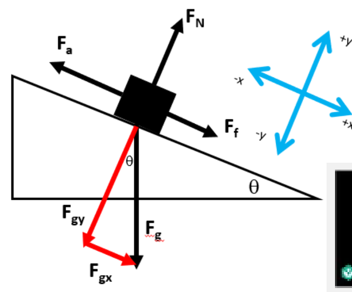


Unit 6: Extension of Kinematics & Dynamics in 2D

Projectile Motion: The only force acting on a launched object is gravity (we assume no air resistance).

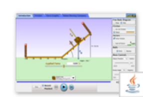
- When launched at an angle, the projectile's velocity will have a horizontal, x, and vertical, y, component. Solving these problems requires the application of kinematic equations in each dimension separately.
- Since gravity only affects the vertical direction, there are no forces, hence no acceleration, in the horizontal direction.
- Horozintal and vertical motion occur within the same time interval, *t*, so time is the link between the motion in each dimension.
- **Range**: The horizontal position where the projectile lands, d_{fx}. The maximum range occurs for angles of 45°, and if landing at the same vertical position as it was launched, complementary angles (angles that add to 90°) have the same range.
- At maximum height the projectile is changing directions so the instantaneous velocity is zero.

Forces on an Incline: If the object is on an incline or ramp the coordinate system is rotated the same degree as the angle the ramp is to the ground, then it is the force of gravity that is broken into components. The trigonometry yields the following based on the plane of the ramp being the x-axis:

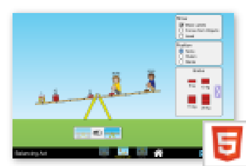


$F_{gx} = F_g sin\theta$
$F_{gy} = F_g cos\theta$

Connected Masses on a Ramp: Break the appropriate forces into components and then apply Newton's 2nd Law but now we include the sum of all the masses that accelerate. Picture the problem stretched out in one dimension to determine the direction of forces. Forces of tension are equal and opposite and cancel out when adding all forces. To find the force of tension, apply Newton's 2nd Law to one of the masses.


Ramp: Forces and Motion

Torque: Think of this as rotational power. When an object is made to rotate around a pivot point by a force acting on a lever arm (think prying something loose or a playground see-saw), that rotation is a result of a torque being applied. It is also the turning or twisting effectiveness of a force. The sign of a torque depends on which way a force would rotate an object. clockwise rotation is negative torque and counterclockwise is positive torque.


Balancing Act

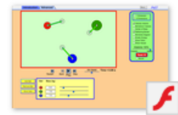
Net Toque: The sum of all the torques acting on an object, incorporating all the rotational directions.

Static Equilibrium: If a system is said to be in static equilibrium, then the net force and net torque are zero. Often, solving such problems requires the use of net torque first because there are too many unknown force variables. By making the pivot point at the location of an unknown force, the net torque analysis eliminates that unknown force from the mathematical equation, allowing for another force to be calculated.

Center of Mass: If able to rotate freely, an object rotates about its center of mass. Objects (like a beam) can be analyzed as if all its mass is contained at one point.

Linear Momentum: An object's tendency to continue on-course, that is, to continue to move in the same direction. Momentum is directly proportional to the object's speed and mass, so the greater the mass and speed, the higher the momentum. The direction of an object's momentum is the same as its velocity, so objects traveling in different directions will have opposite signs.

Elastic Collision: One in which objects separate after colliding. Kinetic energy is conserved.


Collision Lab

Inelastic Collision: One in which objects stick together after colliding. Kinetic energy is not conserved. Collision can be perfectly elastic, inelastic, or anything in-between.

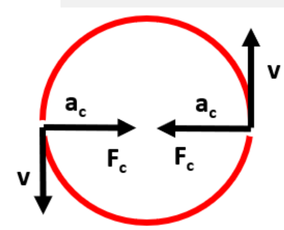
Conservation of Momentum: When two, or more, objects interact (collide) the total momenta before and after the collision is conserved. That means momentum cannot be lost or gained. Momentum is conserved in all dimensions and velocities must be broken up into perpendicular components to allow for the mathematical analysis of each dimension. If an object explodes, the momentum of all the individual pieces must add to zero, assuming the object was at rest before it exploded.

Unit 7: Circular Motion & Universal Gravitation

Uniform Circular Motion: Motion in a circular path at a constant speed, centered at one point. Note that it is not constant velocity because in a circle the direction continually changes. Because the direction continually changes, objects in circular motion are accelerating. The velocity as also called tangential velocity because it is always pointing at a tangent to the circular path.

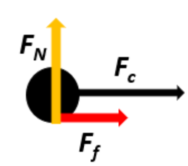
Radial Direction: When considering circular motion, any vector that is on a line that passes through the center of the circle. Vectors do not need to point towards the circle to be considered radial, just have to line up with the center.

Centripetal Acceleration: The acceleration of an object moving in a circle. It always points towards the center of the circle (with is the direction of the change in velocity) and is perpendicular to the object's velocity.

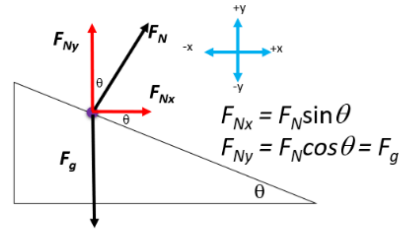


Centripetal Force: Any net force causing circular motion. It is directed towards the center of the circle. For an object to travel a circular path at a specific velocity, a certain amount of centripetal force is required. For example, the force of gravity provides the centripetal force for the Moon to travel about the Earth. Or swing a ball on a rope too fast and the rope could break because it was not designed to provide a centripetal force high enough. Travel around a corner too fast in a car and friction between the road and tires might not be high enough to maintain the circular motion and the car could leave the road.

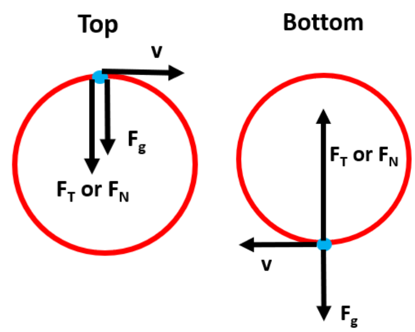
Unbanked Turn: When an object makes a circular turn on horizontal, level ground. The centripetal force for the turn comes from the force of static friction between the object and the ground. If it is a car, for example, it is the force of static friction between the tires and the road.



Banked Turn: When an object travels around a turn and that turn makes an angle with the horizontal, it is possible for the object to navigate the turn without relying on friction. Because it is banked, part of the normal force is directed towards the center of the circle, and this force provides the centripetal force necessary to safely make the turn. When friction is taken into consideration, it is possible to safely make the turn at a higher maximum speed. In that case, there is a component of friction that points towards the center of the circle and one that points down to the Earth (the magnitude of the force of friction points down the banked turn). This is why many highway exit ramps, and race track corners, are banked.



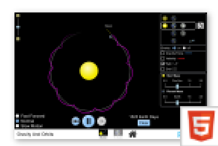
Vertical Circular Motion: This is considered when the object moves in a circle such that the force of gravity acts in the radial direction (so the object changes height above the Earth). The mathematics is similar, the difference being taking into account the force of gravity. The focus of interest will be the motion at the top and bottom of the circle. At the top, the object must have a minimum velocity to maintain the circular motion. At the bottom, the object would have a maximum velocity without breaking the string/rope (string tension is a maximum) or causing a person to suffer an injury (higher force on the body).



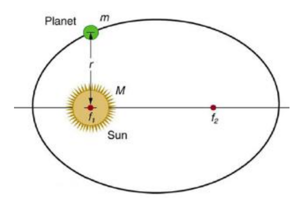
Universal Gravitation: Newton's work showed that the force of gravity is an attractive force proportional to the product of the two interacting masses and inversly proportional to the square of the distance between their centers. For this unit, assume all orbits are circular (the are actually ellipses) and the mass of the object in orbit is much, much lower compared to the mass of the other object (if the masses are comparable, then the object will orbit each other).

- **Orbital Velocity**: The tangential speed of the object about another object. It is perpendicular to the force of gravity.
- **Period**: The time to complete one full orbit.
- **Orbital Radius**: The distance between the centers of mass of each object.
- **Altitude**: The distance above the surface of the object in orbit.

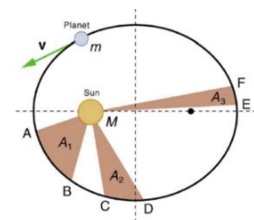
Gravity Force Lab: Basics


Gravity And Orbits

Kepler's 1st Law: The orbit of each planet about the Sun is an ellipse with the Sun at one focus. The Earth, for example, is closest to the Sun in January and farthest away in June. The difference of those distances for the Earth are small compared to the average distance between the Earth and Sun. Some objects, like comets, have very elliptical orbits so their surface temperatures vary greatly.



Kepler's 2nd Law: Each planet moves to that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.



Kepler's 3rd Law: For any system where objects orbit about another object that has a much greater mass, i.e., like the Moon or any satellites orbiting the Earth, the ratio of the squares of the periods for any two satellites is equal to the ratio of the cubes of their average distances from the central body. This comes out of applying the concepts and mathematics of circular motion and Newton's equations for the force of gravity.

Unit 8: Electromagnetic Forces & Fields

Electromagnetic Force: One of the four fundamental forces of nature. It consists of static electricity, moving electricity, and magnetism. All atomic and molecular interactions (like friction, chemical reactions, physical properties like boiling points) are manifestations of the electromagnetic force.

Electric Charges: Protons are positively charged particles and electrons are negatively charged. Objects can become charged by gaining or losing electrons. Protons are fixed within the nucleus of atoms and the atoms, nor protons, are free to move in materials.


Balloons and Static Electricity

Law of Charges: There are two types of charges, positive and negative. Opposite charges attract, like charges repel, and both positively and negatively charged objects attract neutral objects by *polarizing* the charges in neutral objects.

Fundamental Charge, q: q = 1.60 x 10^{-19} C, measured in the SI unit of the coulomb, C. This is the charge of one electron and one proton. Any charged object is an integral multiple of the fundamental charge. Usually, this charge is thought of as an excess or deficit of electrons.

Quarks: While electrons do not have a substructure, high energy particle collision experiments have revealed that protons and neutrons are made of even smaller particles called quarks. There have been six types of quarks discovered, two of which, **Up** and **Down** quarks, make up protons and neutrons. Unexpectically, quarks have been shown to have a partial electrical charge. Up quarks have a charge of $+\frac{2}{3}q$, and down quarks a charge of $-\frac{1}{3}q$.

- Proton = 2 Up + 1 Down = $+\frac{2}{3}q+\frac{2}{3}q-\frac{1}{3}q=+1q$
- Neutron = 1 UP + 2 Down = $+\frac{2}{3}q-\frac{1}{3}q-\frac{1}{3}q=0q$

Law of Conservation of Charge: In any process, the total charge is constant. That means charge cannot be destroyed or created when objects interact.

Antimatter: A particle that has the same properties as a "normal" particle except it has an opposite charge. When a particle and its antimatter counterpart collide, they annihilate one another releasing a burst of energy. The anti-electron, called the positron, behaves like an electron but has a positive charge. The anti-proton is the counterpart to the proton and is made of 2 up antiquarks and 1 down antiquark. Interestingly, one of physics' biggest questions is, "*where is all the antimatter in the universe?*" as particles and anti-particles are created at the same time.

Electrical Conductor: A material that allows for the movement of valence electrons. Electrons collide with atoms or molecules in common conductors, resulting in some energy loss. The amount of energy loss varies by material. Metals and salt water are common examples.

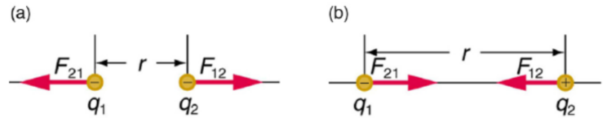
Superconductor: These allow for the movement of charge without any loss of electrical energy.

Electrical Insulator: A material that inhibits the movement of valence electrons. Some common examples are plastic, glass, rubber, and pure water.

Induction: This is a method of creating an electric charge on an object without direct contact. Devices that can charge batteries without contact were developed using this foundational concept (such devices also use *magnetic* induction).

Electric Polarization: When a neutral object is made to have two regions of charge, one positive and one negative. This is usually the result of a charged object being placed near the neutral object. This is the reason charged objects attract neutral objects.

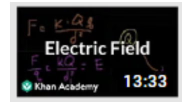

Balloons and Static Electricity



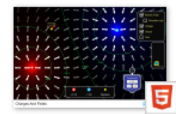
Coulomb's Law: A mathematical foundation for calculating the force of electric attraction or repulsion. Named after Charles Coulomb who spearheaded much of the research of static electricity. The electric force is a vector, and each charged object applies a force on all other charges. Newton's 3rd Law still holds, and a reference charge experiences a force equal and opposite to the force it provides on another charge. (video is electrostatics).

Field: A field is a way of visualizing and tracking a force that acts over a distance (non-contact forces like gravity, electric and magnetic forces). Fields are represented by lines and the more lines per unit area the greater the force acting on a reference, or test, object.

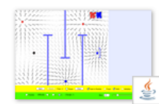
Electric Field: The region around a charged object, Q, that affects the motion of a positive test charge. Mathematically, its strength is the ratio of the Coulomb force to the positive test charge.



Electric Field Lines: Used to visually represent the electric field strength on a positive test charge from a charge, Q. The greater the field strength the more lines per unit area and arrows are used to indicate the direction of the electric force. Since the test charge is always positive, arrows always point toward negative Q-charges and away from positive Q-charges. The five electric field line properties (rules) are summarized below:


Charges and Fields

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.

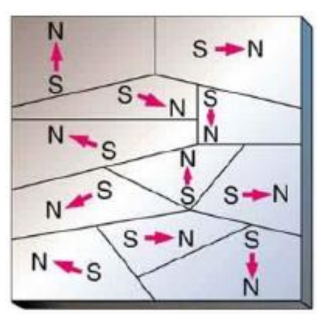

Electric Field Hockey

3. The strength of the field is proportional to the closeness of the field lines, i.e. it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is a tangent to the field line at any point in space.
5. Field lines can never cross. The field is unique at any point, so a test charge does not have a "choice" on how to move.

Electric Current: The rate at which a charge moves, measured in amperes (A, which is C/s).

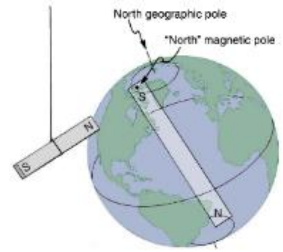
Conventional Current: The direction that a positive charge would flow. In wires, made of metal, positive charges do not move, and it is the negative electrons that flow. Lightning is another example of a discharge of electrons. However, positive ions can move in fluids and biological systems, so it is not only electrons that can create a current.

Magnetism: The result of moving charged particles. Within a solid magnet, this is the result of electron motions within the atom. When electrons are forced through a conducting wire, that motion creates a magnetic field. A magnet is created in a material when its magnetic domains all line up.



Magnetic Domains: Small regions within a materal where there has been a polarization of charge. The existance of these regions do not make a material a magnet, but if all of these regions allign, or are made to allign, then a magnet is created. The poles of a magnet line up with the poles of the magnetic domains and are named north and south poles.

Electromagnetic Force: The name given to all the forces that result from the motion of charged particles.



Magnetic Poles: The opposite ends of a magnet (or magnetic domains) and always occur in pairs. Slicing a magnet into smaller and smaller pieces creates smaller magnets with a north and south pole. Like poles repel and opposite poles attract. The Earth's core produces a large magnetic field with poles near the geographical north and south poles. Note that the geographical north pole and the magnetic south pole. The poles are defined based on what direction the north end of a compass points.

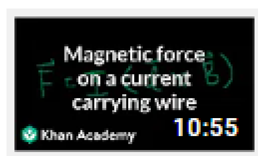
Electromagnet: Created by wrapping an electrically conductive wire around a metal (ferromagnetic) material. A current through the wire will produce a magnetic field and temporarily magnetize the metallic core. Within the core, there is an internal alignment of electron spins. The current creates a magnetic dipole.

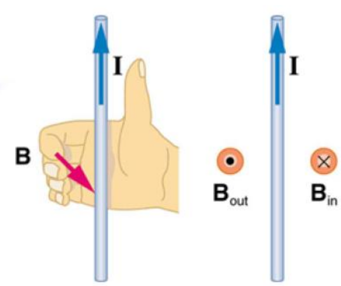
Magnets and Electromagnets

Magnetic Field Lines: A visual representation of the magnetic field strength, the *B-field*. The direction of the field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field strength is represented by the variable ***B*** and is measured in units of Tesla, *T*, or Gauss, *G*. The four rules for magnetic fields are:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line (so to the south pole).
2. The strength of the field is proportional to the closeness of the line. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without a beginning or end. The go from north pole to the south pole. (Note this is different from electric field lines that begin at a positive charge and end at a negative charge.)

