

Solutions ⇒ Probability Review

1. a) $P(\text{ace}) = \frac{4}{52}$
 $= \frac{1}{13}$

b) $P(\text{not ace})$
 $= 1 - P(\text{ace})$
 $= 1 - \frac{4}{52}$
 $= \frac{52}{52} - \frac{4}{52}$
 $= \frac{48}{52}$
 $= \frac{12}{13}$

c) $P(\text{diamond}) = \frac{13}{52}$
 $= \frac{1}{4}$

d) $P(\text{red card}) = \frac{26}{52}$
 $= \frac{1}{2}$

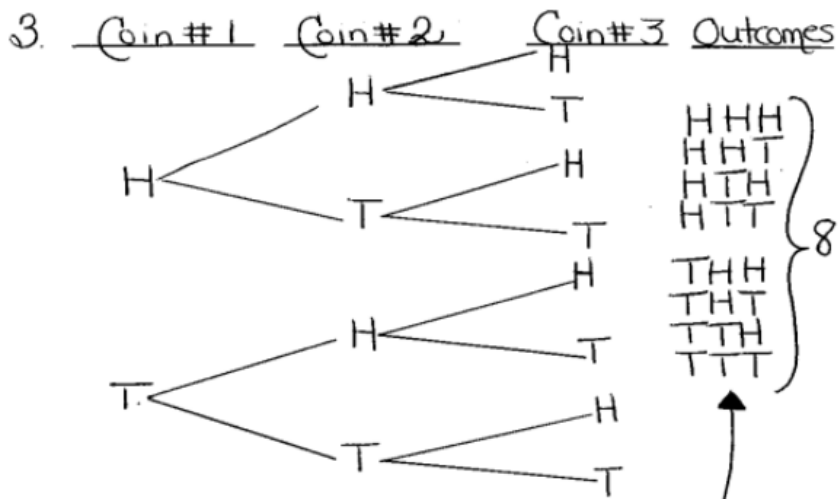
e) $P(4 \text{ of spades}) = \frac{1}{52}$

2 a) $P(\text{blue}) = \frac{3}{18}$
 $= \frac{1}{6}$

b) $P(\text{blue or green})$
 $= P(\text{blue}) + P(\text{green})$
 $= \frac{3}{18} + \frac{6}{18}$
 $= \frac{9}{18}$
 $= \frac{1}{2}$

c) $P(\text{green or red})$
 $= P(\text{green}) + P(\text{red})$
 $= \frac{6}{18} + \frac{4}{18}$
 $= \frac{10}{18}$
 $= \frac{5}{9}$

d) $P(\text{blue or black})$
 $= P(\text{blue}) + P(\text{black})$
 $= \frac{3}{18} + \frac{5}{18}$
 $= \frac{8}{18}$
 $= \frac{4}{9}$



a) The possible outcomes are

b) $P(2 \text{ heads followed by 1 tail}) = \frac{1}{8}$

c) $P(2 \text{ heads and a tail}) = \frac{3}{8}$

d) $P(3 \text{ coins the same}) = \frac{2}{8}$
 $= \frac{1}{4}$

4a) Throwing a 4 with the first die and a 6 with the second \Rightarrow independent

b) Picking a heart or a diamond from a deck of cards \Rightarrow mutually exclusive.

c) Drawing a spade and a heart from the same deck without replacing the card first drawn. \Rightarrow dependent

d) Picking the winner of the Grey Cup and Stanley Cup \Rightarrow independent

e) Rolling a 3 or an odd number on a die
 \hookrightarrow mutually inclusive.

f) Choosing a black marble and a red marble if the marbles are put back in the bag each time \Rightarrow independent.

g) Choosing a diamond or a red card
 \hookrightarrow mutually inclusive.

