

SOLUTIONS \Rightarrow REVIEW
(Fact. Not / Perm. / Comb. / Pas. Δ / Bin. Exp.)

1a) Combination \Rightarrow Order is NOT important.

b) Permutation \Rightarrow Order is important
(1st, 2nd, 3rd place winners)

c) Combination \Rightarrow Order is NOT important.

d) Permutation \Rightarrow Order is important
(President, Vice-President, Secretary)

e) Permutation \Rightarrow Order is important.
(first & second prize winners)

2a) $\frac{5!}{4!} = 5 \times 4$

FALSE

b) $10 \times 9 \times 8 = \frac{10!}{7!}$

TRUE

c) ${}_8P_2 = 56$

TRUE

d) ${}_{100}P_4 = 100 \times 99 \times 98 \times 97$

TRUE

$$3a) 7 \times 6 \times 5$$

$$= \frac{7!}{4!}$$

$$b) 19 \times 9 \times 8 \times 7 \times 6$$

$$= \frac{19! 9!}{18! 5!}$$

$$c) 10 \times 9 \times 8 \times 7 \times 6$$

$$= \frac{10!}{5!}$$

$$d) 30 \times 29 \times 12 \times 11 \times 10 \times 9$$

$$= \frac{30! 12!}{28! 8!}$$

$$4a) nP_r = \frac{n!}{(n-r)!}$$

$${}_8P_3 = \frac{8!}{(8-3)!}$$

$${}_8P_3 = 336$$

$$b) nP_r = \frac{n!}{(n-r)!}$$

$${}_8P_8 = \frac{8!}{(8-8)!}$$

$${}_8P_8 = 40320$$

5. ROUND TABLE!

$$\frac{nP_n}{n}$$

$$= \frac{{}_5P_5}{5}$$

$$= \frac{120}{5}$$

$$= 24$$

$$\underline{\text{OR}} \quad (n-1)!$$

$$= (5-1)!$$

$$= 4!$$

$$= 24$$

6a) Boys Girls 2 boys AND 3 girls

$${}^4C_2 = \frac{4!}{2!(4-2)!} \quad {}^6C_3 = \frac{6!}{3!(6-3)!} \quad 6 \times 20$$
$$= 120.$$

$${}^4C_2 = 6 \quad {}^6C_3 = 20$$

b) Boys Girls 4 boys AND 1 girl

$${}^4C_4 = \frac{4!}{4!(4-4)!} \quad {}^6C_1 = \frac{6!}{1!(6-1)!} \quad 1 \times 6$$
$$= 6$$

$${}^4C_4 = 1 \quad {}^6C_1 = 6$$

c) All Boys

$${}^4C_5 = \frac{4!}{5!(4-5)!}$$

↳ This is IMPOSSIBLE!
You cannot choose a group of 5 boys - there are only 4 boys in total.

$$7. \quad nC_r = \frac{n!}{r!(n-r)!}$$

$${}_{100}C_4 = \frac{100!}{4!(100-4)!}$$

$${}_{100}C_4 = 3\,921\,225$$

$P(\text{guessing the winning number})$

$$= \frac{1}{3\,921\,225}$$

8. a) Row 12 is shown.

The second number is always the same as the row number.

b) Row 13.

$${}_{13}C_0 \quad {}_{13}C_1 \quad {}_{13}C_2 \quad {}_{13}C_3 \quad {}_{13}C_4 \quad {}_{13}C_5 \quad {}_{13}C_6 \quad {}_{13}C_7 \quad {}_{13}C_8 \quad {}_{13}C_9 \quad {}_{13}C_{10} \quad {}_{13}C_{11} \quad {}_{13}C_{12} \quad {}_{13}C_{13}$$

$$= 1 \quad 13 \quad 78 \quad 286 \quad 715 \quad 1287 \quad 1716 \quad 1716 \quad 1287 \quad 715 \quad 286 \quad 78 \quad 13 \quad 1$$

c) Row 11

$${}_{11}C_0 \quad {}_{11}C_1 \quad {}_{11}C_2 \quad {}_{11}C_3 \quad {}_{11}C_4 \quad {}_{11}C_5 \quad {}_{11}C_6 \quad {}_{11}C_7 \quad {}_{11}C_8 \quad {}_{11}C_9 \quad {}_{11}C_{10} \quad {}_{11}C_{11}$$

$$= 1 \quad 11 \quad 55 \quad 165 \quad 330 \quad 462 \quad 462 \quad 330 \quad 165 \quad 55 \quad 11 \quad 1$$