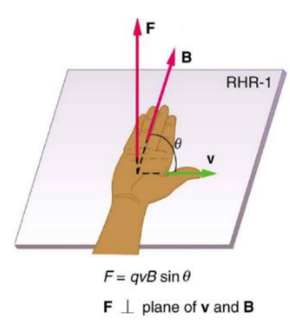
Right Hand Rule 2 (RHR-2): The direction of the magnetic field generated by a current is perpendicular to the direction of the current. The RHR-2 is a way to help remember and visualize the magnetic field (it is called RHR-2 because there is another right-hand-rule for a charge moving through a magnetic field). Picture the wire being held in your right-hand fingers with your thumb pointing in the direction of the current. Your fingers curl in the direction of the magnetic field, ***B***. Note the symbols used to represent the direction of the magnetic field for a current in a wire.





Lorentz Force: This is the name of the force applied on a moving charge from a magnetic field. The force is directly proportional to the charge on the moving object, its velocity and the magnetic field strength. The Lorentz force is perpendicular to the magnetic field and the force on a negative charge is in the exact opposite direction to that on a positive charge.

Right Hand Rule 1 (RHR-1): To determine the direction of the magnetic force on a positive moving charge, you point your thumb of the right hand in the direction of ***v***, the fingers in the direction of ***B***, and a perpendicular to the palm points in the direction of ***F***. There are no magnetic forces on a static (stationary) charge, only moving ones. Electric fields do not affect magnetic fields unless the charge is moving.



Outcome 5.1: Mechanical Waves

Wave Terminology and Properties

1. What does a wave transfer through a medium?

2. Sketch a diagram of a transverse wave and label a crest, trough, rest position, amplitude, and wavelength.

3. Why are longitudinal waves also called pressure waves?

4. How are longitudinal waves used to study the internal structure of the Earth and the Sun?

5. Suppose everyone in the class is talking at once, and at different frequencies, what do all the sound waves have in common?

6. Summarize the effect the force between particles (tension) and particle mass has on wave speed through a medium.

7. What effect does friction have on waves?

Frequency, Period, and the Wave Equation

8. As a wave travels from one material into a different material, what property of the wave will not change?

9. A car tire completes 475 rotations in 12.5 seconds. Calculate the frequency and period of the spinning tire. {T = 0.026 s, f = 38 Hz}

10. On a beach a person watches 75 waves pass by a point in 182 seconds. Calculate the frequency and period of the waves. {f = 0.41 Hz, T = 2.4 s}

11. The period of a water wave is 2.5 seconds.
    a. Calculate the frequency of the waves. {f = 0.4 Hz}
    b. Calculate the length of time for 150 waves to pass by you. {375 seconds}

12. For a particular medium, if the wave frequency increases, how do the period and wavelength change?

13. A wave is created with a frequency of 300 Hz. It has a speed of 1200 m/s.
    a. Calculate the period of the wave. {T = 0.003 s}
    b. Calculate the wavelength of the wave. {λ = 4.0 m}

14. Calculate the speed of waves with a wavelength of 0.78 m and a frequency of 12500 Hz. {v = 9750 m/s}

15. Waves with a wavelength of 0.46 m and a speed of 1465 m/s enter another medium. The speed in the different medium is 2140 m/s. Calculate the wavelength in the different medium. {λ =0.67 m}

16. Radio waves travel at 3.00×10^8 m/s. Calculate the wavelength of radio waves emitted by a radio station operating at 103.1 MHz. {λ = 2.9 m}

17. Two fishing boats are in the water. At a time when one boat is on a crest the other is in a trough and there are three crests between them. The boats are 125 m apart, and the wave period was measured to be 1.75 seconds. Calculate the speed of the waves. {v = 20 m/s}

18. Two boats are 115 m apart. When each boat is on a crest and there are 4 crests between them. A boat goes up-and-down and up again 20 times in 75 seconds. Calculate the speed of the waves. {v = 6.13 m/s}

Doppler Shift

19. An object moves towards a stationary observer.
    a. Will the observed frequency be higher or lower than the source frequency?
    b. What about after the source has moved past the observer?
    c. How did the observed wavelength change for the case of approaching and receding?

20. What causes a sonic boom?

21. Calculate the observed frequency of a 6500 Hz source moving 95 m/s in each case below. Take the speed of sound as 325 m/s.
    a. Towards a stationary observer. {f_{obs} = 9185 Hz}
    b. Away from a stationary observer. {f_{obs} = 5030 Hz}

22. A fire truck emits a frequency of 7850 Hz and travels 22.4 m/s. A stationary observer is on the same road. The speed of sound is 346 m/s. Calculate the frequency heard by the observer if the vehicles are:
    a. Approaching {f_{obs} = 8383 Hz}
    b. Receding. {f_{obs} = 7373 Hz}

23. Someone standing on the road hears the frequency of a police car to be 17 700 Hz. The police car is driving 38.6 m/s towards the observer. The speed of sound is 334 m/s.
    a. Calculate the frequency of the siren. {f_s = 15 654 Hz}
    b. Calculate the observed siren frequency when the police car is moving receding. {f_{obs} = 14 033 Hz}
    c. Calculate the wavelength of sound when the car is receding. {λ_{obs} = 0.024 m}

24. The speed of sound in the air is 350 m/s. A plane approaches a stationary observer. The emitted frequency of the plane is 13 500 Hz, the observed frequency is 21 000 Hz.
    a. Calculate the observed wavelength of the sound. {λ_{obs} = 0.017 m}
    b. Calculate the speed of the plane. {v_s = 125 m/s}
    c. Calculate the observed frequency once the plane has passed the observer. {f_{obs} = 9947 Hz}
    d. Calculate the wavelength of sound when the plane is receding. {λ_{obs} = 0.035 m}

25. A plane traveling 112 m/s emits a frequency of 14 600 Hz and approaches a stationary observer. The observed frequency is 20 817 Hz. Calculate the speed of sound. {v_w = 375 m/s}

26. A stationary observer at an air show watches a fighter plane approach at one-third the speed of sound. The plane emits a sound frequency of 7500 Hz. Calculate the observed frequency by the person. {f_{obs} = 11 250 Hz}

Wave Reflection, Interference, and Standing Waves

27. Describe the properties of a reflected wave if initially it had undergone:
    a. Free-end reflection.
    b. Fixed-end reflection.

28. What is the principle of superposition?

29. Define constructive and destructive interference.

30. Define a standing wave and provide an example of such a wave in 1, 2 and 3 dimensions.

31. Define nodes and antinodes.

32. A standing wave pattern in a 0.86 m long string has 5 total nodes including a node at each end. Waves are made with a frequency of 1245 Hz. Calculate the speed of the waves. {v = 535 m/s}

33. A standing wave pattern in a 1.64 m long string has 8 total nodes including a node at each end. The wave speed in the string is 425 m/s, calculate the frequency of the waves. {f = 907 Hz}

Expert Level Questions

34. A stationary observer hears a frequency of 16 750 Hz from an approaching speeding race car. Once receding, the observed frequency is 12 360 Hz. The speed of sound is 342 m/s, calculate the speed of the car. {v_s = 51.6 m/s}

35. What fraction of the speed of sound must a source be moving so that a stationary observer hears a frequency four times greater than the source? {$v_s = \frac{3}{4}v_w$}

36. What fraction of the speed of sound must a source be moving so that a stationary observer hears a frequency two-thirds that of the source? {$v = \frac{1}{2}v_w$}

37. A 1.5 m long string is secured at both ends. A frequency of 250 Hz results in a standing wave with one antinode.
    a. Calculate the wavelength if the frequency is increased to 1500 Hz. {λ = 0.5 m}
    b. How many antinodes would appear in the string? {6 antinodes}

Outcome 5.2: Electromagnetic Spectrum and Refraction

Short Answer

1. What creates an electromagnetic wave?

2. What two types of fields make an electromagnetic wave?

3. List the electromagnetic spectrum from lowest to highest energy, highest to lowest frequency, and shortest to longest wavelength.

4. Define refraction.

5. Is it possible for a light to enter water from the air and not change direction? Provide a brief explanation.

Index of Refraction

6. What is the index of refraction a measurement of?

7. Calculate the speed of light in flint glass. {$v = 1.82 \times 10^8$ m/s}

8. Calculate the speed of light in ruby. {$v = 1.95 \times 10^8$ m/s}

9. Calculate the index of refraction for a material in which light travels 2.75×10^8 m/s. {$n = 1.09$}

10. Calculate the index of refraction for a material in which light travels 1.21×10^8 m/s. {$n = 2.48$}

11. How many times faster does light travel in glycerin than in zircon? {1.31 times faster}

12. By how much does the speed of light decrease in diamond when light enters diamond from water? {1.01×10^8 m/s}

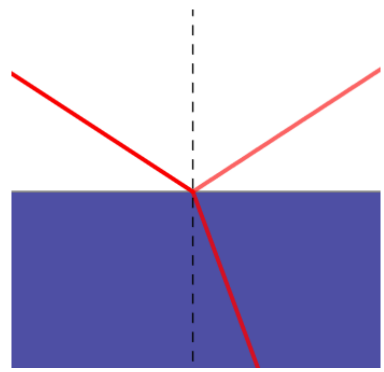
13. Calculate the wavelength of yellow light in ethyl alcohol if $f = 7.05 \times 10^{14}$ Hz. {$\lambda = 3.12 \times 10^{-7}$ m or 312 nm}

14. In the air, yellow light has a wavelength of 589 nm. Calculate its wavelength in sodium chloride. {$\lambda = 382$ nm}

15. Calculate the wavelength of yellow light in diamond, if its wavelength in a perfect vacuum is 589 nm. {$\lambda = 243$ nm}

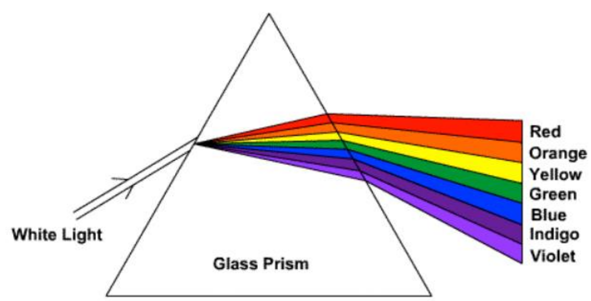
Snell's Law

16. In the diagram to the right, label each of the following if the top part is the first medium:
    a. Refracted ray
    b. Angle of incidence
    c. Reflected ray
    d. Normal
    e. Incident ray
    f. Boundary
    g. Angle of refraction
    h. Angle of reflection



17. An angle of refraction in a medium is measured to be greater than the angle of incidence. Knowing that, compare the indexes of refraction and the speed of light between the two media.

18. Do all the colors of light travel at the same speed in glass? If not, which color has the highest/lowest index of refraction?



19. Light travels from air into flint glass with an angle of incidence of 25°. Calculate the angle of refraction. {$\theta_2 = 15°$}

20. Light is traveling from an unknown medium into diamond. The angle of incidence is 42° and the angle of refraction is 28°. Calculate the index of refraction of the unknown medium. {$n_? = 1.7$}

21. Light travels from Plexiglas into ruby. Calculate the angle of incidence if the angle of refraction is measured to be 16.8°. {$\theta_1 = 17.1°$}

22. Light travels from water into an unknown material. The angle of incidence is 72° and the angle of refraction is 51°. Calculate the speed of light in the unknown material. {$v = 1.84 \times 10^8$ m/s}

23. The speed of light in a certain material is 9.68×10^7 m/s. If light enters that material from crown glass with an angle of incidence of 33.5°, calculate the angle of refraction. {$\theta_2 = 15.7°$}

24. Calculate the speed of light in a material that is measured to have an angle of refraction of 38°. The angle of incidence, in water, was 46°. {$v = 1.93 \times 10^8$ m/s}

25. Calculate the angle of refraction for light traveling from carbon disulfide into ice if the angle of incidence is 75°.

Maximum Refracted Angle and Critical Angle

26. Explain what the critical angle is and when it can occur.

27. Light is traveling from air into glycerin, calculate the largest possible angle of refraction. {$\theta_{max} = 43°$}

28. Light is traveling from water into crown glass, calculate the largest possible angle of refraction. {$\theta_{max} = 61°$}

29. Calculate the critical angle for flint glass into air. {$\theta_c = 37°$}

30. Calculate the critical angle for diamond into water. {$\theta_c = 33°$}

31. Calculate the critical angle for calcium chloride into glycerin. {$\theta_c = 73°$}

Applications of Refraction

32. What causes lateral displacement of light traveling through a rectangular piece of glass?

33. Why does an object in a liquid, like a spoon in water, look bent or distorted?

34. How does refraction impact a sunrise and sunset?

35. Explain, in general, why light doesn't leave a fiber optic cable until it reaches the end of the cable.

36. Why is the Moon red during a lunar eclipse? Why does it not appear to take another color?

37. How did the human eye evolve to make use of refraction? How are corrective lenses implemented?

38. Why does a rear-view mirror show a dimmer image of what is behind a car when the mirror is tilted down?

39. How does a mirage of a puddle appear to be on the ground a large distance in front of you when walking or driving?

Expert Level Questions

40. Light travels through a 0.10 m wide rectangular block of flint glass from air. It enters with an angle of incidence of 30°. Calculate the lateral displacement of the emerging light ray. {d = 0.023 m}

41. A 0.17 m rectangular block of diamond laterally displaces light as it enters and exits the diamond from air. The angle of incidence is 60°. Calculate the lateral displacement of the emerging light ray. {d = 0.010 m}

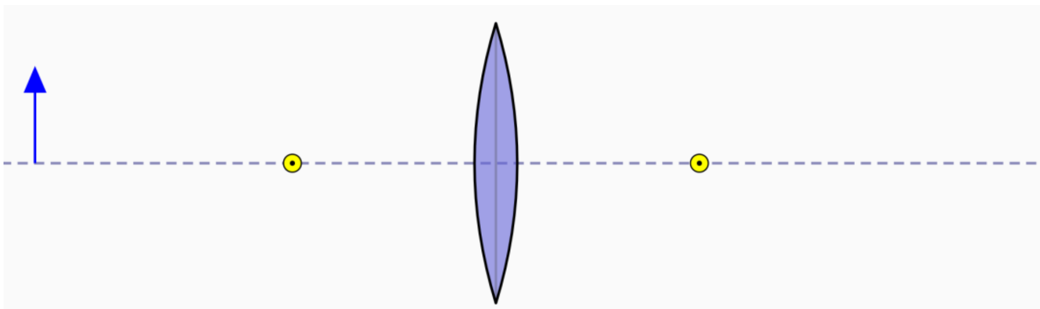
Outcome 5.3 Spherical Lenses

1. Explain real and virtual images.

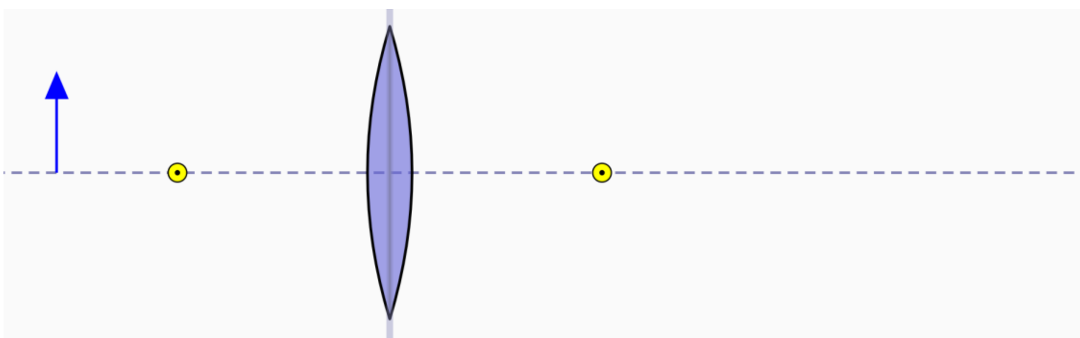
2. Define focal point and focal length.

3. Use a ruler and pencil to draw the two (of the three) principal rays to find the location of the image given all of the lenses are converging.