$$5.$$

$$a) P(\text{red and rolling 6}) = P(\text{red}) \times P(\text{rolling 6}) = \frac{26}{52} \times \frac{1}{6} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$b) P(\text{spade and rolling 4}) = P(\text{spade}) \times P(\text{rolling 4}) = \frac{13}{52} \times \frac{1}{6} = \frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$$

$$c) P(\text{face card and rolling less than 3}) = P(\text{f.c.}) \times P(<3) = \frac{12}{52} \times \frac{2}{6} = \frac{3}{13} \times \frac{1}{3} = \frac{3}{39} = \frac{1}{13}$$

$$d) P(\text{not black and not rolling a 4}) = P(\text{not black}) \times P(\text{not rolling 4}) = \frac{26}{52} \times \frac{5}{6} = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$

$$6.$$

$$a) P(\text{green and green}) = P(\text{green}) \times P(\text{green} | \text{green}) = \frac{7}{15} \times \frac{6}{14} = \frac{7}{15} \times \frac{3}{7} = \frac{21}{105} = \frac{1}{5}$$

$$b) P(\text{green and blue and red}) = P(\text{green}) \times P(\text{blue} | \text{green}) \times P(\text{red} | \text{green \& blue}) = \frac{7}{15} \times \frac{5}{14} \times \frac{3}{13} = \frac{105}{2730} = \frac{1}{26}$$

$$c) P(\text{blue and blue and blue and blue and blue}) = P(b) \times P(b|b) \times P(b|2b) \times P(b|3b) \times P(b|4b) = \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \times \frac{2}{12} \times \frac{1}{11} = \frac{1}{3} \times \frac{2}{7} \times \frac{3}{13} \times \frac{1}{6} \times \frac{1}{11} = \frac{6}{18018} = \frac{1}{3003}$$

$$\begin{aligned}
 7. \quad & \text{a) } P(\text{odd or } 3) \\
 & = P(\text{odd}) + P(3) - P(\text{odd and } 3) \\
 & = \frac{3}{6} + \frac{1}{6} - \frac{1}{6} \\
 & = \frac{4}{6} - \frac{1}{6} \\
 & = \frac{3}{6} \\
 & = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 & \text{b) } P(\text{even or number greater than } 4) \\
 & = P(\text{even}) + P(\# > 4) - P(\text{even and } \# > 4) \\
 & = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} \\
 & = \frac{5}{6} - \frac{1}{6} \\
 & = \frac{4}{6} \\
 & = \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 & \text{c) } P(6 \text{ or } 4) \\
 & = P(6) + P(4) \\
 & = \frac{1}{6} + \frac{1}{6} \\
 & = \frac{2}{6} \\
 & = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 & \text{d) } P(\text{greater than } 3 \text{ or less than } 4) \\
 & = P(\text{greater than } 3) + P(\text{less than } 4) \\
 & = \frac{3}{6} + \frac{3}{6} \\
 & = \frac{6}{6} \\
 & = 1 \leftarrow \text{CERTAIN!}
 \end{aligned}$$

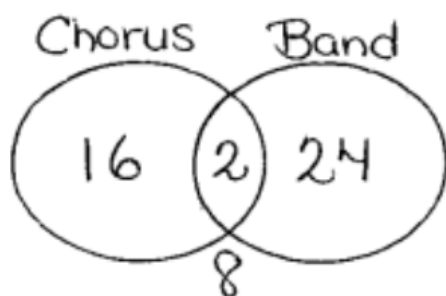
$$\begin{aligned}
 & \text{e) } P(\text{even or odd}) \\
 & = P(\text{even}) + P(\text{odd}) \\
 & = \frac{3}{6} + \frac{3}{6} \\
 & = \frac{6}{6} \\
 & = 1 \leftarrow \text{CERTAIN!}
 \end{aligned}$$

8. 50 total students
- 18 take Chorus
 - 26 take Band
 - 2 take both Chorus and Band.

Conclusion: $18 - 2 = 16$ take only Chorus
 $26 - 2 = 24$ take only Band.

