Solving Rational Equations

Solve for x

$$\frac{x+6}{x^2-4}=\frac{2}{x-2}+\frac{x}{x+2}$$

Multiply by the LCD: $(x-2)(x+2)$

$$\frac{(x+6)(x-2)(x+2)}{(x-2)(x+2)}=\frac{2(x-2)(x+2)}{(x-2)}+\frac{x(x-2)(x+2)}{(x+2)}$$

$$x+6=2(x+2)+x(x-2)$$

$$x+6=2x+4+x^2-2x$$

$$x+6=x^2+4$$

$$0=x^2-x-2 \quad \leftarrow \text{Factor}$$

$$0=(x-2)(x+1)$$

$x-2=0$ | $x+1=0$

$x=2$ | $x=-1$ ✓

$$\frac{x+6}{x^2-4}=\frac{2}{x-2}+\frac{x}{x+2}$$

Test $x=2$

$$\frac{(2)+6}{(2)^2-4}=\frac{2}{(2)-2}+\frac{(2)}{(2)+2}$$

$$\frac{8}{0}=\frac{2}{0}+\frac{2}{4}$$

$x=2$ is not a solution LHS $\neq$ RHS

Test $x=-1$

$$\frac{(-1)+6}{(-1)^2-4}=\frac{2}{(-1)-2}+\frac{(-1)}{(-1)+2}$$

$$\frac{5}{-3}=\frac{2}{-3}+\left(\frac{-1}{1}\right)$$

$$-\frac{5}{3}=-\frac{2}{3}-\frac{3}{3}$$

$$-\frac{5}{3}=-\frac{5}{3}$$

$x=-1$ is a solution LHS = RHS

Solving Irrational Equations

Solve for x

$$\left(\sqrt{2x+7}\right)^2 = (x-4)^2$$

$$2x+7 = x^2-8x+16$$

$$0 = x^2-10x+9$$

$$0 = (x-1)(x-9)$$

$x-1=0$ | $x-9=0$

$x=1$ | $x=9$

$x=1$ ↑ extraneous root

$x=9$ is a solution

Test $x=1$

$$\sqrt{2(1)+7} = (1)-4$$

$$\sqrt{2+7} = -3$$

$$\sqrt{9} = -3$$

$$3 \neq -3$$

Test $x=9$

$$\sqrt{2(9)+7} = (9)-4$$

$$\sqrt{18+7} = 5$$

$$\sqrt{25} = 5$$

$$5 = 5 \checkmark$$

Solving Irrational Equations

Solve for x

$$\left(\sqrt{3x-5}\right)^2=\left(3+\sqrt{x-2}\right)^2$$ * Square both sides

$3x-5 = 9+6\sqrt{x-2}+(x-2)$

$3x-5 = 7+6\sqrt{x-2}+x$ * Isolate the radical

Factor ↘ $2x-12 = 6\sqrt{x-2}$

$\frac{2(x-6)}{2} = \frac{6\sqrt{x-2}}{2}$ * Divide by 2

$(x-6)^2 = (3\sqrt{x-2})^2$ * Square both sides

$x^2-12x+36 = 9(x-2)$

$x^2-12x+36 = 9x-18$

$x^2-21x+54 = 0$ * Factor

$(x-18)(x-3) = 0$

$x-18=0$ | $x-3=0$

$x=18$ | $x=3$ → Extraneous Root

Solution

$$\sqrt{3x-5}=3+\sqrt{x-2}$$

Test $x=18$

$\sqrt{3(18)-5} = 3+\sqrt{(18)-2}$

$\sqrt{54-5} = 3+\sqrt{16}$

$\sqrt{49} = 3+4$

$7 = 7$

Test $x=3$

$\sqrt{3(3)-5} = 3+\sqrt{(3)-2}$

$\sqrt{9-5} = 3+\sqrt{1}$

$\sqrt{4} = 3+1$

$2 \neq 4$

Homework