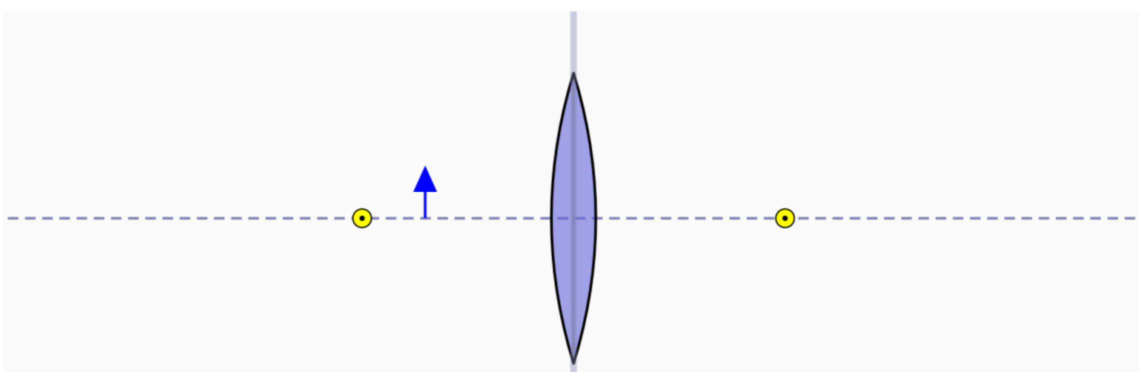
a.

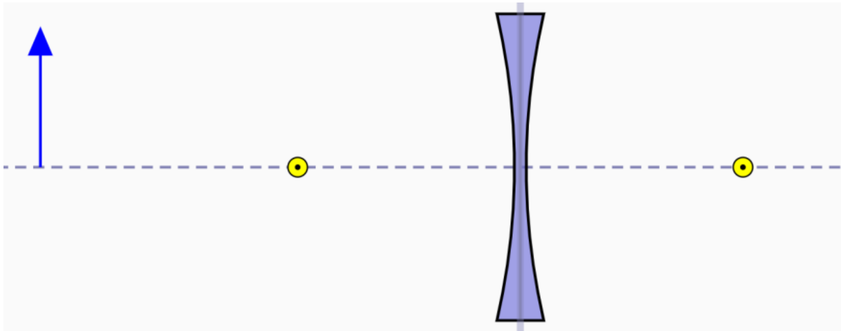
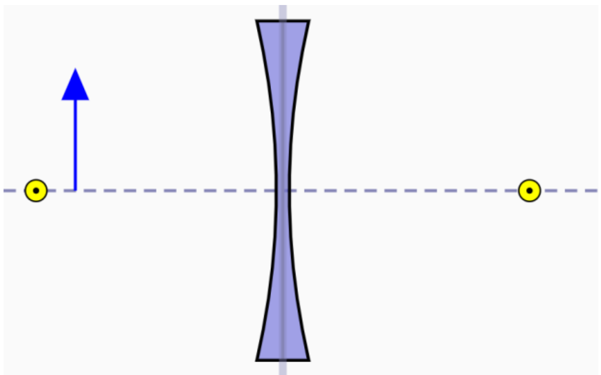


b.



c.



4. Use a ruler and pencil to draw the two (of the three) principal rays to find the location of the image given all the lenses are diverging.
   a.
   
   b.
   

Mathematical Problems

5. A converging lens has a focal length of 20 cm. Calculate the image distance if the object is placed 50 cm from the lens. {d_i = 33 cm}

6. An object is 8.0 cm high and placed 80 cm in front of a converging lens that has a focal length of 25 cm.
   a. Calculate the distance to the image. {d_i = 36 cm}
   b. Calculate the height of the image. {h_i = -3.6 cm}

7. A 10 cm high pencil is placed 60 cm in front of a diverging lens with a focal length of –20 cm.
   a. Calculate the distance to the image of the pencil. {d_i = –15 cm}
   b. Calculate the height of the pencil's image. {h_i = 2.5 cm}

8. A camera has a converging lens with a focal length of 5.0 cm. A 25 cm high candle is placed 100 cm from the lens.
   a. Calculate the distance to the image of the candle. {d_i = 5.3 cm}
   b. Calculate the height of the candle's image. {h_i = –1.3 cm}
   c. Calculate the magnification. {m = –0.052}

9. A lens with a focal length of 15 cm is 42 cm from a screen.
   a. Calculate the location of the object to form an image on the screen. {d_o = 23 cm}
   b. Calculate the magnification. {m = –1.83}

10. A diverging lens has a focal length of –21 cm. An image is formed at –8.7 cm from the lens.
    a. Calculate the location of the object. {d_o = 15 cm}
    b. Calculate the magnification. {m = 0.58}

11. An object is to be 35 cm from a lens must form a real image 56 cm from the lens.
    a. Calculate the focal length required. {f = 21.5 cm}
    b. Calculate the magnification. {m = –1.6}

12. A child wants to magnify a flower by a factor of 15. The magnification glass has a focal length of 7.5 cm.
    a. Calculate the object and image distance to get that magnification if the image is *virtual*. {d_o = 7.0 cm, di = –105 cm}
    b. Calculate the object and image distance to get that magnification if the image is *real*. {d_o = 8.0 cm, di = 120 cm}

13. An ant is to be magnified by a factor of 37. The converging lens used has a focal length of 25 cm.
    a. Calculate the object and image distance to get that magnification if the image is *virtual*. {d_o = 24.3 cm, di = –900 cm}
    b. Calculate the object and image distance to get that magnification if the image is *real*. {d_o = 25.7 cm, di = 950 cm}

Lens Maker's Equation

14. Calculate the focal length of a lens with index of refraction (n_{lens}) equal to 1.6 which has a radius of curvature of 6.0 cm on one side and 12 cm on the other side. Both sides are convex and take n_o = 1. {f = 6.7 cm}

15. A crystal glass lens is plano-concave in shape with a radius of curvature of 12 cm. Calculate its focal length taking n_o = 1. {f = –22 cm}

16. A meniscus lens has a concave radius of 25 cm and a convex radius of 18 cm. Calculate the focal length if the lens is made of flint glass and take n_o = 1. {f = 99 cm}

17. A double convex lens crafted from ruby has radii of curvatures of 11 cm.
    a. Calculate the focal length in air. {f = 10.2 cm}
    b. Calculate the focal length in water. {f = 34.8 cm}

18. A converging meniscus flint glass lens has a focal length of 26.5 cm. Calculate its convex radius if its concave radius is 8.0 cm. {R = 5.5 cm}

19. Calculate the radii of curvature for a double convex lens made of Plexiglas that has a focal length of 30 cm. {R = 30.6 cm}

20. Calculate the index of refraction of a diverging meniscus lens of f = –15.5 cm. It has a concave radius of 5.0 cm and a convex radius of 12 cm. Take n_o = 1. {n_{lens} = 1.55}

21. A plano-concave flint glass lens has a focal length of –16 cm in a perfect vacuum. Material of what index of refraction should the lens be placed into to have a focal length of +16 cm? {n_o = 4.71}

22. An object 2.0 cm high is placed 8.0 cm from a double convex lens made of quartz. The radii of curvature of the lens are 20 cm and 5 cm.
    a. Calculate the location of the image. {d_i = –100 cm}
    b. Calculate the height of the image. {h_i = 25 cm}

23. In air, a fused quartz lens has a focal length of +17 cm. Calculate the focal length if it is submerged in a liquid with n = 1.66. {f = –65 cm}

24. Submerged in a liquid with n = 1.99, a Plexiglas lens has a focal length of +33 cm. Calculate its focal length in air. {f = –15.6 cm}

Expert Level Questions

25. A candle is placed 36 cm from a screen. Calculate where between the candle and the screen should a converging lens with a focal length of 8.0 cm be placed to have a focused image on the screen. {d_o = 24 cm or d_o = 12 cm}

26. An object is to be enlarged on a screen. The distance between the object and screen is 50 cm and the focal length of the converging lens is 10 cm. Calculate the location of the lens from the object to create the enlarged, focused, image. {d_o = 13.8 cm}

27. A person has two identically shaped convex lenses, one is made of Plexiglas and the other is made of zircon. Which lens has the greater focal length, and by what factor? {Plexiglas by a factor $f_{plex} = 1.8f_{zircon}$}

28. A quartz lens is converging in a perfect vacuum. By what factor does its focal length change when submerged in carbon disulfide? {$f_{new} = –4.36f_{old}$}

29. Given the focal length, f, of a lens and the desired magnification, m, derive the following formula for object distance: $d_o = \frac{f(m-1)}{m}$

Outcome 6.1: Qualitative and quantitative analysis of a projectile on earth.

1. Neglecting air friction, how does increasing the mass of a projectile affect its maximum height and range if no other initial conditions are changed?

2. What is the only force acting on a projectile?

3. What is the acceleration of a projectile in the vertical (y) and horizontal (x) directions?

4. Mars has an acceleration due to gravity that is about 1/3rd that of Earth. How would the trajectory on Mars compare to that on Earth for the same initial velocity and neglecting air resistance? Support your answer with an explanation.

Projectile Launched Horizontally

5. A marble rolls of a 1.8 m high table and lands 3.2 m from the base of the table. Calculate the velocity with which the marble left the table. {v_x = 5.3 m/s}

6. An airplane is traveling parallel to the ground at 55 m/s and an altitude of 925 m. The pilot is to drop a supply crate, from rest, into a target area.
    a. At what horizontal distance before the target drop zone, should the pilot release the crate? {756 m}
    b. Calculate the vertical velocity with which the create strikes the target. {v_{fy} = 134 m/s}
    c. Calculate the velocity with which the crate strikes the target. {v_f = 146 m/s 68° down from the horizontal}

7. A ball is thrown horizontally at 18 m/s from a location 12 m above the ground.
    a. Calculate the time for the ball to hit the ground. {t = 1.6 s}
    b. Calculate the horizontal travel distance in that time. {d_{fx} = 28 m}

Projectile Launched at an Angle

8. An archer shoots an arrow at 159 m/s at an angle of 10° up from the horizontal. The arrow hits a target located at the same height it was fired (so d_{oy} and d_{fy} are zero)
    a. Calculate the time it takes the arrow to land. {t = 5.6 s}
    b. Calculate the maximum height reached by the arrow. {d_{fy} = 39 m}
    c. Calculate the range to the target. {d_{fx} = 877 m}

9. A ball is kicked from the ground with a velocity of 16.0 m/s, 35° up from the horizontal.
    a. Calculate the time to reach maximum height. {t = 0.935 s}
    b. Calculate the maximum height. {d_{fy} = 4.3 m}
    c. Calculate the time to land back on the ground. {t = 1.87 s}
    d. Calculate the range. {d_{fx} = 24.5 m}
    e. Calculate the velocity 1.2 seconds after the kick. {v_f = 13.3 m/s, 11° down from the horizontal}

10. A ball is thrown from a height of 2.1 m. 2.5 seconds later, the ball is caught at a height of 0.75 m, 15 m horizontally from where it was thrown.
    a. Calculate the initial velocity of the ball. {v_o = 13.2 m/s, 63° up from the horizontal}
    b. Calculate the maximum height of the ball. {d_{fy} = 9.1 m}

11. A rock is thrown from a cliff 51 m high with an initial velocity of 37 m/s, 35° up from the horizontal.
    a) Calculate the time to reach maximum height. {t = 2.16 s}
    b) Calculate the maximum height. {dfy = 74 m}
    c) Calculate the vertical velocity when the rock is 39 m above the ground. {vfy = –26.2 m/s}
    d) Calculate the velocity that the rock strikes the ground. {vf = 48.7 m/s, 52° down from the horizontal}
    e) Calculate the time for the rock to be located 23 m from the base of the cliff. {t = 0.76 s}
    f) Calculate the time for the rock to be 42 m above the ground. {t = 4.7 s}

12. A ball is thrown to a location (d_{fx}, d_{fy}) = (42 m, 3.5 m) from position (0, 0). The launch angle was 55° up from the horizontal.
    a. Calculate the time to reach that position. {t = 3.4 s}
    b. Calculate the launch velocity. {v_o = 21.5 m/s}

13. A cannonball is fired at 165 m/s, 57° up from the horizontal. Calculate the time for the cannonball to be 575 m above the ground. {t = 5.1 s and 23 s}

14. A person is atop a 75 m cliff and throws a rock 20° *down* from the horizontal. The rock is to land on the ground, 45 m from the base of the cliff.
    a. Calculate the launch velocity necessary. {v_o = 13.8 m/s}
    b. Calculate the time to reach the final location. {t = 3.46 s}

15. A projectile fired at an angle strikes a target 8.42 seconds later. The range was 1260 m and the projectile landed at the same height it was launched (so d_{oy} and d_{fy} are zero). Calculate the launch velocity of the projectile. {v = 155 m/s 15° up from horizontal}

16. A ball is thrown from the top of a building towards the wall of a second building 15.2 m away. The initial velocity 6.5 m/s at an angle of 20° down from the horizontal. How far below its original position did the ball hit the other building? {d_f = 35.9 m below launch height}

17. A hockey puck is hit at a 45° angle and lands 35 m away. Calculate the launch velocity of the puck. {v_o = 19 m/s}

18. A baseball is hit and just clears the fence that is 3.5 m high and 110 m from the position where the ball was hit. Calculate the initial speed of the ball if the launch angle was 52° above the horizontal. {v = 33 m/s}

Expert Level Questions

19. Calculate the angle to hit a golf ball so that it will land in the hole 156 m away. The ball is hit with an initial velocity of 42 m/s and the hole is the same height as the golf ball (so d_{oy} and d_{fy} are zero). {θ = 30° or 60°}

20. Calculate the angle(s) to kick at ball with 22 m/s of speed, such that it lands 35 m away. {θ = 23° or 67°}

21. Calculate the angle(s) to shoot a cannonball with 120 m/s, such that it lands 1250 m away. {θ = 29° or 61°}

22. A cannon is located on a cliff 51 m above the ground and launches a cannonball at 82 m/s. Calculate the angle(s) necessary to hit a target on the ground located 700 m from the base of the cliff. {θ = 33° or 53°}

23. An arrow is to hit a target located at (d_{fx}, d_{fy}) = (72 m, 23 m) from initial position (0, 0). The launch speed is 65 m/s. Calculate the launch angle(s). {θ = 23° or 85°}

24. You need to launch a T-shirt to a fan located 4.8 m above you and a horizontal distance from you of 12 m. Your shirt cannon fires at 15 m/s. Calculate the angle up from the horizontal to shoot the T-shirt towards the lucky fan. {θ = 40° or 71°}

25. Show that when using the quadratic formula to solve $d_f = d_o + v_o t + ½at^2$ for time, t, the determinant (what is under the square-root symbol) is equal to $v_o^2 + 2a(d_f – d_o)$.

26. Assume $d_{oy} = d_{fy}$, if you double the launch speed of a projectile, by what factor will that affect the travel time?

27. Derive a formula for the trajectory (d_{fy} as a function of d_{fx}, or y(x) if you prefer) of an object launched from position (0,0) with v_o = 75 m/s, 30° up from the horizontal.

Math Challenge Questions (Not Assessed)

28. Suppose you are trying to throw a ball at the exact spot your friend will be while she is running. You both start at the same position, and she runs at a constant 4.1 m/s in a straight line for 3.0 seconds before you throw the ball at 25° up from the horizontal. The ball is caught at the same height it was thrown. Calculate the speed at which you need to throw the ball. {v_o = 15 m/s}

29. Assume $d_{oy} = d_{fy}$, if you double the launch angle of a projectile {keeping $0 \leq \theta \leq 45°$}, by what factor will that effect the travel time?

30. Show mathematically that two complementary angles will result in an equal range for the case $d_{oy} = d_{fy}$.

31. Derive a formula for the necessary launch angle if the target coordinates are (x, y) relative to the launch position and initial velocity *v*.

Outcome 6.2: Applying Newton's laws of motion for objects on an incline plane.

