Questions from Homework

⑰ j) $f(x)=\dfrac{6x^3-30x^2-84x}{2x^3+3x^2+x}$

$$=\frac{6x(x^2-5x-14)}{x(2x^2+3x+1)}$$

$$=\frac{6x(x-7)(x+2)}{x(2x^2+2x+x+1)}$$

$$=\frac{6x(x-7)(x+2)}{x[2x(x+1)+1(x+1)]}$$

$$=\frac{\cancel{6x}(x-7)(x+2)}{\cancel{x}(2x+1)(x+1)}$$

$2x+1=0$

$2x=-1$

$x=-1/2$

① Roots

$x=-2,7$

② V.A.

$x=-1,-1/2$

③ H.A.

$y=\frac{6}{2}\to 3$

④ Hole:

$X=0$

Rational Functions Continued

We will explore the properties of rational functions of this form so we can predict the locations of vertical, horizontal, and oblique asymptotes. We will also be able to identify the roots of the function and any other points of discontinuity (holes).

$$y = \frac{(x+2)(x+3)}{(x-1)}$$

$$y = \frac{(x+2)(x+3)}{(x+2)}$$

$$y = \frac{x^2+2x-3}{x^2+3x-4}$$

$$f(x) = \frac{4x}{x^2-4}$$

Roots
Are given by the zeroes of the numerator.

Vertical Asymptotes:
Are given by the zeroes of the denominator

Horizontal Asymptotes:
If the numerator and denominator have the same degree, then the horizontal asymptote is given by the quotient of the leading coefficients of the numerator and denominator.

If the degree of the denominator is greater than that of the numerator, then the horizontal asymptote is given by $y = 0$.

If the degree of the denominator is less than that of the numerator, then there is no horizontal asymptote (*an oblique asymptote exists).*

Holes
Occur when the same factor is in the numerator and the denominator.

Oblique Asymptotes: "x"
A line with a finite, non-zero slope that a graph approaches at extreme values but never crosses. They occur when the degree of the numerator is one greater than the degree of the denominator and can be determined by dividing the numerator by the denominator (ignoring the remainder).

We can use the factor theorem (long division) or synthetic substitution

Ex: $f(x) = \dfrac{x^2+7x+12}{x+5} = \dfrac{(x+4)(x+3)}{x+5}$

① Roots $x=-4,-3$

② V.A. $x=-5$

③ O.A. $y=x+2$

④ Holes: None

Divide:

$$\begin{array}{r l}
 & \boxed{x+2} \leftarrow \text{Oblique Asymptote} \\
x+5 \,\big) & x^2+7x+12 \\
 & -(x^2+5x) \\
 & \quad\quad 2x+12 \\
 & \quad -(2x+10) \\
 & \quad\quad\quad 2R
\end{array}$$

$$x^2+2x+3 \,\big)\overline{2x^3-x^2+3x-4}$$

This table below shows whether a rational function has a horizontal or oblique asymptote.

Type of Equation	Horizontal Asymptote	Oblique Asymptote
Degree of numerator is equal to degree of denominator	Given by quotient of leading coefficients in numerator and denominator	
Degree of numerator is less than degree of denominator	$y = 0$	
Degree of numerator is one more than degree of denominator		The equation can be found by examining the quotient of numerator and denominator (ignoring the remainder)

Examples

Determine any vertical, horizontal, and oblique asymptotes, and any roots or holes for the following rational functions.

$$f(x)=\frac{x^2+2x-3}{x+1} = \frac{(x+3)(x-1)}{x+1}$$

$$\begin{array}{r} \boxed{x+1} \rightarrow \text{O.A.} \\ x+1 \overline{)\,x^2+2x-3} \\ -(x^2+x) \downarrow \\ x-3 \\ -(x+1) \\ -4R \end{array}$$

① Roots
$x=-3, 1$

② V.A.
$x=-1$

③ O.A.
$y=x+1$

④ Holes:
None

$$f(x)=\frac{x^2+2x-3}{x+2} = \frac{(x+3)(x-1)}{x+2}$$

$$\begin{array}{r} \boxed{x} \rightarrow \text{O.A.} \\ x+2 \overline{)\,x^2+2x-3} \\ -(x^2+2x) \downarrow \\ -3R \end{array}$$

