

Solutions

4. What is the numerical value for each rational expression? Identify any non-permissible values.

$$\begin{aligned} \text{a) } & \frac{2s-8s}{s} \\ &= \frac{-6s}{s} \\ &= -6, s \neq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{5x-3}{3-5x} \\ &= \frac{5x-3}{-5x+3} \\ &= \frac{5x-3}{-1(5x-3)} \\ &= \frac{1}{-1} \\ &= -1, x \neq \frac{3}{5} \end{aligned}$$

$$\begin{aligned} & 3-5x \neq 0 \\ & \frac{3}{5} \neq \frac{5}{5}x \\ & \frac{3}{5} \neq x \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{2-b}{4b-8} \\ &= \frac{-b+2}{4b-8} \\ &= \frac{-1(b-2)}{4(b-2)} \\ &= \frac{-1}{4}, b \neq 2 \end{aligned}$$

$$\begin{aligned} & 4b-8 \neq 0 \\ & \frac{4b}{4} \neq \frac{8}{4} \\ & b \neq 2 \end{aligned}$$

Solutions

9. Simplify each product. Determine all non-permissible values.

$$\begin{aligned}
 \text{a) } & \frac{2\cancel{p}}{r} \times \frac{10q}{8\cancel{p}} \\
 &= \frac{(2)(10q)}{(r)(8)} \\
 &= \frac{20q}{8r} \\
 &= \frac{5q}{2r}, r \neq 0, p \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & 4\cancel{m^3} \times \frac{1}{16\cancel{m}^4} \\
 &= \frac{4\cancel{m}^2}{16t^3} \\
 &= \frac{m^2}{4t^3}, m \neq 0, t \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{3a+3b}{8} \times \frac{4}{a+b} \\
 &= \frac{3(\cancel{a+b})}{8} \times \frac{4}{\cancel{a+b}} \\
 &= \frac{(3)(4)}{8} \\
 &= \frac{12}{8} \\
 &= \frac{3}{2}, a \neq -b
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{x^2-4}{x^2+25} \times \frac{2x^2+10x}{x^2+2x} \\
 &= \frac{(x-2)(\cancel{x+2})}{x^2+25} \times \frac{2x(\cancel{x+5})}{x(\cancel{x+2})} \\
 &= \frac{2(x-2)(x+5)}{x^2+25}, x \neq 0, -2
 \end{aligned}$$

Solutions

$$\begin{aligned}
 e) & \frac{d^2+3d+2}{2d+2} \times \frac{2d+6}{d^2+5d+6} \\
 & = \frac{\cancel{(d+1)}\cancel{(d+2)}}{\cancel{2}(d+1)} \times \frac{\cancel{2}\cancel{(d+3)}}{\cancel{(d+2)}(d+3)} \\
 & = 1, d \neq -1, -2, -3
 \end{aligned}$$

$$\begin{aligned}
 f) & \frac{y^2-8y-9}{y^2-10y+9} \times \frac{y^2-9y+8}{y^2-1} \times \frac{y^2-25}{5-y} \\
 & = \frac{\cancel{(y+1)}\cancel{(y-9)}}{\cancel{(y+1)}\cancel{(y-9)}} \times \frac{\cancel{(y-1)}(y-8)}{\cancel{(y-1)}\cancel{(y+1)}} \times \frac{(y-5)(y+5)}{-y+5} \\
 & = \frac{(y-8)}{(y-1)} \times \frac{\cancel{(y-5)}(y+5)}{-1\cancel{(y-5)}} \\
 & = \frac{(y-8)(y+5)}{-1(y-1)} \\
 & = \frac{-1(y-8)(y+5)}{(y-1)}, y \neq 1, 9, -1, 5
 \end{aligned}$$

10. Divide. Express answers in simplest form. Identify any non-permissible values.

$$\begin{aligned}
 a) & 2t \div \frac{1}{4} \\
 & = 2t \times \frac{4}{1} \\
 & = \frac{8t}{1} \\
 & = 8t
 \end{aligned}$$

Solutions

$$\begin{aligned}
 \text{b) } & \frac{a^3}{b^4} \div \frac{a^3}{b^3} \\
 &= \frac{\cancel{a^3}}{b^4} \times \frac{\cancel{b^3}}{\cancel{a^3}} \\
 &= \frac{1}{b}, \quad a \neq 0, b \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{7}{x^2-y^2} \div \frac{-35}{x-y} \\
 &= \frac{7}{(\cancel{x-y})(x+y)} \times \frac{\cancel{x-y}}{-35} \\
 &= \frac{7}{-35(x+y)} \\
 &= \frac{-1}{5(x+y)}, \quad x \neq \pm y
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{3a+9}{a-3} \div \frac{a^2+6a+9}{a-3} \\
 &= \frac{3(a+3)}{a-3} \div \frac{(a+3)(a+3)}{a-3} \\
 &= \frac{3(\cancel{a+3})}{\cancel{a-3}} \times \frac{\cancel{a-3}}{(a+3)(\cancel{a+3})} \\
 &= \frac{3}{a+3}, \quad a \neq \pm 3
 \end{aligned}$$