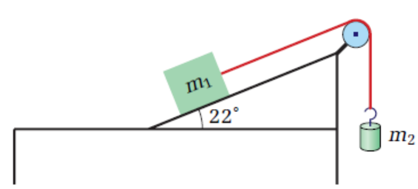
1. A 33 kg block is sliding down a 35° incline. The coefficient of kinetic friction is 0.13.
   a. Calculate the applied force up the ramp necessary so the block accelerates with a magnitude of 0.75 m/s^2 down the ramp. {$|F_a| = 126$ N}
   b. Recalculate to determine the applied force required to accelerate the block up the ramp at 0.75 m/s^2. {$|F_a| = 245$ N}

2. A 25 kg box is placed on a 33° incline. The coefficient of kinetic friction is 0.38. Calculate the acceleration of the box. {$|a| = 2.2\,\frac{m}{s^2}$}

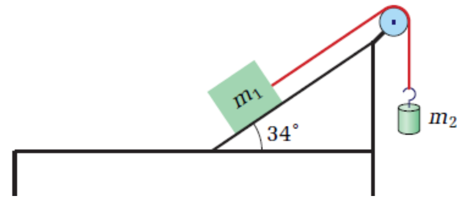
3. A counterweight is used to slide an object up an inclined plane that makes a 42° angle with the horizontal. The mass on the ramp is 40 kg. The coefficient of kinetic friction on the plane is 0.33.
   a. For the acceleration of the object to be 0.72 m/s^2 up the ramp, what must be the mass of the counterweight? {M_2 = 43 kg}
   b. Calculate the tension in the string. {$|F_T| = 388$ N}

4. A counterweight is used to help slide a 70 kg object *down* an inclined plane of 38°. The counterweight has a mass of 25 kg. The coefficient of kinetic friction on the plane is 0.21.
   a. Calculate the acceleration of the masses. {$|a| = 0.66\,\frac{m}{s^2}$}
   b. Calculate the tension in the string. {$|F_T| = 262$ N}

5. The block in the diagram has a mass of 14.5 kg and the freely hanging object has a mass of 8.5 kg. The coefficient of kinetic friction between the block and the ramp is 0.18 and the object will move up the ramp.
   a. Calculate the acceleration of the masses. {$|a| = 0.26\,\frac{m}{s^2}$}
   b. Calculate the tension in the string. {$|F_T| = 81$ N}

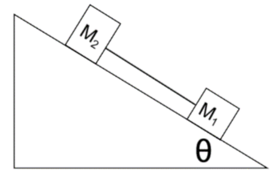


6. The block in the diagram has a mass of 7.25 kg and the hanging object has a mass of 5.95 kg. μ_s = 0.47 and μ_k = 0.12.



   a. What force, directed up the ramp, would you have to apply to the block to make the objects start to move? $\{|F_a| = 9.1\ \text{N}\}$
   b. Calculate the acceleration of the masses assuming the same applied force up the ramp. $\{|a| = 1.6\ \frac{m}{s^2}\}$
   c. Calculate the tension in the string while the masses are accelerating. $\{|F_T| = 49\ \text{N}\}$

7. The masses in the diagram are connected by a massless string {compared to the mass of the objects}. M_1 = 19 kg, M_2 = 13 kg, θ = 24°, μ_{k1} = 0.15, and μ_{k2} = 0.22.



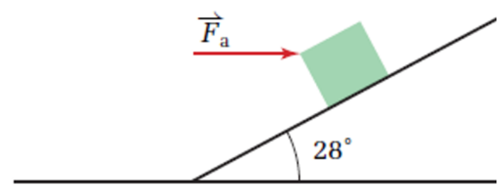
   a. Calculate the acceleration of the masses. $\{|a| = 2.4\ \frac{m}{s^2}\}$
   b. Calculate the tension in the string. $|F_T| = 4.7\ \text{N}\}$

Expert Level Questions

8. An inclined ramp is to be used to slide an object down at a constant velocity. The coefficient of kinetic friction is 0.38. Calculate the angle required for this to happen. {θ = 21°}

9. A 61 kg container sits on a ramp.
   a. If the coefficient of static friction is 0.68, at what angle of the ramp would the container just start to slide? {θ= 34°}
   b. If the coefficient of kinetic friction is 0.18, what would be the acceleration of the container at that angle? $\{|a| = 4.0\ \text{m/s}^2\}$

10. A new worker in a warehouse is pushing an 85 kg crate up a ramp. μ_s = 0.75. Instead of pushing directly up the ramp, the worker is pushing parallel to the ground.

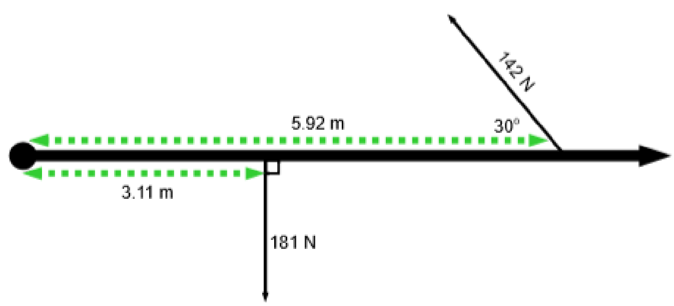


   a. Calculate the applied force necessary to overcome static friction. $\{|F_a| = 1778\ \text{N}\}$
   b. An experienced worker tells the new worker to push directly up, parallel to the ramp. With this revelation, calculate the applied force necessary to overcome static friction. $\{|F_a| = 944\ \text{N}\}$

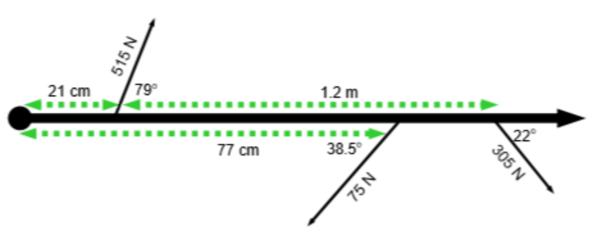
Outcome 6.3: Learn and apply the concept of net torque to solve static equilibrium problems.

Net Torque

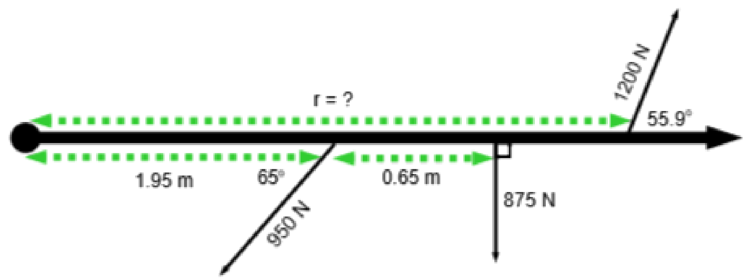
1. Calculate the net torque. {τ_{net} = –143 Nm}



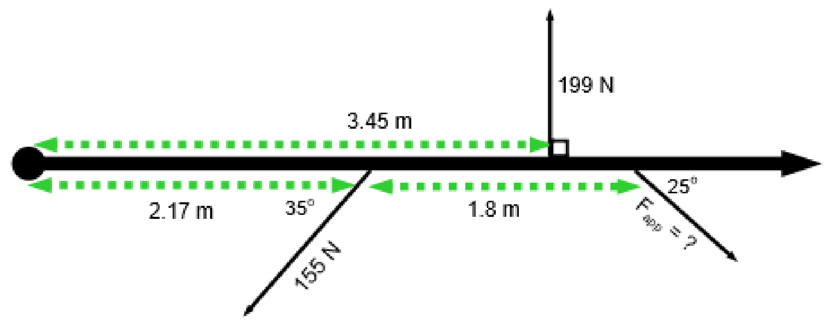
2. Calculate the net torque. {τ_{net} = –91 Nm}



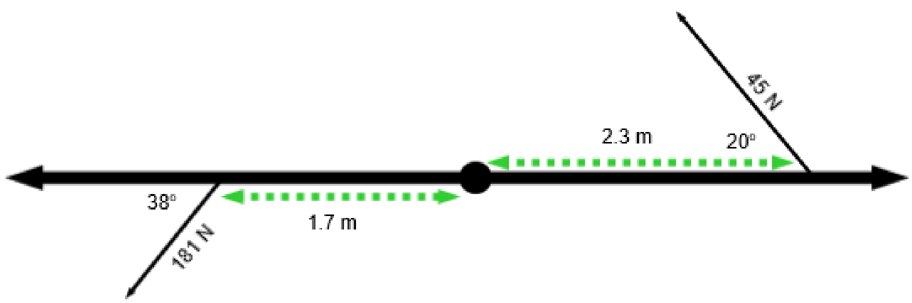
3. The net torque is zero, calculate the missing quantity. {r = 3.98 m}



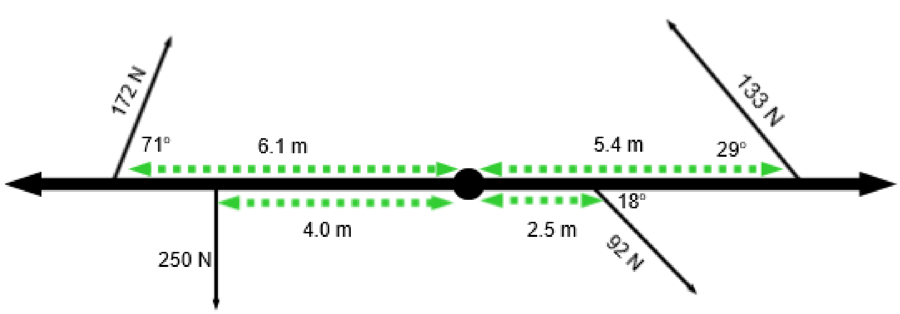
4. The net torque is zero, calculate the missing quantity. {F_{app} = 294 N}



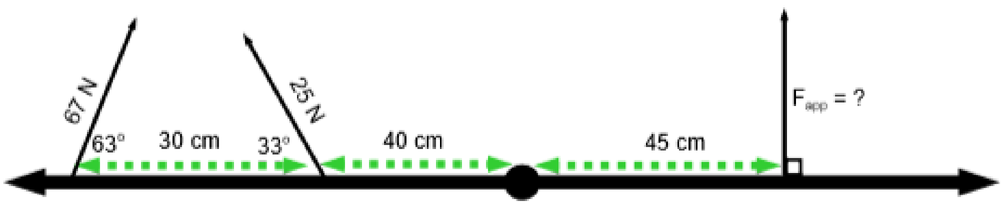
5. Calculate the net torque. {τ_{net} = 224 Nm}



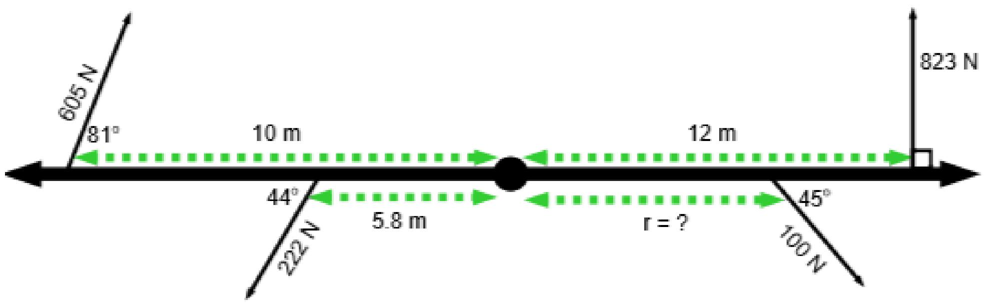
6. Calculate the net torque. {τ_{net} = 285 Nm}



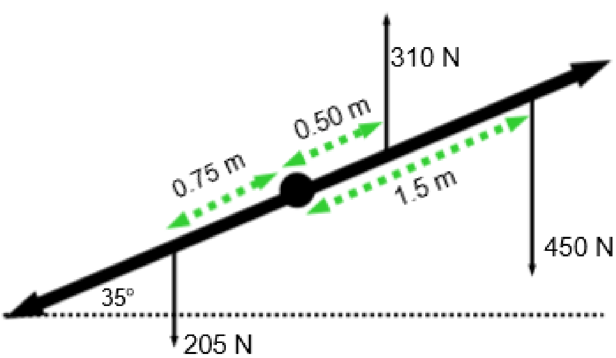
7. The net torque is zero, calculate the missing quantity. {F_{app} = 105 N}



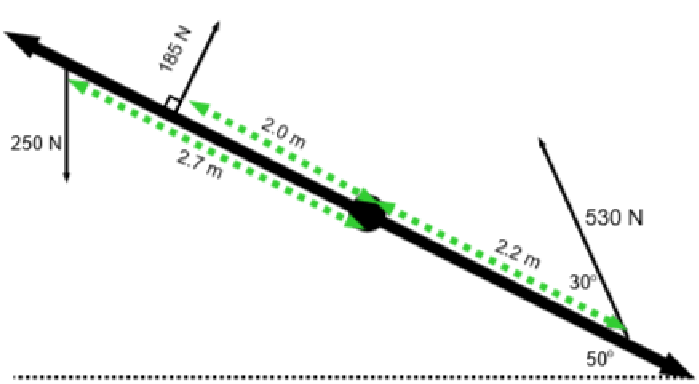
8. The net torque is zero, calculate the missing quantity; image not to scale. {r = 67.8 m}



9. Calculate the net torque if all the forces are perpendicular to the horizontal dashed line. {τ_{net} = –300 Nm}

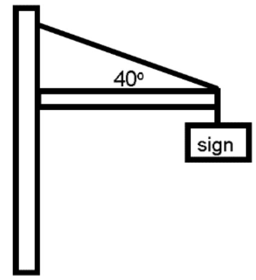


10. Calculate the net torque. The left-most force is perpendicular to the horizontal. {τ_{net} = 647 Nm}

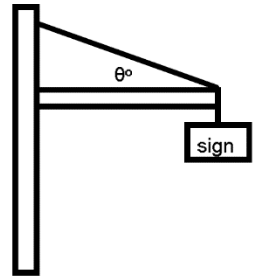


Static Equilibrium

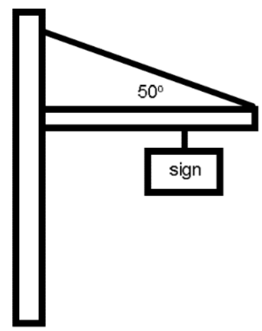
11. The beam has a mass of 170 kg and is 6.0 m long. The sign has a mass of 75 kg and is located at the end of the beam.
    a. Calculate the tension in the wire. {F_T = 2442 N}
    b. Calculate the force acting on the hinge, where the beam meets the pole. {F = 2048 N, 24° up from the horizontal}



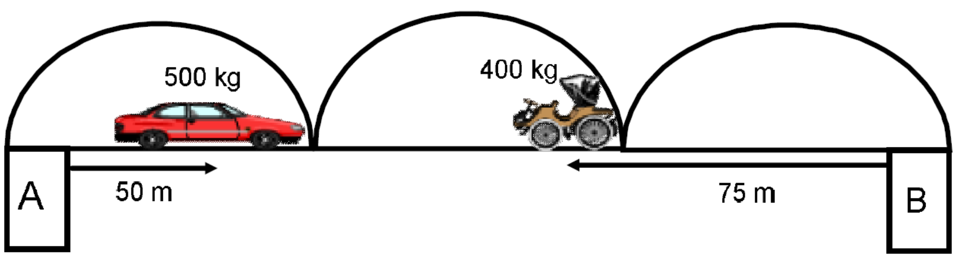
12. The cable in the diagram to the right will break if the tension reaches 1500 N. The beam is 15 m long and has a weight of 1050 N. The sign has a weight of 500 N.
    a. Calculate the smallest angle that can be made with the horizontal. {θ = 43°}
    b. Calculate the force acting on the hinge using the angle found above. {F = 1217 N, 26° up from the horizontal}



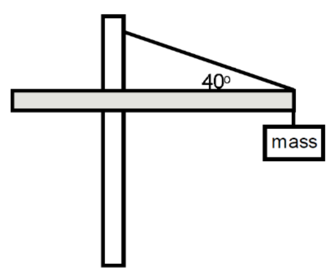
13. The cable will break under a stress of 2300 N. The beam is 150 kg and 8.0 m long. The object is located 5.5 m from the left end of the beam.
    a. Calculate the maximum mass that can be supported by the cable. {m = 152 kg}
    b. Calculate the force acting on the hinge. {F = 1904 N, 39° up from the horizontal}

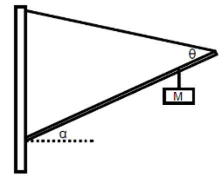


14. The bridge in the diagram below spans 225 m and has a mass of 8500 kg. Calculate the force each column supports. {F_A = 46942 N; F_B = 45272 N}

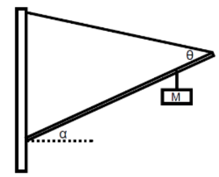


15. The boom of a crane {the beam} has a mass of 625 kg. The hinge is located 10 m from the left end and 15 m from the right end. The mass at the right end is 300 kg.
    a. Calculate the tension in the cable. {F_T = 6168 N}
    b. Calculate the force on the hinge. {F_h = 6960 N; 47° up from horizontal}
    c. If this beam did not extend to the left the tension would have been 7440 N. Explain why the tension is lower by having part of the boom on the opposite side of the pivot point.