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$$a) \quad n=4 \\ j=1$$

$$n=4 \\ j=2$$

$$\begin{aligned} & nC_{j-1} \\ &= 4C_{1-1} \\ &= 4C_0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} & nC_{j-1} \\ &= 4C_{2-1} \\ &= 4C_1 \\ &= 4 \end{aligned}$$

$$b) \quad n=12 \\ j=1$$

$$n=12 \\ j=2$$

$$n=12 \\ j=3$$

$$\begin{aligned} & nC_{j-1} \\ &= 12C_{1-1} \\ &= 12C_0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} & nC_{j-1} \\ &= 12C_{2-1} \\ &= 12C_1 \\ &= 12 \end{aligned}$$

$$\begin{aligned} & nC_{j-1} \\ &= 12C_{3-1} \\ &= 12C_2 \\ &= 66 \end{aligned}$$

$$c) \quad n=18 \\ j=7$$

$$d) \quad n=15 \\ j=8$$

$$\begin{aligned} & nC_{j-1} \\ &= 18C_{7-1} \\ &= 18C_6 \\ &= 8008 \end{aligned}$$

$$\begin{aligned} & nC_{j-1} \\ &= 15C_{8-1} \\ &= 15C_7 \\ &= 6435 \end{aligned}$$

$$10. a) (x+2)^6$$

$$= {}_6C_0(x)^6(2)^0 + {}_6C_1(x)^5(2)^1 + {}_6C_2(x)^4(2)^2 + {}_6C_3(x)^3(2)^3 + {}_6C_4(x)^2(2)^4 + {}_6C_5(x)^1(2)^5 + {}_6C_6(x)^0(2)^6$$

$$= (1)(x^6)(1) + (6)(x^5)(2) + (15)(x^4)(4) + (20)(x^3)(8) + (15)(x^2)(16) + (6)(x)(32) + (1)(1)(64)$$

$$= 1x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$$

$$b) (x+3)^4$$

$$= {}_4C_0(x)^4(3)^0 + {}_4C_1(x)^3(3)^1 + {}_4C_2(x)^2(3)^2 + {}_4C_3(x)^1(3)^3 + {}_4C_4(x)^0(3)^4$$

$$= (1)(x^4)(1) + (4)(x^3)(3) + (6)(x^2)(9) + (4)(x)(27) + (1)(1)(81)$$

$$= 1x^4 + 12x^3 + 54x^2 + 108x + 81$$

$$c) (x-4)^5$$

$$\hookrightarrow (x+(-4))^5$$

$$= {}_5C_0(x)^5(-4)^0 + {}_5C_1(x)^4(-4)^1 + {}_5C_2(x)^3(-4)^2 + {}_5C_3(x)^2(-4)^3 + {}_5C_4(x)^1(-4)^4 + {}_5C_5(x)^0(-4)^5$$

$$= (1)(x^5)(1) + (5)(x^4)(-4) + (10)(x^3)(16) + (10)(x^2)(-64) + (5)(x)(256) + (1)(1)(-1024)$$

$$= 1x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$$

$$d) (x-6)^3$$

$$\hookrightarrow (x+(-6))^3$$

$$= {}_3C_0(x)^3(-6)^0 + {}_3C_1(x)^2(-6)^1 + {}_3C_2(x)^1(-6)^2 + {}_3C_3(x)^0(-6)^3$$

$$= (1)(x^3)(1) + (3)(x^2)(-6) + (3)(x)(36) + (1)(1)(-216)$$

$$= 1x^3 - 18x^2 + 108x - 216$$

$$11. (x+3)^{20}$$

$$= {}_{20}C_0(x)^{20}(3)^0 + {}_{20}C_1(x)^{19}(3)^1 + {}_{20}C_2(x)^{18}(3)^2 \dots$$

$$= (1)(x^{20})(1) + (20)(x^{19})(3) + (190)(x^{18})(9) \dots$$

$$= 1x^{20} + 60x^{19} + 1710x^{18} \dots$$