Polar Coordinates

1. **Convert 4 - 3i to Polar form**

$a+bi \longrightarrow r\text{cis}\theta$

Find the radius ***r,*** using the Pythagorean relationship $r = \sqrt{x^2 + y^2}$

Find the related angle, α, using $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

★

180-α	α
180+α	360-α

Remember from last semester

The polar form is $r\text{cis}\theta$

2. **Convert 2cis47° to rectangular form**

$r\text{cis}\theta \longrightarrow a+bi$

$a = r\cos\theta$

$b = r\sin\theta$

① $4-3i$

$a=4$
$b=-3$ } Quad 4

① $r=\sqrt{(4)^2+(-3)^2}$

$r=\sqrt{25}$

$r=5$

② $\alpha=\tan^{-1}\left(\frac{3}{4}\right)$

$\alpha=36.9°$

③ Quad 4

$\theta=360°-\alpha$

$\theta=360°-36.9$

$\theta=323.1°$

④ $\boxed{5\text{cis}323.1°}$

② $2\text{cis}47°$

$r=2$

$\theta=47°$

① $a=r\cos\theta$

$=2\cos47°$

$=1.36$

② $b=r\sin\theta$

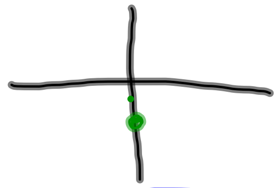
$=2\sin47°$

$=1.46$

③ $\boxed{1.36+1.46i}$

19 c) $0-2i$

$a+bi \longrightarrow r\text{cis}\theta$



① $r=\sqrt{0^2+(-2)^2}$

$r=\sqrt{4}$

$r=2$

② $\alpha=\tan^{-1}\left(\frac{2}{0}\right)$

*

$\alpha=270°$

③ $\boxed{2\text{cis}270°}$

Operations with Complex Numbers in Polar Form

Multiply the following complex numbers

$$(2+3i)(3+i) = 6+11i+3(i^2) = 3+11i$$

$$\left(\sqrt{13}\operatorname{cis}56.3^\circ\right)\left(\sqrt{10}\operatorname{cis}18.4^\circ\right) = \left(\sqrt{130}\operatorname{cis}74.7^\circ\right)$$

Convert all complex numbers from rectangular form to Polar form.

① $r=\sqrt{(2)^2+(3)^2}$ ② $\alpha=\tan^{-1}\left(\frac{3}{2}\right)$ ③ Quad I ④ $\sqrt{13}\operatorname{cis}56.3^\circ$

$r=\sqrt{13}$

$\theta=\alpha$

$\alpha=56.3^\circ$

$\theta=56.3^\circ$

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① $r=\sqrt{(3)^2+(1)^2}$ ② $\alpha=\tan^{-1}\left(\frac{1}{3}\right)$ ③ Quad I ④ $\sqrt{10}\operatorname{cis}18.4$

$r=\sqrt{10}$

$\theta=\alpha$

$\alpha=18.4^\circ$

$\theta=18.4^\circ$

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① $r=\sqrt{(3)^2+(11)^2}$ ② $\alpha=\tan^{-1}\left(\frac{11}{3}\right)$ ③ Quad I ④ $\sqrt{130}\operatorname{cis}74$

$r=\sqrt{130}$

$\theta=\alpha$

$\alpha=74.7^\circ$

$\theta=74.7^\circ$

Do the same for the following complex numbers

$$(1+4i)(3-2i)$$

You may have noticed a shortcut when multiplying complex numbers in Polar form.

angles θ

- When Multiplying, *multiply* the "r" values and *add* the arguments.
- When Dividing you *divide* the "r" values and *subtract* the arguments

Argument:

The angle from the positive real axis to the position vector representing a complex number in the complex plane. If the number is written in ***polar form*** as ***rcis***θ then θ is the argument and "r" is the modulus.

Examples

$$(2cis150°)(3cis200°) = 6cis350°$$

$$(2\sqrt{2}cis60°)(3\sqrt{8}cis240°) = 6\sqrt{16}\,cis300° = 24cis300°$$

$$(3cis150°)(5cis240°) = 15cis390° = 15cis30°$$

$$* = 15cos30 + 15i\sin 30°$$

$$\frac{45cis120°}{3cis190°} = 15cis(-70°) = 15cis290°$$

De Moivre's Theorem

* Complex #'s must be in polar form

$$(r cis\theta)^n = r^n cis n\theta$$

$$(1+i\sqrt{3})^9$$

$a=1 \quad b=\sqrt{3}$

① $r=\sqrt{(1)^2+(\sqrt{3})^2}$ ② $\alpha=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ ③ Quad I ④ $2cis60°$

$r=\sqrt{4}$ $\quad$ $\theta=\alpha$

$\alpha=60°$ $\quad$ $\theta=60°$

$r=2$

$(2cis60°)^9 = 2^9 cis(9\cdot 60) = 512cis540°$

Polar form $= \boxed{512cis\,180°}$

If the question asked you to express your answer in rectangular form

① $a = 512\cos180°$ ② $b=512\sin180°$ ③ $-512+0i$

$a=-512$ $\quad$ $b=0$

Rectangular form $= \boxed{-512}$

$512cis180°$

$= 512\cos180° + 512i\sin180°$

$= 512(-1) + 512i(0)$

$= \boxed{-512}$ Rectangular Form

Homework