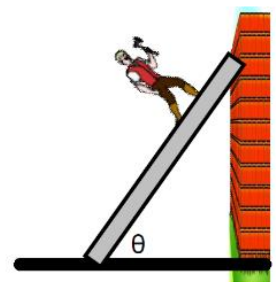




16. The cable in the diagram will break if the tension reaches 1800 N. The beam is 15 m long and has a mass of 60 kg and makes an angle of $\alpha = 30^{o}$ with the horizontal. The cable makes an angle of 60^{o} with the beam and the hanging mass is located 10 m from the hinge.
    a. Calculate the maximum mass that can be attached. {m = 230 kg}
    b. Calculate the force on the hinge for that mass. {F = 2493 N, 51^{o} counterclockwise from the horizontal or 21^{o} counterclockwise from the beam}



17. The cable in the diagram will break if the tension reaches 1074 N. The beam is 12 m long, has a mass of 72 kg and makes an angle of $\alpha = 40^{o}$ with the horizontal. The hanging mass is located 8.5 m from the hinge and has a mass of 144 kg.
    a. Calculate the minimum angle that can be used to attach the cable to the beam. {$\theta = 75^{o}$}
    b. Calculate the force on the hinge for that angle. {F = 1742 N, 60^{o} up from the horizontal or 20^{o} up from the beam}



18. Calculate the coefficient of static friction necessary for the ladder not to slip along the ground. The ladder has a mass of 32 kg, the person has a mass of 44 kg, the ladder is 7.0 m long, the man is 5.5 m from the base of the ladder. The ladder makes an angle of 55^{o} with the ground. There is no friction between the ladder and the wall. {$\mu_s = 0.47$}

19. A 4.0 m long 25 kg ladder leans up against a wall and makes a 60^{o} angle with the ground. A 75 kg person is 3.0 m up the ladder. The coefficient of static friction between the ladder and the wall is 0.72. For the ladder not to move, calculate the coefficient of static friction between the ladder and the ground. {$\mu_s = 0.35$}

Expert Level Question

20. A 10 m long 20 kg ladder rests against the side of a building. A 64 kg person needs to be 7.5 m up the ladder. The coefficient of static friction between the ground and the ladder is 0.46 and between the building and the ladder it is 0.58. Calculate the smallest angle that will support the person without slipping. {$\theta = 53^{o}$}

Outcome 6.4: Collisions and Explosions in 1D and 2D.

1-Dimension

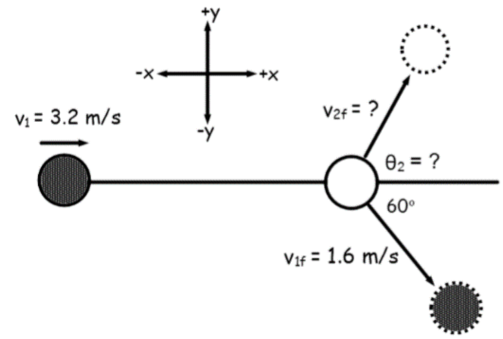
1. Two cars are heading directly towards each other. The mass of car A is 375 kg and a velocity of +1.8 m/s. The mass of car B is 422 kg with a velocity of –1.4 m/s. When they collide, they fuse together and continue to move in a straight line. Calculate their velocity immediately after they collide. {***v*** = +0.11 m/s}

2. A 1385 kg cannon containing a 58.5 kg cannon ball is at rest. The cannon fires the ball with a velocity of 49.8 m/s, calculate the initial velocity of the cannon the instant after it fires the cannon ball. {***v*** = –2.7 m/s}

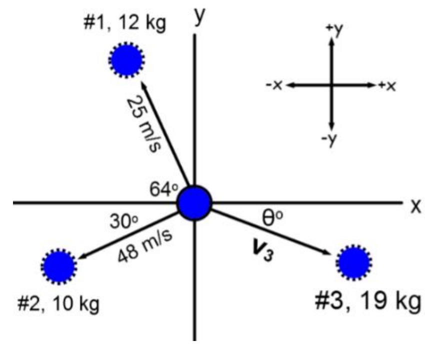
3. A 0.72 kg object moving 0.89 m/s [E] collides with a 2.21 kg object moving 0.46 m/s [W]. After the collision, the larger object moves 0.20 m/s [E]. Calculate the velocity of the smaller object. {v = –1.15 m/s [E]}

2-Dimensions

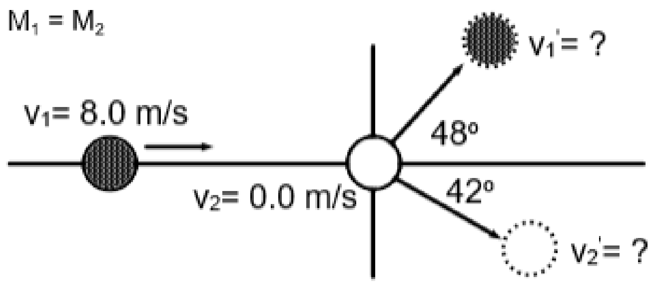
4. A pool ball traveling to the right strikes another, stationary ball. Each ball has the same mass. Use the information given in the diagram to calculate the velocity of pool ball #2. {$\boldsymbol{v_{f2}}$ = 2.77 m/s at 30° up from +x-axis}



5. An object explodes into three pieces. Given the information in the diagram below, calculate the velocity of the third piece. Note: the diagram may not be accurate in depicting the direction (quadrant) of the third piece. {$\boldsymbol{v_3}$ = 28.8 m/s at 3.1° down from +x-axis}

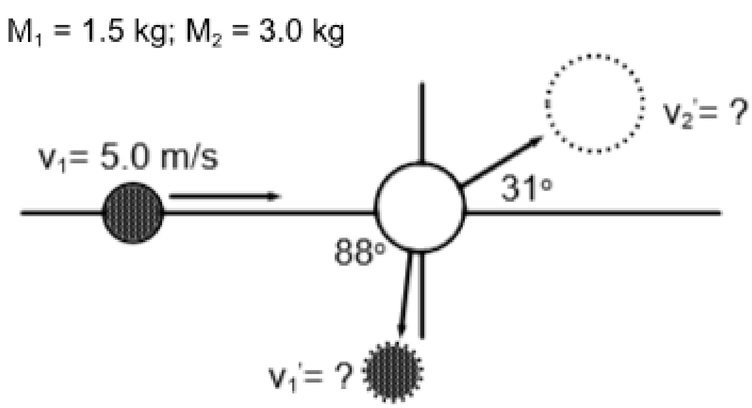


6. Calculate the velocity of each object after the collision.



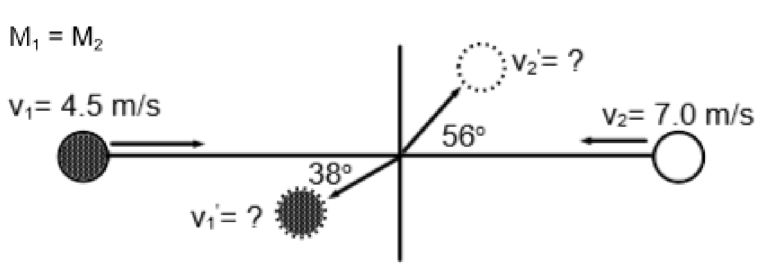
Answers
v_1' = 5.3 m/s
v_2' = 6.0 m/s

7. Calculate the velocity of each object after the collision.



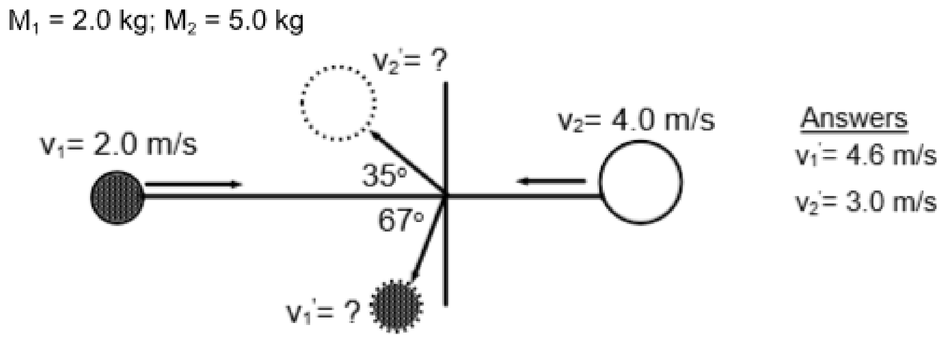
Answers
v_1' = 2.9 m/s
v_2' = 2.9 m/s

8. Calculate the velocity of each object after the collision.



Answers
v_1' = 6.7 m/s
v_2' = 5.0 m/s

9. Calculate the velocity of each object after the collision.



Outcome 7.1: Qualitatively and quantitatively analyze circular motion using vectors and Newton's laws.

Horizontal, Uniform Circular Motion

1. A plane flying at a constant speed in a circular path of radius 5500 m completes one revolution every 485 seconds. Calculate the centripetal acceleration of the plane. {a_c = 0.922 m/s^2}

2. A 1.2 kg stone attached to a 2.0 m long string is whirled in a horizontal circle. At what speed must the stone move for its centripetal acceleration to equal 9.81 m/s^2. {v = 4.4 m/s}

3. A mass of 0.50 kg is tied to one end of a rope and is swung in a circle of radius 1.0 m. The speed is 4.0 m/s. Calculate centripetal force. {F_c = 8.0 N}

4. A communications satellite has a period of 5600 s and an orbital radius of 6.8 x 10^6 m. If the mass is 2000 kg, calculate the centripetal force keeping the satellite in orbit. {F_c = 17 000 N}

5. A 1.5 kg ball on a string is swung in a horizontal circle. The string will break under a tension of 350 N. Calculate the maximum velocity of the ball if the string is,
   a. 0.5 m long {v = 10.8 m/s}
   b. 1.5 m long {v = 18.7 m/s}

6. A 50.0 kg satellite that is 7.00 x 10^6 m from the center of the Earth is orbiting at a speed of 1000 m/s. Calculate:
   a. The time it takes to orbit the Earth once. {T = 44 000 s}
   b. The centripetal acceleration of the satellite. {a_c = 0.143 m/s^2}
   c. The centripetal force that keeps the satellite in orbit. {F_c = 7.14 N}

7. A boy holds a 0.25 kg object 6.00 m from the center of a merry-go-round by means of a string. The toy has a tangential velocity of 3.0 m/s.
   a. How long does it take the merry-go-round to make one revolution? {T = 12.6 s}
   b. Calculate the tension in the string. {F_T = 0.375 N}

8. The Moon has a mass of 7.4 x 10^{22} kg. It is 4.0 x 10^8 m from the center of the Earth and orbits the Earth every 2.4 x 10^6 seconds. Calculate:
   a. The velocity of the Moon. {v = 1050 m/s}
   b. The force needed to keep the Moon in orbit. {F_c = 2.03 x 10^{20} N}

9. A 1200 kg is traveling through a 90° corner with a radius of 15.9 m. It takes the car 2.50 seconds to make the turn. Calculate:
    a. The velocity of the car. {v = 10.0 m/s}
    b. The centripetal acceleration of the car. {a_c = 6.28 m/s^2}
    c. The centripetal force acting on the car. {F_c = 7500 N}
    d. The maximum speed the car could make the corner at if the maximum force available between the tires and the pavement is 5000 N due to ice. {v = 8.14 m/s}

10. A 25 kg object is moving along a circular path at a constant speed of 4.0 m/s and completes one trip around the circle in 5.0 seconds. The coefficient of static friction is 0.75.
    a. Calculate the radius of the circle. {r = 3.2 m}
    b. Calculate the acceleration of the object. {a_c = 5.0 m/s^2}
    c. Calculate the centripetal force and the force of static friction. {F_c = 125 N; F_f = 184 N}
    d. Calculate the farthest position the object can be placed and keep the circular motion {r = 4.7 m}

11. A 1750 kg truck is traveling through a 90° section of a circle with a radius of 24.8 m. It takes the truck 3.17 seconds to make the turn driving at its maximum safe velocity. Calculate the coefficient of static friction between the tires and the road. {μ_s=0.62}

12. An amusement park ride consists of a large cylinder that rotates around a vertical axis. People stand on a ledge inside. When the rotational speed is high enough, the ledge drops away and people "stick" to the wall. If the period of rotation is 2.5 seconds and the radius is 3.5 m, calculate the minimum coefficient of friction required to keep the riders from sliding. {μ_s = 0.44}

13. A circular platform is designed to spin horizontally (around a vertical axis) with a period of 1.6 seconds. How far from the center should a mass be placed to not fly off if the coefficient of static friction is 0.57? {r = 0.36 m}

14. A mass is to be placed 1.75 m from the center of a spinning platform. The coefficient of static friction is 0.41. Calculate the maximum frequency of the spinning platform such that the mass would not move. {f = 0.24 Hz}

15. A car rounds a 92 m radius corner without losing traction at a speed of 26 m/s. Calculate the coefficient of static friction between the tires and the road? {μ_s = 0.75}

16. The maximum speed a car can have to safely take a turn is 30 m/s. On a cold morning ice has reduced the coefficient of static friction by a factor of three. Calculate the new maximum speed for the car to safely navigate the same turn under icy conditions. {v = 17 m/s}

Banked Curves – No Friction

17. Without relying on friction, calculate the maximum speed a car can travel around a 120 m curve and banked at 18°. {v = 20 m/s}

18. Without relying on friction, calculate the angle necessary to bank a curve of 150 m such that cars can safely navigate it traveling 25 m/s. {θ = 23°}

19. A turn is to be banked at 25° for vehicles to navigate, without relying on friction, at 30 m/s. Calculate the required radius of the turn. {r = 197 m}

20. A 1575 kg airplane makes a 1325 m radius turn at 125 m/s.
    a. Calculate the bank-angle the airplane makes with the horizontal. {θ = 50°}
    b. Calculate the centripetal force acting on the airplane. {F_c = 18 573 N}

21. For comfort, a passenger airplane keeps a low bank-angle of 12°. A 737 plane is traveling 225 m/s and needs to adjust course.
    a. Calculate the turning radius required. {r = 24 278 m}
    b. Calculate the centripetal force necessary for a 60 500 kg 737 plane to make that turn. {Fc = 126 156 N}
    c. Calculate the centripetal force experienced by a 75 kg passenger. {F_c = 156 N}

22. A 125 m turn is to be constructed on Earth and on Mars. The maximum velocity for each turn is 15 m/s.
    a. Calculate, for each planet, the angles the turn needs to be banked at for vehicles to navigate the turn without relying on friction. {θ_{Earth} = 10°, θ_{Mars} = 26°}
    b. Why is a larger angle required for a banked turn on a planet with a lower acceleration due to gravity?

Banked Turns - Including Friction

23. A 75 m turn is banked at 20°. The coefficient of static friction between the tires and road is 0.55. Calculate the maximum velocity to safely navigate the turn. {v = 29 m/s}

24. Vehicles will be taking a banked turn at 28 m/s. The bank will be made at an angle of 15° with the ground. With a coefficient of static of 0.65, calculate the required radius of the turn. {r = 72 m}

25. Calculate the minimum coefficient of static friction necessary for a truck to safely navigate an 87 m radius turn that is banked at 18° while driving 32 m/s. {μ = 0.63}

26. A race car track needs to build banked turns. The cars will have a maximum speed of 56 m/s and a coefficient of static friction of 0.74 between the tires and the ground. The radius of curvature for the turns is 120 m. Calculate the bank-angle necessary for the cars to safely navigate the corners. {θ = 33°}

Vertical Circular Motion

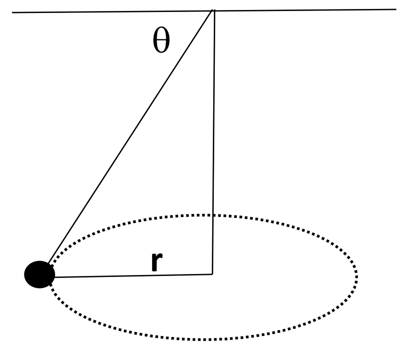
27. A rope has a length of 1.2 m. At the end of the rope is a 10.5 kg mass.
    a. Calculate the minimum velocity to swing the mass in a complete vertical circle. {v = 3.4 m/s}
    b. If the rope breaks under a tension of 750 N, calculate the maximum velocity. {v = 8.6 m/s}

28. The fastest a 350 kg motorcycle can travel is 35 m/s. Calculate the radius of the largest vertical circle that the motorcycle can safely travel around. {r = 125 m}

29. A rope will break under a tension of 1250 N. The desired speed of a 7.25 kg mass at the bottom of a vertical circle is 16 m/s. A piece of rope 1.2 m long is used. With support from calculations, will the rope break before the mass reaches 16 m/s?