$50 - 16 - 24 - 2 = 8$ do not take either Chorus or Band.

a) Venn Diagram



b) 8 students are not enrolled in either Chorus or Band.

c) i) $P(\text{band}) = \frac{26}{50}$
 $= \frac{13}{25}$

ii) $P(\text{not band})$
 $= 1 - P(\text{band})$
 $= 1 - \frac{26}{50}$
 $= \frac{50 - 26}{50}$
 $= \frac{24}{50}$
 $= \frac{12}{25}$

iii) $P(\text{band or chorus})$
 $= \frac{18}{50} + \frac{26}{50} - \frac{2}{50}$
 $= \frac{44}{50} - \frac{2}{50}$
 $= \frac{42}{50}$ or $\frac{21}{25}$

iv) $P(\text{band and chorus})$
 $= \frac{2}{50}$
 $= \frac{1}{25}$

$$\begin{aligned}
 9.a) P(\text{goalie and goalie}) &= P(\text{goalie}) \times P(\text{goalie} | \text{goalie}) \\
 &= \frac{6}{30} \times \frac{5}{29} \\
 &= \frac{1}{5} \times \frac{5}{29} \\
 &= \frac{5}{145} \\
 &= \frac{1}{29}
 \end{aligned}$$

$$\begin{aligned}
 b) P(\text{forward and forward}) &= P(\text{forward}) \times P(\text{forward} | \text{forward}) \\
 &= \frac{10}{30} \times \frac{9}{29} \\
 &= \frac{1}{3} \times \frac{9}{29} \\
 &= \frac{9}{87} \\
 &= \frac{3}{29}
 \end{aligned}$$

$$\begin{aligned}
 c) P(\text{forward and defenseman}) &= P(\text{forward}) \times P(\text{defenseman} | \text{forward}) \\
 &= \frac{10}{30} \times \frac{14}{29} \\
 &= \frac{1}{3} \times \frac{14}{29} \\
 &= \frac{14}{87}
 \end{aligned}$$

$$\begin{aligned}
 d) P(\text{goalie and forward}) &= P(\text{goalie}) \times P(\text{forward} | \text{goalie}) \\
 &= \frac{6}{30} \times \frac{10}{29} \\
 &= \frac{1}{5} \times \frac{10}{29} \\
 &= \frac{10}{145} \\
 &= \frac{2}{29}
 \end{aligned}$$

$$\begin{aligned}
 e) P(\text{defensemen and goalie}) &= P(\text{defensemen}) \times P(\text{goalie} | \text{defenseman}) \\
 &= \frac{14}{30} \times \frac{6}{29} \\
 &= \frac{7}{15} \times \frac{6}{29} \\
 &= \frac{42}{435} \\
 &= \frac{14}{145}
 \end{aligned}$$

$$\begin{aligned}
 10. \text{ a) } & P(\text{black or even}) \\
 &= P(\text{black}) + P(\text{even}) - P(\text{black and even}) \\
 &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\
 &= \frac{15}{18} - \frac{3}{18} \\
 &= \frac{12}{18} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & P(\text{green or 4}) \\
 &= P(\text{green}) + P(4) - P(\text{green and 4}) \\
 &= \frac{6}{18} + \frac{3}{18} - \frac{1}{18} \\
 &= \frac{9}{18} - \frac{1}{18} \\
 &= \frac{8}{18} \\
 &= \frac{4}{9}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & P(\text{black or blue}) \\
 &= P(\text{black}) + P(\text{blue}) \\
 &= \frac{1}{3} + \frac{1}{3} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & P(\text{blue or black or green}) \\
 &= P(\text{blue}) + P(\text{black}) + P(\text{green}) \\
 &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\
 &= \frac{3}{3} \\
 &= 1 \leftarrow \text{CERTAIN!}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } & P(\text{black or greater than 3}) \\
 &= P(\text{black}) + P(>3) - P(\text{black and } >3) \\
 &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\
 &= \frac{15}{18} - \frac{3}{18} \\
 &= \frac{12}{18} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned} \text{f) } P(\text{green or black}) &= P(\text{green}) + P(\text{black}) \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{g) } P(\text{blue or odd}) &= P(\text{blue}) + P(\text{odd}) - P(\text{blue and odd}) \\ &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\ &= \frac{15 - 3}{18} \\ &= \frac{12}{18} \\ &= \frac{2}{3} \end{aligned}$$