Advanced Mathematics with Calculus 120

Limits

1. Evaluate the following limits if they exist. If a limit does not exist provide a reason to support your claim.

(a) $\lim\limits_{x \to 0} \dfrac{\dfrac{2}{x+2} - 1}{x}$

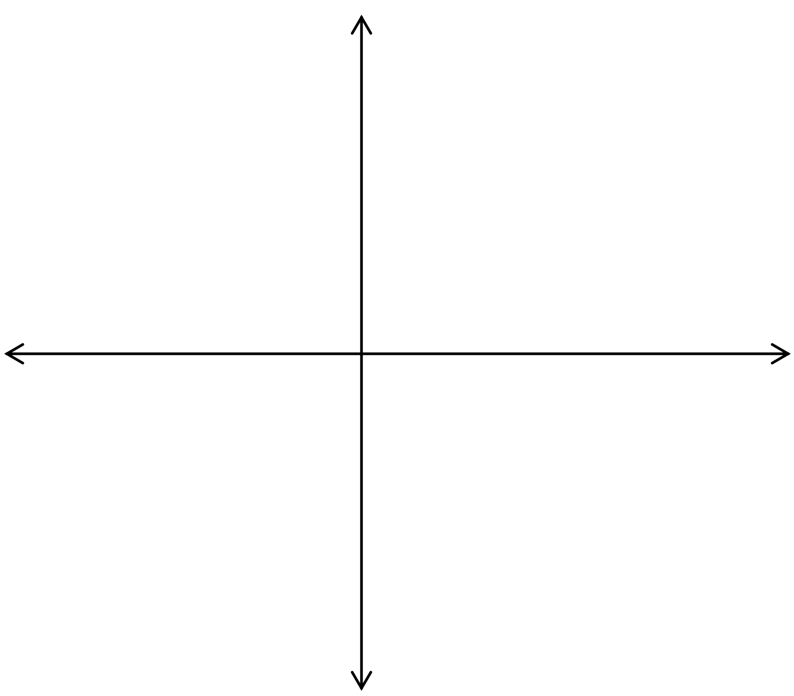
(b) $\lim\limits_{x \to \infty} \dfrac{(2-3x^2)^2}{6x^4 - 7x^2 - 5}$

(c) $\lim\limits_{x \to 1} \dfrac{(x+2)^3 - 27}{x-1}$

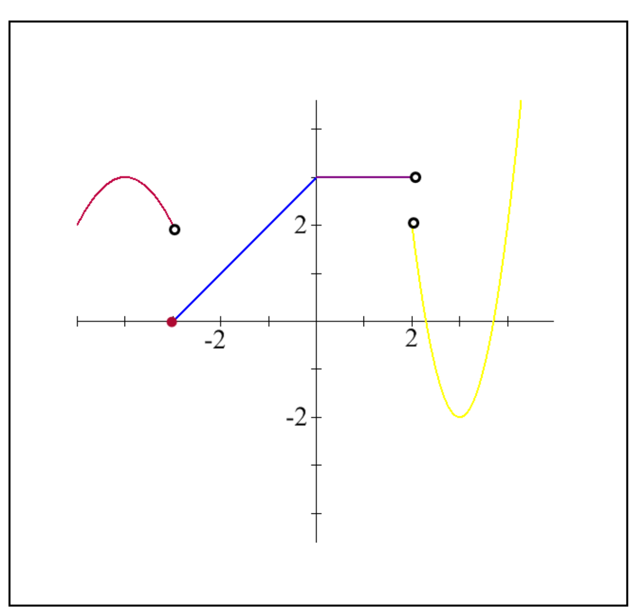
(d) $\lim\limits_{x \to 7} \dfrac{\sqrt{x+9} - 4}{x-7}$

2. Given the function … $f(x) = \begin{cases} 3 - x & \text{if } x < -1 \\ 4 & \text{if } -1 \le x < 2 \\ 6 & \text{if } x = 2 \\ (x-2)^2 + 4 & \text{if } x > 2 \end{cases}$

Draw a sketch of *f(x)* and list any point(s) of discontinuity



3. The following is a graph of *f*(*x*):



Evaluate each of the following:

(a) $\lim\limits_{x \to -3^+} f(x) =$___ (b) $\lim\limits_{x \to -3^-} f(x) =$____

(c) $f(-3) =$____ (d) $\lim\limits_{x \to 2^-} f(x) =$____

(e) $\lim\limits_{x \to 2^+} f(x) =$___ (f) $f(2) =$____

(g) $\lim\limits_{x \to 3} f(x) =$___ (h) $f(3) =$____

4. Differentiate the following functions using the *limit definition of the derivative:*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

a) $f(x) = x^2 + 4x + 2$

b) $f(x) = \dfrac{2x-1}{4x}$

5. Find the slope of the tangent to the curve at the given point.

a) $f(x) = 3x^2 + \dfrac{5}{x} - 4$ at x = *-2*

b) $f(x) = \dfrac{4}{x^3}$ at x = *3*

6. Find the equation of the tangent line to the curve at the given point.

a) $y = 3x^2 + 5x$ at *(2, 22)*

b) $y = 2x^2 - 6\sqrt{x}$ *at (4, 20)*

7. Find the derivative: *Express answers with positive exponents!*

a) $f(x) = 3x^5 + \sqrt[3]{x}$

b) $f(x) = \sqrt[5]{x^2}$

c) $g(x) = 4x^3 - \dfrac{6}{x^2} + 14x$

d) $y = (3x^2 - 5)^2$