Expert Level Questions

30. A 1.47 kg mass is suspended on a string (schematic to the right). A person starts swinging it in a horizontal circle with a radius of 0.25 m and a period of 0.83 seconds. Calculate the force of tension in the string and the angle the string makes with the horizontal.



31. An object starts at rest and is subjected to a force over 7.00 m. After the object is free of that force, a 224 N force can turn the object through a 90° corner with a radius of 0.25 m. Calculate the initial force. {F_o = 4.0 N}

32. A 3.50 mass with a kinetic energy of 43.75 J goes through a 90° section of a circle with a radius of 1.25 m. If the centripetal force acting on the mass in the corner is 70.0 N, calculate the velocity of the mass as it leaves the corner. {5.00 m/s}

33. A model train car is coasting (no driving force) on a circular track. Initially the centripetal force necessary to keep the car going around the track is 5.0 N. If after the car has gone halfway around the circle the necessary centripetal force has been reduced to 1.86 N. Calculate the force of friction acting on the car. {F_f = 0.50 N}

Outcome 7.2: Quantitatively apply Newton's law of gravitation and Kepler's 3rd law of planetary motion.

1. Consider the Earth, Moon, and Sun system.
    a. Calculate the gravitational force between the Earth and the Sun. {Fg = 3.58×10^{22}}
    b. Calculate the gravitational force between the Earth and the Moon. {Fg = 1.99×10^{20}}

2. How far apart would you have to place two 7.0 kg bowling balls so that the force of gravity between them would be 1.25×10^{-4} N? Would it be possible to place them that close together? {r = 0.0051 m}

3. Calculate the gravitational force between the electron and the proton in a hydrogen atom if they are 5.30×10^{-11} meters apart. {$F_g = 3.61 \times 10^{-47}$ N}

4. On Venus, a person with a mass of 68 kg would weigh 572 N. Calculate the mass of Venus if its radius is 6.31×10^{6} meters. {M = 5.0×10^{24} kg}

5. In an experiment, an 8.0 kg lead sphere is brought close to a 1.5 kg mass. The gravitational force between them is 1.28×10^{-8} N. Calculate the separation of the masses. {r = 0.25 m}

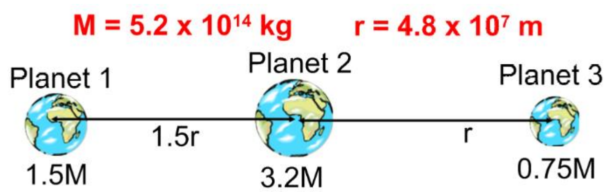
6. Along a line connecting Earth and the Moon, at what distance from Earth's center would an object have to be located so that the gravitational attractive force of Earth on the object was equal in magnitude and opposite in direction from the gravitational attractive force of the Moon on the object? {3.46×10^{8} m or 0.9xEarth-Moon Dist}

7. Calculate how far above the Earth's surface you must be to experience an acceleration due to gravity of 0.5g? {altitude = 2.64×10^{6} m}

8. Two planets are separated by 2.5×10^{8} m. One planet is 8 times as massive as the other. Calculate where an object would have to be positioned to experience a force of gravity from the larger planet that is twice the smaller. {1.67×10^{8} m from 8M planet's center}

9. Calculate:
    a. The acceleration of the Moon from the Earth. {$a_m = 2.7\times10^{-3}$ m/s^2}
    b. The acceleration of the Earth from the Moon. {$a_E = 3.3\times10^{-5}$ m/s^2}

10. Calculate the net force on each planet in the images below. The answers are to the right of the image.



- F_{net1} = 18109 N to the right
- F_{net2} = 2087 N to the right
- F_{net3} = 20200 N to the left

11. Jupiter's moon Io orbits Jupiter once every 1.769 days. Its average orbital radius is 4.216 x 10^8 m. Calculate Jupiter's mass. {M = 1.9 x 10^{27} kg}

12. Some weather satellites orbit the Earth every 90.0 minutes. Calculate the altitude of such a satellite. {alt = 263000m}

13. Calculate the speed of the Moon in its orbit about the Earth. {v = 1019 m/s}

14. Calculate the speed of the Earth around the Sun. {v = 29750 m/s}

15. A satellite orbits the Moon 60 km above its surface.
    a. Calculate the satellite's orbital period. {T = 6848 s, or 114 minutes}
    b. Calculate the satellite's orbital velocity. {v = 1651 m/s}

16. A star at the edge of the Andromeda galaxy is measured to be orbiting the center of that galaxy at 200 km/s. The star is 7.45 x 10^{17} meters from the galaxy's center. Calculate the mass of the Andromeda galaxy. {4.47 x 10^{38} kg}

17. The polar-orbiting environmental satellites orbit at a low altitude. The satellites orbit 14.1 times a day.
    a. Calculate the satellite's altitude. {alt = 860 000 m}
    b. Calculate the satellite's orbital speed. {v = 7423 m/s}

18. The altitude of the International Space Station is 226 km.
    a. Calculate the ISS' orbital speed. {v = 7770 m/s}
    b. Calculate the ISS' period in minutes. {T = 5340 s, or 89 minutes}

19. A geosynchronous satellite orbits directly above the same location of the Earth/s surface. Calculate the orbital speed and altitude of such a satellite. {v = 3072 m/s; r = 3.59 x 10^7 m}

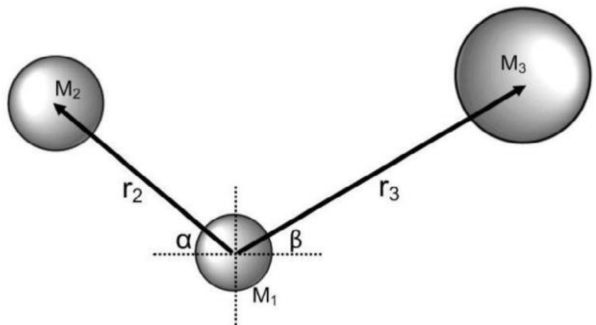
Expert Level Questions

20. Even though the Sun-Earth force of gravity is about 200 times that of the Moon-Earth, the Moon is responsible for our tides. A tidal force is a measure of the *change in force with distance*. Calculate the magnitude of the tidal force between the Sun – Earth and the Moon – Earth systems. {$Tide_{Earth\text{-}Sun}$ = 4.70 x 10^{11} N/m; $Tide_{Earth\text{-}Moon}$ = 1.03 x 10^{12} N/m}

21. The radius of Uranus is 4.3 times the radius of Earth. The mass of Uranus is 14.7 times the Earth's Mass. How does the gravitational attractive force on Uranus' surface compare to that on Earth's? {$F_{Uranus} = 0.8F_{Earth}$}

22. Suppose you launch a projectile at an angle at the Earth's surface. Then you launch the same projectile, at the same angle, on a planet that is 12 times the mass of the Earth and 2.8 times Earth's radius. How many times higher and farther (or lower and shorter) would it travel on the new planet. {0.65xHeight and range}

23. Calculate the net force of gravity on M_1 in the diagram below. $M_1 = 5M_{Earth}$, $M_2 = 3.6M_{Earth}$, $M_3 = 8.1M_{Earth}$, r_2 = 12 Au, r_3 = 18 Au, $\alpha = 36^o$, and $\beta = 21^o$. {$F_{net} = 1.74 \times 10^{16}$ N [80° up from the negative x-axis]}



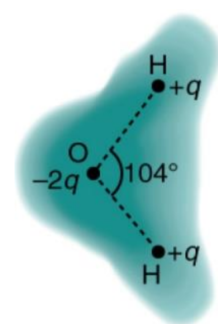
Outcome 8.1: Qualitatively and quantitatively analyze electric charge and electric fields, including Coulomb's law.

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near the knob?

3. Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

4. The image shows the charge distribution of a water molecule, which is a polar molecule because it has an inherent separation of charge. With that in mind, explain what effect humidity has on removing excess charge from objects.

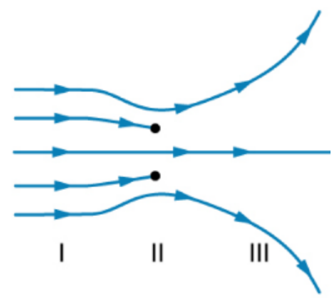


5. In terms of Coulomb's Law, why is a polar molecule attracted by both positive and negative charges?

6. Water, a polar molecule, has a higher melting point {0°C} than carbon dioxide {–57°C}, a non-polar molecule. Use the concept of electric forces to explain why this is.

7. Why must the test charge, *q*, in the definition of the electric field be vanishingly small?

8. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?



9. An electric field extending over three regions is shown in the diagram.
   a. Are there any isolated charges? If so, in what region and what are their signs?
   b. Where is the field the strongest?
   c. Where is it weakest?
   d. Where is the field the most uniform?

10. How many electrons are needed to form a charge of -2.00 nC? {1.25×10^{10} e^-}

11. How many electrons must be removed from a neutral object to leave a net charge of 0.500 µC? {3.13×10^{12} e^-}

12. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved? {q = 600 C}

13. Suppose an object has 1.00×10^{16} protons and a net charge of +192 µC.
    a. How many fewer electrons are there than protons? {1.2×10^{15} fewer e^-}
    b. If you paired them up, what fraction of the protons would have no electrons? {$\#e^-/\# p^+ = 0.12$}

14. Calculate the repulsive force between two –30.0 nC charges that are 0.08 m apart? {**F** = 1.26×10^{-3} N}

15. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three? {**F** = 0.556 N}

16. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased? {**r** decreased by a factor of 5}

17. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them? {r = 0.007 m}

18. A test charge of +2.0 µC is placed halfway between a charge of +6.0 µC and another of +4.0 µC separated by 0.10 m.
    a. Calculate the magnitude of the net force on the test charge. {$\mathbf{F_{net}}$ = 14 N}
    b. What is the direction of this force {away from or toward the +6 µC charge}? {away from the +6.0 µC charge}

19. Suppose two equal, but opposite, charges of q = 1.0 C are separated by 1000 m {1.0 km}.
    a. Calculate the electrostatic force of attraction between them. {**F** = 8990 N}
    b. If the objects could accelerate towards each other, would it be constant or change?
    c. Is 1.0 C a significant amount of charge?

20. Two point charges totaling +8.00 µC exert a repulsive force of 0.150 N on one another when separated by 0.500 m.
    a. Calculate the charge on each. {q_1 = 7.44 µC, q_2 = 0.560 µC}
    b. Calculate the charge on each if the force was attractive and still totaled +8.0 µC. {q_1 = 8.49 µC, q_2 = –0.49 µC}

21. Point charges of 5.00 µC and –3.00 µC are placed 0.250 m apart.
    a. Where can a third charge be placed so that the net force on it is zero? {not possible, explain why}
    b. Recalculate if both charges are positive. {**r** = 0.141 m from 5.00 µC charge}

22. Calculate the magnitude and direction of an electric field that exerts a 2.00×10^{-5} N upward force on a –1.75 µC charge. {E = 11.4 N/C [down]}

23. What is the magnitude and direction of the force exerted on a 3.50 µC charge by a 250 N/C electric field that points due east? {**F** = 8.75×10^{-4} N [E]}

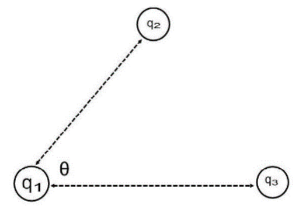
24. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 µC. {1.12×10^{4} N/C}

25. Calculate the initial {from rest} acceleration of a proton in a 5.00×10^{6} N/C electric field {such as one created by a Van de Graaf} {**a** = 4.78×10^{14} m/s^2}

26. Separately, sketch the electric field lines near a point charge +q and a point charge of –3.00q.

Expert Level Question

27. Three charges are arranged as depicted below. Given the information, calculate the net electrostatic force acting on q_1. q_1 = –7.25 µC, q_2 = –5.0 µC, q3 = 9.5 µC, r_{12} = 1.75 m, r_{13} = 2.7 m, θ = 25°. {F_{net} = 0.047 N [75° down from negative x-axis]}



Outcome 8.2: Quantitatively analyze electric circuits.

1. List the four parts required to make a circuit:

2. Write the unit's name and symbol next to each quantity:

Current ________________ Voltage ________________ Resistance ________________

3. What quantity is the measurement of charge per second?

4. The energy per unit charge is what quantity?

5. Define direct and alternating currents.

6. What is meant by *electrical resistance*?

7. What is the purpose of resistors in a circuit?

8. Summarize the properties of a series circuit.

9. Summarize the properties of a parallel circuit.

Equivalent Resistance and Ohm's Law

10. Three resistors are connected in series: 15 Ω, 33 Ω, and a 78 Ω. Calculate the equivalent resistance. {R_{eq} = 126 Ω}

11. Calculate the equivalent resistance of 26 Ω and 37 Ω resistors connected in parallel. {R_{eq} = 15.3 Ω}

12. The equivalent resistance of four resistors connected in parallel is 7.35 Ω. Calculate the value of the fourth resistor if the other three are 29 Ω, 42 Ω, and 31 Ω. {R_4 = 22 Ω}

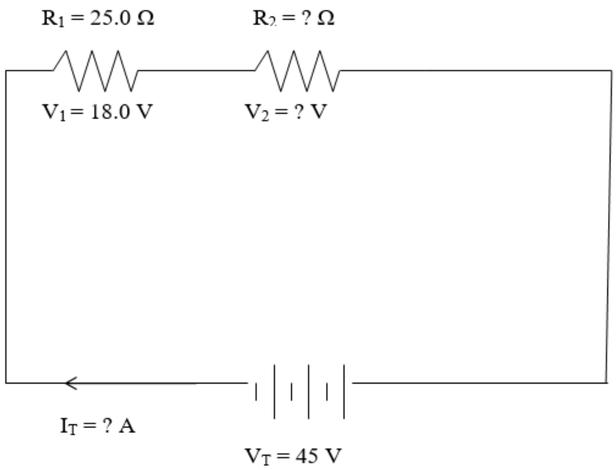
13. What current comes from a 24 V battery if the resistance of a circuit is 42 Ω?

14. Calculate the voltage drop of a 120 Ω resistor if 0.75 A of current goes through it.

15. What resistance is necessary to have a current of 1.6 A from a battery of 30 V?

Circuit Diagrams and V-I-R Tables

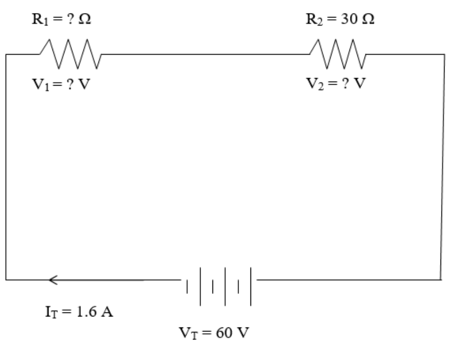
16.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
Totals	V_T=	I_T=	R_{eq}=

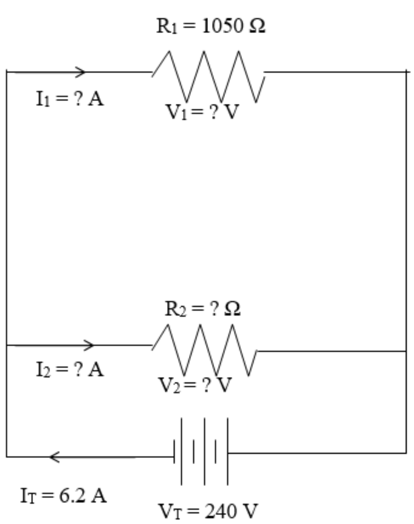
17.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
Totals	V_T=	I_T=	R_{eq}=

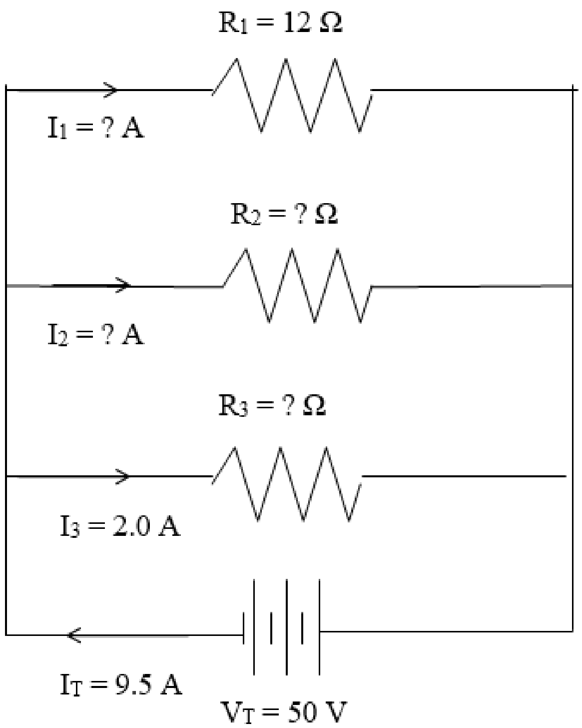
18.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
Totals	V_T=	I_T=	R_{eq}=