Solutions

11. Multiply or divide as indicated.
Express answers in simplest form.
Determine all non-permissible values.

$$\begin{aligned}
 \text{a) } & \frac{9}{2m} \div \frac{3}{m} \times \frac{m}{3} \\
 & = \frac{9}{\cancel{2m}} \times \frac{\cancel{m}}{3} \times \frac{m}{3} \\
 & = \frac{9(m)}{(2)(3)(3)} \\
 & = \frac{9m}{18} \\
 & = \frac{m}{2}, m \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{x^2-3x+2}{x^2-4} \times \frac{x+3}{x^2+3x} \div \frac{1}{x+2} \\
 & = \frac{(x-1)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} \times \frac{\cancel{x+3}}{x\cancel{(x+3)}} \times \cancel{x+2} \\
 & = \frac{x-1}{x}, x \neq \pm 2, 0, -3
 \end{aligned}$$

15. Add or subtract. Express answers in simplest form. Identify any non-permissible values.

$$\begin{aligned}
 \text{a) } & \frac{4x-3}{6} - \frac{x-2}{4} \\
 & = \frac{2(4x-3)}{12} - \frac{3(x-2)}{12} \\
 & = \frac{2(4x-3)-3(x-2)}{12} \\
 & = \frac{8x-6-3x+6}{12} \\
 & = \frac{5x}{12}
 \end{aligned}$$

Solutions

$$\begin{aligned}
 \text{b) } & \frac{2y-1}{3y} + \frac{y-2}{2y} - \frac{y-8}{6y} \\
 & = \frac{2(2y-1)}{6y} + \frac{3(y-2)}{6y} - \frac{y-8}{6y} \\
 & = \frac{4y-2}{6y} + \frac{3y-6}{6y} - \frac{y-8}{6y} \\
 & = \frac{4y-2+3y-6-y+8}{6y} \\
 & = \frac{6y}{6y} \\
 & = 1, y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{9}{x-3} + \frac{7}{x^2-9} \\
 & = \frac{9}{x-3} + \frac{7}{(x-3)(x+3)} \\
 & = \frac{9(x+3)}{(x-3)(x+3)} + \frac{7}{(x-3)(x+3)} \\
 & = \frac{9x+27+7}{(x-3)(x+3)} \\
 & = \frac{9x+34}{(x-3)(x+3)}, x \neq \pm 3
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{a}{a+3} - \frac{a^2-3a}{a^2+a-6} \\
 & = \frac{a}{a+3} - \frac{a(a-3)}{(a+3)(a-2)} \\
 & = \frac{a(a-2)}{(a+3)(a-2)} - \frac{a(a-3)}{(a+3)(a-2)} \\
 & = \frac{a^2-2a-a^2+3a}{(a+3)(a-2)} \\
 & = \frac{a}{(a+3)(a-2)}, a \neq -3, 2
 \end{aligned}$$

Solutions

20. Solve each rational equation. Identify all non-permissible values.

a) $\frac{s-3}{s+3} = 2$

$$\cancel{s+3} \left[\frac{s-3}{\cancel{s+3}} \right] = s+3[2]$$

$$s-3 = 2(s+3)$$

$$s-3 = 2s+6$$

$$-3-6 = 2s-s$$

$$-9 = s, s \neq -3$$

b) $\frac{x+2}{3x+2} = \frac{x+3}{x-1}$

$$\cancel{(3x+2)}(x-1) \left[\frac{x+2}{\cancel{3x+2}} \right] = (3x+2)\cancel{(x-1)} \left[\frac{x+3}{\cancel{x-1}} \right]$$

$$(x-1)(x+2) = (3x+2)(x+3)$$

$$x^2+2x-1x-2 = 3x^2+9x+2x+6$$

$$x^2+x-2 = 3x^2+11x+6$$

$$0 = 3x^2 - x^2 + 11x - x + 6 + 2$$

$$0 = 2x^2 + 10x + 8$$

$$0 = 2(x^2 + 5x + 4)$$

$$0 = 2(x+1)(x+4)$$

$$0 = x+1 \text{ or } 0 = x+4$$

$$-1 = x$$

$$-4 = x, x \neq -\frac{2}{3}, 1$$

Solutions

$$c) \quad \frac{z-2}{z} + \frac{1}{5} = \frac{-4}{5z}$$

$$5z \left[\frac{z-2}{z} \right] + 5z \left[\frac{1}{5} \right] = 5z \left[\frac{-4}{5z} \right]$$

$$5(z-2) + z(1) = -4$$

$$5z - 10 + z = -4$$

$$6z = -4 + 10$$

$$\frac{6z}{6} = \frac{6}{6}$$

$$z = 1, z \neq 0$$