① Roots
$x=-3, 1$

② V.A.
$x=-2$

③ O.A.
$y=x$

④ Holes:
None

Factored form

$$f(x)=\frac{x^2+5x+6}{(x+2)^2} = \frac{x^2+5x+6}{x^2+4x+4} = \frac{\cancel{(x+2)}(x+3)}{(x+2)^{\cancel{2}}}$$

① Roots:
$x=-3$

② V.A.
$x=-2$

③ H.A.
$y=1$

④ Holes:
None

$$f(x)=\frac{x^2+5x+6}{2x^3+6x^2+4x}$$

Graphing Rational Functions

$$f(x)=\frac{x^2+2x-3}{x+1} = \frac{(x+3)(x-1)}{x+1}$$

$$\begin{array}{r} \boxed{x+1} \rightarrow \text{O.A.} \\ x+1 \overline{) x^2+2x-3} \\ -(x^2+x) \downarrow \\ x-3 \\ -(x+1) \\ \hline -4R \end{array}$$

① Roots
$x=-3, 1$

② V.A.
$x=-1$

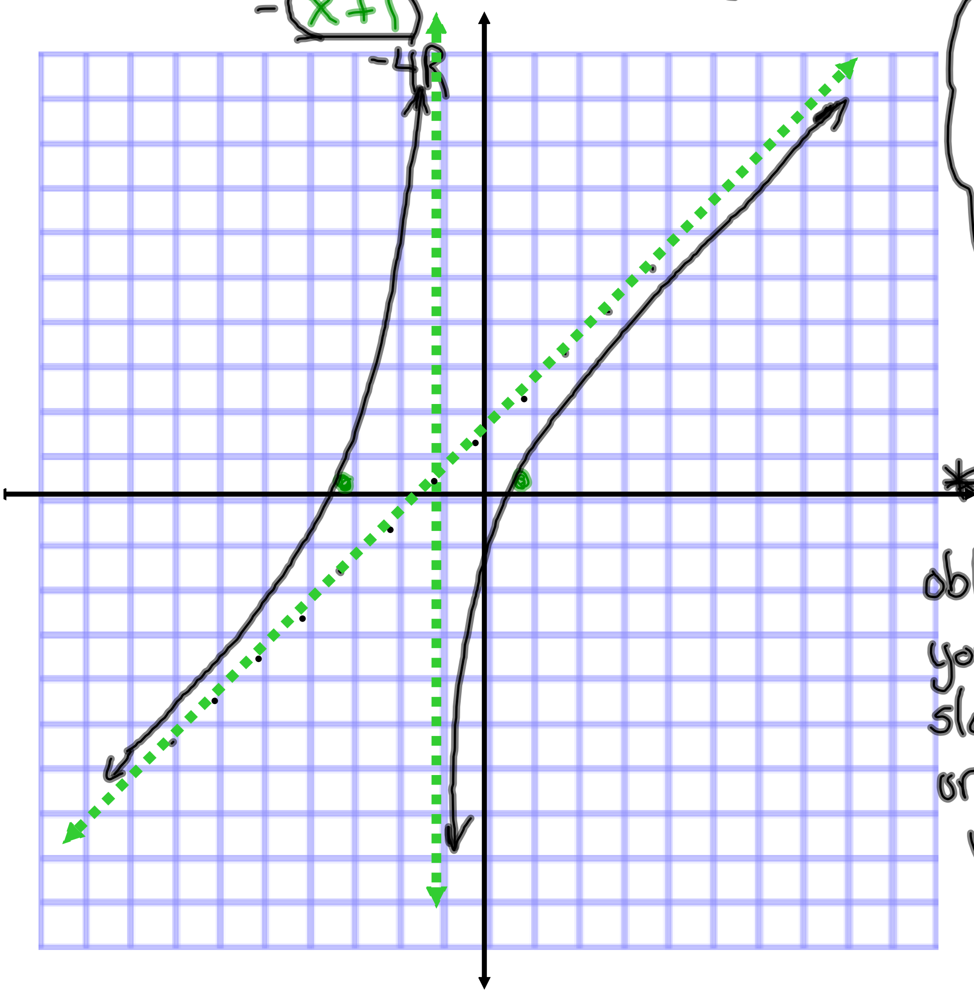
③ O.A.
$y=x+1$

$m=1$
$b=1$

or

x	y
-2	-1
-1	0
0	1
1	2
2	3

④ Holes:
None



* To graph your oblique asymptote you can use the slope y-int method or a table of values.

Homework