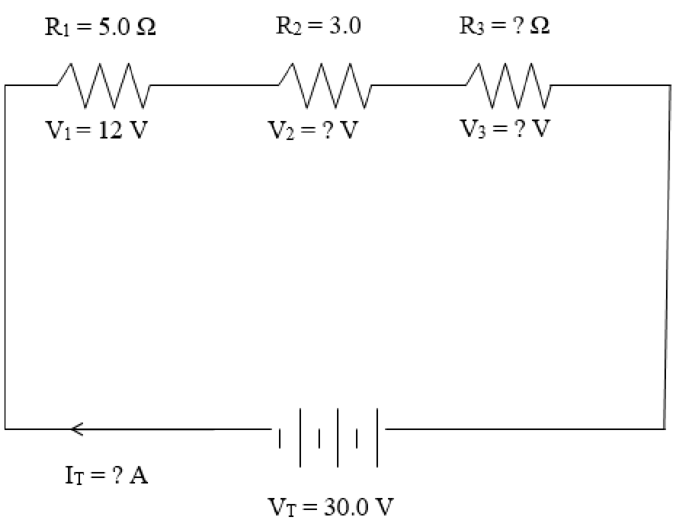
19.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

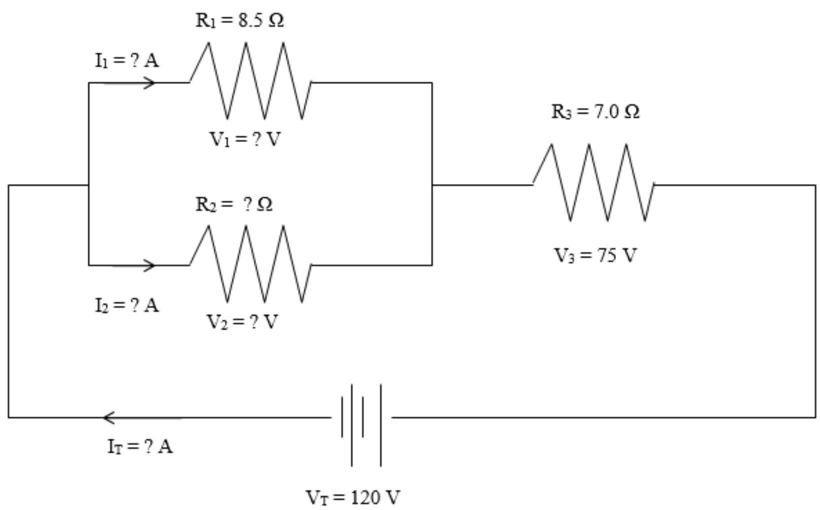
20.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

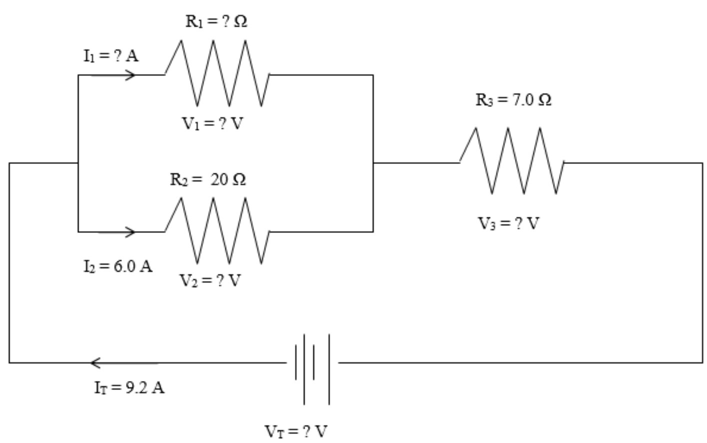
21.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

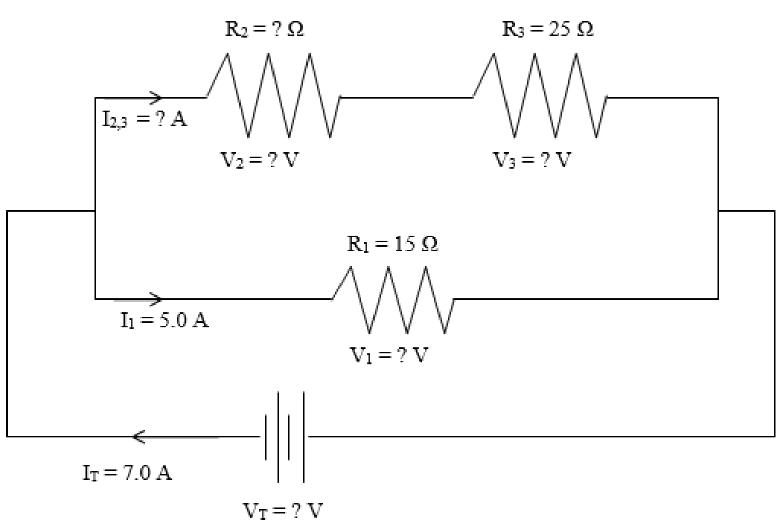
22.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

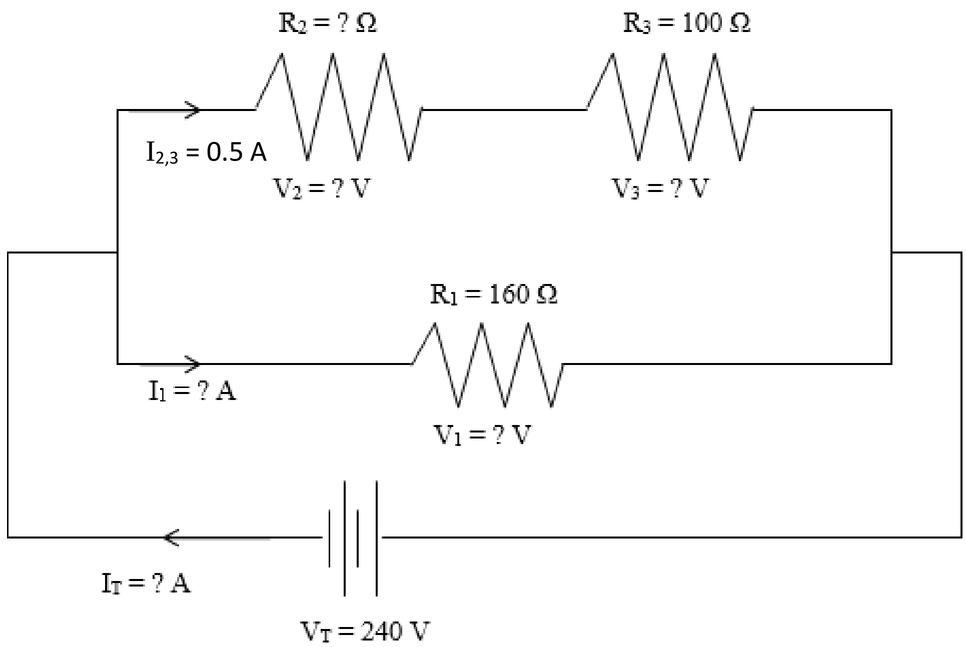
23.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

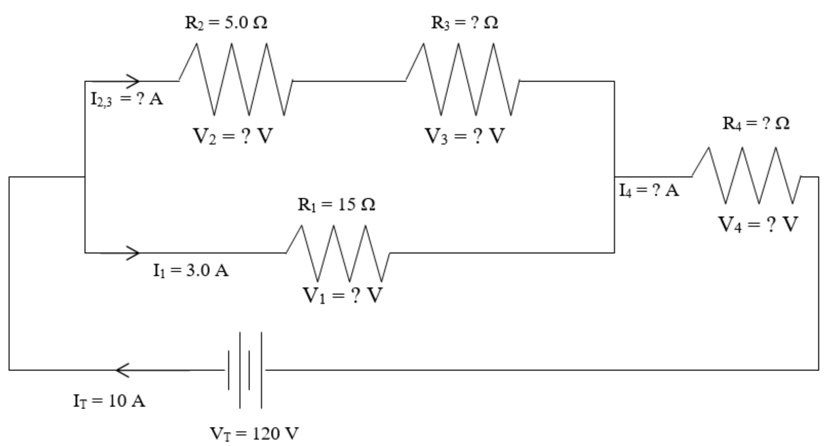
24.



V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
Totals	V_T=	I_T=	R_{eq}=

25.

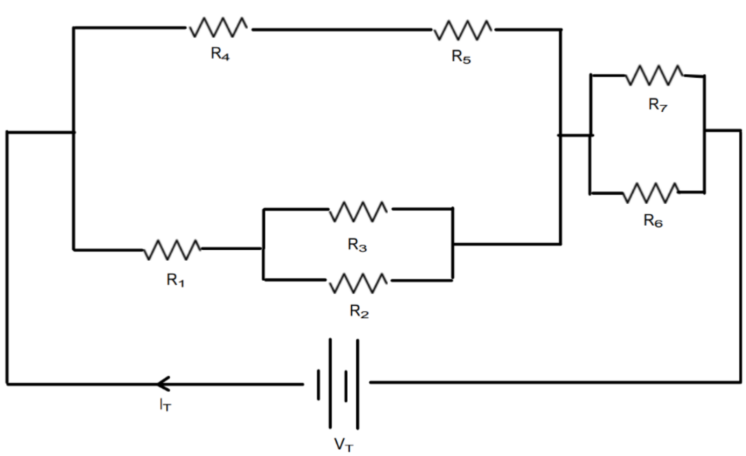


23.

V-I-R Table

Resistor #	V (V)	I (A)	R (Ω)
1			
2			
3			
4			
Totals	V_T=	I_T=	R_{eq}=

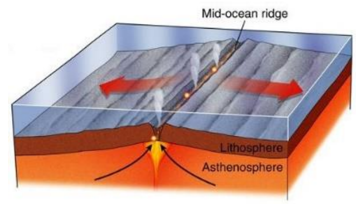
26.



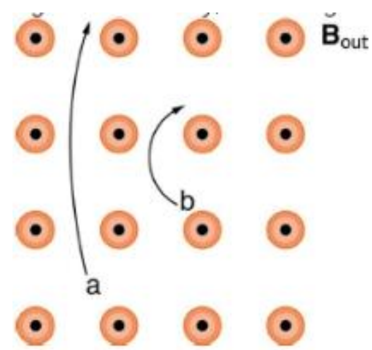
#	V (V)	I (A)	R (Ω)
1			3.0
2			8.0
3			4.5
4			2.5
5			5.5
6			6.0
7			6.0
	V_T = 60	I_T =	R_{eq} =

Outcome 8.3: Qualitatively analyze magnetism; including magnetic poles, fields, and moving charges.

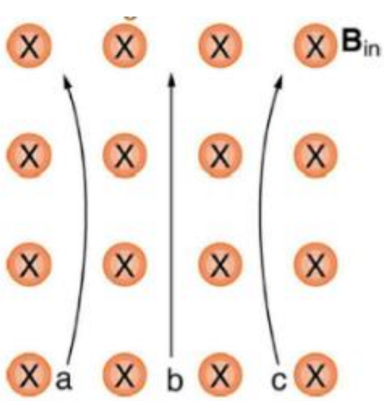
1. Volcanic and other such activity at the mid-Atlantic ridge extrude material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give a historical record?



2. List the ways in which magnetic field lines and electric field lines are similar and how they differ.
3. Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet?
4. Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? In no, where is it the greatest?
5. If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in the region is necessarily zero?
6. How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?
7. Describe how the Earth's magnetic field can shield us from high-energy, cell damaging, charged particles from space.
8. What direction would a proton be deflected if it approached perpendicular to the equator? What about electrons?
9. Refer to the image on the right to answer the following:
   a. Which of the particles has the greatest velocity, assuming they have identical charges and masses?
   b. Which of the particles has the greatest mass, assuming all have identical charges and velocities?



10. What are the signs of the charges on the particles in the image to the left?



Unit 1 – Kinematic Equations

$$v = \frac{d}{t}$$

$$\vec{v}_{avg} = \frac{\vec{d}_f - \vec{d}_o}{t}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{t}$$

$$\vec{d}_f = \vec{d}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_f{}^2 = \vec{v}_o{}^2 + 2a(\vec{d}_f - \vec{d}_o)$$

Variable	Quantity	Unit
v	Average/Instantaneous speed	m/s
$\vec{v}_{avg}$	Average velocity	m/s
$\vec{v}_f$	Final velocity	m/s
$\vec{v}_o$	Initial velocity	m/s
$\vec{d}_f$	Final position	m
$\vec{d}_o$	Initial position	m
d	Distance	m
$\vec{a}$	Acceleration	m/s^2
t	Time	s

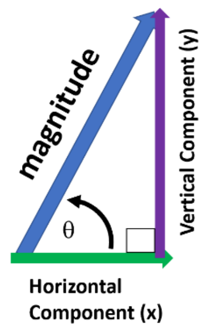
Unit 2 – Vector Components

Vertical component = Magnitude × sin θ

Horizontal component = Magnitude × cos θ

Magnitude = $\sqrt{(horizontal\ component)^2 + (vertical\ component)^2}$

Angle, $\theta = \tan^{-1}\left(\frac{vertical\ component}{horizontal\ component}\right)$



Unit 3 – Dynamics

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_{net} = \sum Forces$$

$$|\vec{F}_f| = \mu|\vec{F}_N|$$

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_{net} = \sum \text{connected masses} \times \vec{a}$$

Variable	Quantity	Unit
m	Mass	kg
$\vec{F}_g$	Force of gravity	N
$\vec{g}$	Acceleration due to gravity 9.81 for objects on Earth	m/s^2
$\vec{F}_{net}$	Net force	N
$\vec{F}_f$	Force of friction	N
$\vec{F}_N$	Normal force	N
μ	Coefficient of friction	none

Location	$\lvert\vec{g}\rvert$
Earth (avg.)	9.81
Moon	1.64
Mars	3.72
Jupiter	25.9
North Pole	9.8322
Equator	9.7805
Mt. Everest	9.7647
Mariana Trench	9.8331
ISS	9.0795

Unit 4 – Mechanical Energy

$$W = F_{\parallel}(\vec{d})$$

$$W_{nc} = F_{\parallel}(d)$$

$$E_g = mgh$$

$$E_k = \frac{1}{2}mv^2$$

$$E_e = \frac{1}{2}kx^2$$

$$\vec{F}_s = kx$$

$$\Delta E_T = 0$$

$$\Delta E_T = W_{nc}$$

Variable	Quantity	Unit
W	Work	J
$F_{\parallel}$	Force parallel to motion	N
E_g	Gravitational potential energy	J
g	Acceleration due to gravity	m/s^2
h	Change in height	m
E_k	Kinetic energy	J
E_e	Elastic potential energy	J
$\vec{F}_s$	Restoring force	N
k	Spring constant	N/m
x	Stretch or compression length	m
ΔE_T	Change in total mechanical energy	J
W_{nc}	Work done by non-conservative forces	J

Unit 5 – Wave Phenomena

$$T = \frac{1}{f}$$

$$v = f\lambda$$

$$f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$$

Refraction

$$n = \frac{c}{v}$$

$$n_i \sin\theta_i = n_R \sin\theta_R$$

Substance	Index of Refraction (*n*)
vacuum	1.00000
gases at 0°C, 1.013 × 10⁵ Pa	
hydrogen	1.00014
oxygen	1.00027
air	1.00029
carbon dioxide	1.00045
liquids at 20°C	
water	1.333
ethyl alcohol	1.362
glycerin	1.470
carbon disulfide	1.632

Substance	Index of Refraction (*n*)
solids at 20°C	
ice (at 0°C)	1.31
quartz (fused)	1.46
optical fibre (cladding)	1.47
optical fibre (core)	1.50
Plexiglas™ or Lucite™	1.51
glass (crown)	1.52
sodium chloride	1.54
glass (crystal)	1.54
ruby	1.54
glass (flint)	1.65
zircon	1.92
diamond	2.42

