Questions From Homework

④ d) $y = \sqrt[3]{x}$, $(\overset{x_1}{-8}, \overset{y_1}{-2})$

$y = x^{1/3}$

① Find derivative:

$y' = \frac{1}{3}x^{-2/3}$

$$\boxed{y' = \frac{1}{3x^{2/3}}}$$

② Sub in x-value (x = -8)

$y' = \frac{1}{3(-8)^{2/3}}$

$y' = \frac{1}{3(4)}$

$$\boxed{y' = \frac{1}{12}}$$

③ Find the equation:

$y - y_1 = m(x - x_1)$

$12 \cdot y + 12 \cdot 2 = 12 \cdot \frac{1}{12}(x+8)$

$12y + 24 = 1(x+8)$

$12y + 24 = x + 8$

$-x + 12y + 16 = 0$

$$\boxed{x - 12y - 16 = 0}$$

⑤ $f(x) = \boxed{\frac{1}{x}}$ | $f(x+h) = \boxed{\frac{1}{x+h}}$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

multiply by:
$(x)(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{x - (x+h)}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x)(x+h)} = \boxed{\frac{-1}{x^2}}$$

② f) $y = \frac{2}{x^2} = 2x^{-2}$

$y' = -4x^{-3} = \frac{-4}{x^3}$

j) $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$

Example:

Find the slope of the tangent line to the graph of the given function at the given x value.

$$g(x) = \sqrt[5]{x} \qquad x = 32$$

$$g(x) = x^{1/5}$$

$$g'(x) = \frac{1}{5}x^{-4/5}$$

$$g'(x) = \frac{1}{5x^{4/5}}$$

$$g'(32) = \frac{1}{5(32)^{4/5}}$$

$$= \frac{1}{5(16)}$$

$$\boxed{= \frac{1}{80}}$$

① Slope (-192)
② Point $(-2, 64)$

Example:

Find the equation of the tangent line to the curve $f(x) = x^6$ at the point (-2, 64)

Remember that the equation of a line is found by using the point-slope formula… $y - y_1 = m(x - x_1)$

The curve is the graph of the function $f(x) = x^6$ and we know that the slope of the tangent line at (-2, 64) is the derivative $f'(-2)$

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

$f(x) = x^6$

① Find Derivative

$f'(x) = 6x^5$

② Sub in x-value $(x=-2)$

$f'(-2) = 6(-2)^5$

$= 6(-32)$

$\boxed{= -192}$

Slope of my tangent line

③ Find the equation:

$y - y_1 = m(x - x_1)$

slope

$y - 64 = -192(x + 2)$

$y - 64 = -192x - 384$

$\boxed{192x + y + 320 = 0}$

Sums and Differences

- These next rules say that <u>the derivative of a sum (difference) of functions is the sum (difference) of the derivatives</u>:

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x)=2x^4+\sqrt{x}$

$f(x)=2x^4+x^{1/2}$

$f'(x)=8x^3+\frac{1}{2}x^{-1/2}$

$f'(x)=8x^3+\frac{1}{2x^{1/2}}=8x^3+\frac{1}{2\sqrt{x}}$

2. $f(x)=6x^4-5x^3-2x+17$

$f'(x)=24x^3-15x^2-2x^0+0$

$f'(x)=24x^3-15x^2-2$

3. $f(x)=\left(2x^3-5\right)^2$

$f(x)=(2x^3-5)(2x^3-5)$

$f(x)=4x^6-20x^3+25$

$f'(x)=24x^5-60x^2$

Homework