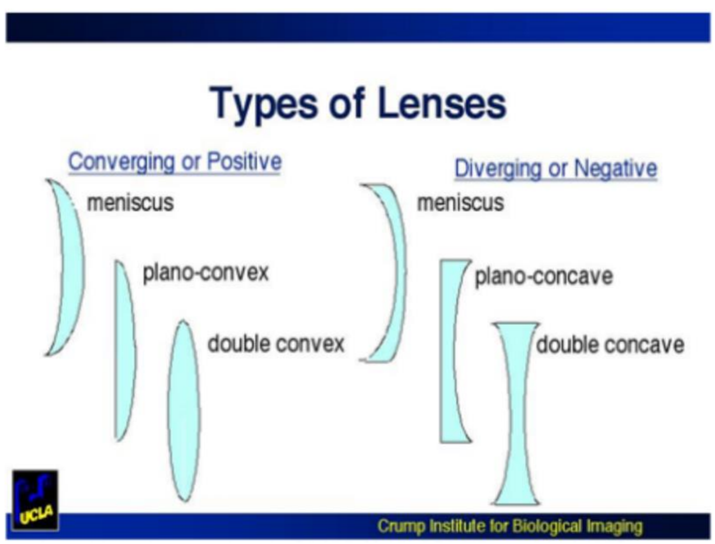
Spherical Lenses

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Variable	Quantity	Unit
Waves		
T	Period	s
f	Frequency	Hz
v	Wave speed	m/s
λ	Wavelength	m
f_{obs}	Observed frequency	Hz
f_s	Source frequency	Hz
v_w	Speed of sound	m/s
v_s	Speed of the source	m/s
Refraction		
n	Index of refraction	none
c	3.00×10^8; Speed of light in a vacuum	m/s
n_i	Incident-ray medium index of refraction	none
θ_i	Angle of incidence	degrees
n_R	Refracted-ray medium index	none
θ_R	Angle of refraction	degrees
Spherical Lenses		
f	Focal length	cm, m
d_o	Object distance	cm, m
d_i	Image distance	cm, m
m	magnification	none
h_i	Image height	cm, m
h_o	Object height	cm, m
n_{lens}	Lens refractive index	none
n_o	Surrounding medium refractive index	none
R_1	Radius of curvature of first side of lens	cm, m
R_2	Radius of curvature of second side of lens	cm, m



Unit 6 – Kinematics & Dynamics in 2D

$\vec{\tau} = rF_{\perp}$

$\vec{\tau}_{net} = \sum \text{Torques}$

$\vec{p} = m\vec{v}$

$\vec{p}_{oT} = \vec{p}_{fT}$

Variable	Quantity	Unit
$\vec{\tau}$	Torque	Nm
r	Distance from pivot (lever arm)	m
$F_{\perp}$	Perpendicular force component	N
$\vec{\tau}_{net}$	Net torque	Nm
$\vec{p}$	momentum	$\frac{kg \cdot m}{s}$
m	mass	kg
$\vec{v}$	velocity	m/s
$\vec{p}_{oT}$	Total initial momenta	$\frac{kg \cdot m}{s}$
$\vec{p}_{fT}$	Total final momenta	$\frac{kg \cdot m}{s}$

Unit 7 – Circular Motion and Universal Gravitation

Circular Motion

$v = \frac{2\pi r}{T}$

$a_c = \frac{v^2}{r}$

$F_c = \frac{mv^2}{r}$

$T = \frac{1}{f}$

$v = \sqrt{rg \tan\theta}$

Universal Gravitation

$\vec{F}_g = G\frac{m_1 m_2}{r^2}$

$|\vec{g}| = G\frac{m}{r^2}$

$v^2 = G\frac{m}{r}$

$\frac{Gm}{4\pi^2} = \frac{r^3}{T^2}$

Variable	Quantity	Unit
v	Circular or orbital speed	m/s
r	Circular or orbital radius; distance between centers of mass	m
T	Period	s
a_c	Centripetal acceleration	m/s^2
F_c	Centripetal force	N
f	Frequency	Hz
m	Mass or planet mass	kg
g	Acceleration due to gravity	m/s^2
μ_s	Coefficient of static friction	none
θ	Bank angle	degrees
$\vec{F}_g$	Force of gravity	N
G	6.673 x 10^{-11}, Universal gravitational constant	$N\frac{m^2}{kg^2}$

Inner Solar System Data					
Quantity	**Sun**	**Mercury**	**Venus**	**Earth**	**Moon**
Mass (kg)	1.99×10^{30}	3.3×10^{23}	4.9×10^{24}	5.98×10^{24}	7.36×10^{22}
Radius (m)	6.96×10^{8}	2.5×10^{6}	6.1×10^{6}	6.38×10^{6}	1.74×10^{6}
Orbital Radius (m)	N/A	5.8×10^{10}	1.1×10^{11}	1.49×10^{11}	3.84×10^{8}
Period of Orbit	N/A	88 Earth days	225 Earth days	365.25 Earth days	27.3 Earth days
Rotation	Varies	58 Earth days	243 Earth days	24 hours	27.3 Earth days

Outer Solar System Data					
Quantity	**Mars**	**Jupiter**	**Saturn**	**Uranus**	**Neptune**
Mass (kg)	6.4×10^{23}	2.0×10^{27}	5.7×10^{26}	8.7×10^{25}	1.0×10^{26}
Radius (m)	3.4×10^{6}	7.1×10^{7}	6.0×10^{7}	2.6×10^{7}	2.5×10^{7}
Orbital Radius (m)	2.3×10^{11}	7.8×10^{11}	1.4×10^{12}	2.9×10^{12}	4.5×10^{12}
Period of Orbit	1.88 Earth years	11.9 Earth years	29 Earth years	84 Earth years	164 Earth years
Rotation	24 hours	10 hours	11 hours	17 hours	16 hours

Unit 8 – Electric Circuits, Fields, and Electromagnetic Forces

$$q = Ne$$

$$\vec{F}_q = k\frac{q_1 q_2}{r^2}$$

$$\vec{E} = \frac{\vec{F}_q}{q}$$

$$\left|\vec{E}\right| = k\frac{q}{r^2}$$

$$\vec{E}_q = k\frac{q_1 q_2}{r}$$

$$V = \frac{E_q}{q}$$

$$V = k\frac{q}{r}$$

$$I = \frac{\Delta q}{\Delta t}$$

$$R = \rho\frac{L}{A}$$

$$V = IR$$

$$P = IV$$

$$R_{eqs} = R_1 + R_2 + \cdots + R_n$$

$$C_{eqp} = C_1 + C_2 + \cdots + C_n$$

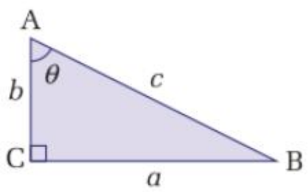
$$\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Variable	Quantity	Unit
q	Electric charge	C
N	Number of charges	none
e	1.602×10^{-19}, Elementary charge	C
$\vec{F}_q$	Electrostatic Force	N
k	8.988×10^{9}, Coulomb's constant	$N\frac{m^2}{C^2}$
r	Charge separation	m
$\vec{E}_q$	Electric potential energy	J
V	Electric potential difference	V
I	Electric current	A
R	Electrical Resistance	Ω
ρ	Resistivity	Ω·m
L	Length of wire	m
A	Cross sectional area of wire	m^2
P	Electrical power	W
R_{eqs}	Equivalent resistance (series)	Ω
C_{eqp}	Equivalent capacitance (parallel)	F
R_{eqp}	Equivalent resistance (parallel)	Ω
C_{eqs}	Equivalent capacitance (series)	F

Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{a}{b}$$

Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the roots of it can be found using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \qquad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u}$$

- **Pythagorean Identities**

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

The Metric System: Fundamental and Derived Units

Metric System Prefixes

Prefix	Symbol	Factor
tera	T	1 000 000 000 000 = 10^{12}
giga	G	1 000 000 000 = 10^{9}
mega	M	1 000 000 = 10^{6}
kilo	k	1000 = 10^{3}
hecto	h	100 = 10^{2}
deca	da	10 = 10^{1}
		1 = 10^{0}
deci	d	0.1 = 10^{-1}
centi	c	0.01 = 10^{-2}
milli	m	0.001 = 10^{-3}
micro	μ	0.000 001 = 10^{-6}
nano	n	0.000 000 001 = 10^{-9}
pico	p	0.000 000 000 001 = 10^{-12}
femto	f	0.000 000 000 000 001 = 10^{-15}
atto	a	0.000 000 000 000 000 001 = 10^{-18}

Fundamental Physical Quantities and Their SI Units

Quantity	Symbol	Unit	Symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
absolute temperature	T	Kelvin	K
electric current	I	ampère (amp)	A
amount of substance	mol	mole	mol

Derived SI Units

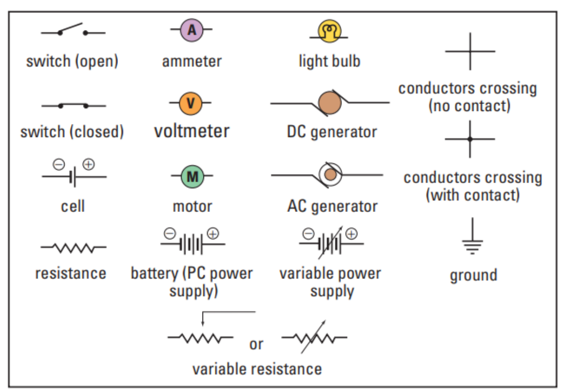
Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	m^2	
volume	V	cubic metre	m^3	
velocity	v	metre per second	m/s	
acceleration	a	metre per second per second	m/s^2	
force	F	newton	N	$kg \cdot m/s^2$
work	W	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
energy	E	joule	J	$N \cdot m$, $kg \cdot m^2/s^2$
power	P	watt	W	J/s, $kg \cdot m^2/s^3$
density	ρ	kilogram per cubic metre	kg/m^3	
pressure	p	pascal	Pa	N/m^2, $kg/(m \cdot s^2)$
frequency	f	hertz	Hz	s^{-1}
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	$A \cdot s$
electric potential	V	volt	V	W/A, J/C, $kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A, $kg \cdot m^2/(C^2 \cdot s)$
magnetic field intensity	B	tesla	T	$N \cdot s/(C \cdot m)$, $N/(A \cdot m)$
magnetic flux	Φ	weber	Wb	$V \cdot s$, $T \cdot m^2$, $m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	s^{-1}
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	˚C	T˚C = (T + 273.15) K
		atomic mass unit	u	$1u = 1.660\ 566 \times 10^{-27}$ kg
		electron volt	eV	$1\ eV = 1.602 \times 10^{-19}$ J

Physical Constants and Data

Fundamental Physical Constants

Quantity	Symbol	Accepted value
speed of light in a vacuum	c	2.998×10^8 m/s
gravitational constant	G	$6.673 \times 10^{-11}\ N \cdot m^2/kg^2$
Coulomb's constant	k	$8.988 \times 10^9\ N \cdot m^2/C^2$
charge on an electron	e	1.602×10^{-19} C
rest mass of an electron	m_e	9.109×10^{-31} kg
rest mass of a proton	m_p	1.673×10^{-27} kg
rest mass of a neutron	m_n	1.675×10^{-27} kg
Planck's constant	h	$6.626 \times 10^{-34}\ J \cdot s$

Electric Circuit Symbols

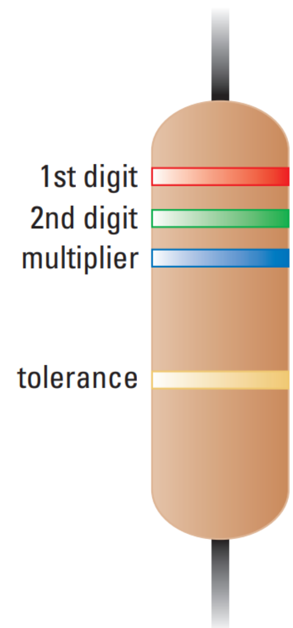


Other Physical Data

Quantity	Symbol	Accepted value
standard atmospheric pressure	P	1.013×10^5 Pa
speed of sound in air		343 m/s (at 20°C)
water: density (4°C)		1.000×10^3 kg/m^3
latent heat of fusion		3.34×10^5 J/kg
latent heat of vaporization		2.26×10^6 J/kg
specific heat capacity (15°C)		4186 J/(kg°C)
kilowatt hour	E	3.6×10^6 J
acceleration due to Earth's gravity	g	9.81 m/s^2 (standard value; at sea level)
mass of Earth	m_E	5.98×10^{24} kg
mean radius of Earth	r_E	6.38×10^6 m
mean radius of Earth's orbit	R_E	1.49×10^{11} m
period of Earth's orbit	T_E	365.25 days or 3.16×10^7 s
mass of Moon	m_M	7.36×10^{22} kg
mean radius of Moon	r_M	1.74×10^6 m
mean radius of Moon's orbit	R_M	3.84×10^8 m
period of Moon's orbit	T_M	27.3 days or 2.36×10^6 s
mass of Sun	m_s	1.99×10^{30} kg
radius of Sun	r_s	6.96×10^8 m

Resistor Colour Codes

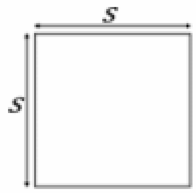
Colour	Digit represented	Multiplier	Tolerance
black	0	$\times 1$	
brown	1	$\times 1.0 \times 10^1$	
red	2	$\times 1.0 \times 10^2$	
orange	3	$\times 1.0 \times 10^3$	
yellow	4	$\times 1.0 \times 10^4$	
green	5	$\times 1.0 \times 10^5$	
blue	6	$\times 1.0 \times 10^6$	
violet	7	$\times 1.0 \times 10^7$	
gray	8	$\times 1.0 \times 10^8$	
white	9	$\times 1.0 \times 10^9$	
gold		$\times 1.0 \times 10^{-1}$	5%
silver		$\times 1.0 \times 10^{-2}$	10%
no colour			20%



GEOMETRY SHAPES AND SOLIDS

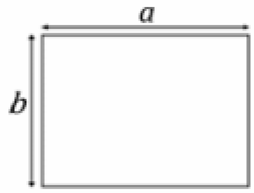
SQUARE

$P = 4s$
$A = s^2$



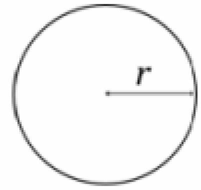
RECTANGLE

$P = 2a + 2b$
$A = ab$



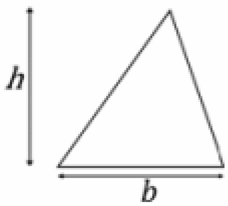
CIRCLE

$P = 2\pi r$
$A = \pi r^2$



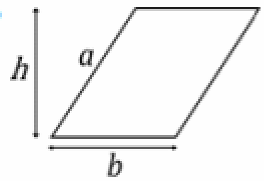
TRIANGLE

$P = a + b + c$
$A = \frac{1}{2} bh$



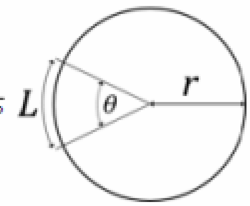
PARALLELOGRAM

$P = 2a + 2b$
$A = bh$



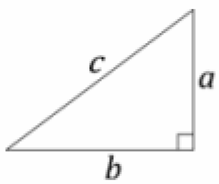
CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$



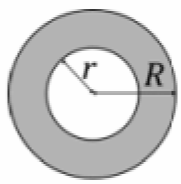
PYTHAGOREAN THEOREM

$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$



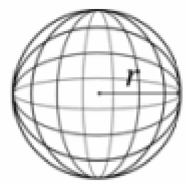
CIRCULAR RING

$A = \pi(R^2 - r^2)$



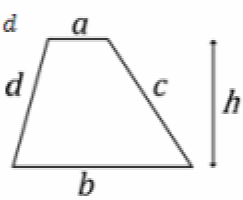
SPHERE

$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$



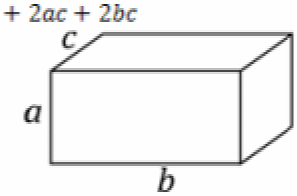
TRAPEZOID

$P = a + b + c + d$
$A = h\frac{a+b}{2}$



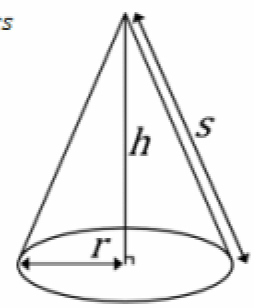
RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$
$V = abc$



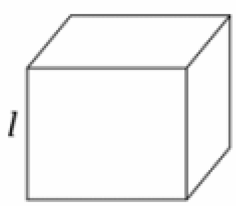
RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



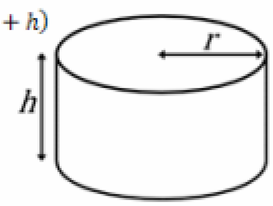
CUBE

$A = 6l^2$
$V = l^3$



CYLINDER

$A = 2\pi r(r + h)$
$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$