SOLUTIONS => REVIEW
(Fact. Not / Perm. / Comb. / Pas. Δ / Bin. Exp.

1a) Combination => Order is NOT important.

b) Permutation => Order is important
(1^{st}, 2^{nd}, 3^{rd} place winners)

c) Combination => Order is NOT important.

d) Permutation => Order is important
(President, Vice-President, Secretary)

e) Permutation => Order is important.
(first & second prize winners)

2a) $\frac{5!}{4!} = 5 \times 4$

FALSE

b) $10 \times 9 \times 8 = \frac{10!}{7!}$

TRUE

c) ${}_8P_2 = 56$

TRUE

d) ${}_{100}P_4 = 100 \times 99 \times 98 \times 97$

TRUE

3a) $7 \times 6 \times 5$

$= \frac{7!}{4!}$

b) $19 \times 9 \times 8 \times 7 \times 6$

$= \frac{19!\,9!}{18!\,5!}$

c) $10 \times 9 \times 8 \times 7 \times 6$

$= \frac{10!}{5!}$

d) $30 \times 29 \times 12 \times 11 \times 10 \times 9$

$= \frac{30!\,12!}{28!\,8!}$

4a) ${}_nP_r = \frac{n!}{(n-r)!}$

${}_8P_3 = \frac{8!}{(8-3)!}$

${}_8P_3 = 336$

b) ${}_nP_r = \frac{n!}{(n-r)!}$

${}_8P_8 = \frac{8!}{(8-8)!}$

${}_8P_8 = 40\,320.$

5. ROUND TABLE!

$$\frac{{}_nP_n}{n} \quad \underline{\underline{\text{OR}}} \quad (n-1)!$$

$$= \frac{{}_5P_5}{5} \qquad = (5-1)!$$

$$= \frac{120}{5} \qquad = 4!$$

$$= 24. \qquad = 24.$$

6a) <u>Boys</u> <u>Girls</u> 2 boys <u>AND</u> 3 girls

$_4C_2 = \frac{4!}{2!(4-2)!}$ $_6C_3 = \frac{6!}{3!(6-3)!}$ 6×20

$= 120.$

$_4C_2 = 6$ $_6C_3 = 20$

b) <u>Boys</u> <u>Girls</u> 4 boys <u>AND</u> 1 girl

$_4C_4 = \frac{4!}{4!(4-4)!}$ $_6C_1 = \frac{6!}{1!(6-1)!}$ 1×6

$= 6$

$_4C_4 = 1$ $_6C_1 = 6$

c) All Boys

$${}_4C_5 = \frac{4!}{5!(4-5)!}$$

↳ This is IMPOSSIBLE!
You cannot choose a group of 5 boys - there are only 4 boys in total.

7. $_nC_r = \dfrac{n!}{r!(n-r)!}$

$_{100}C_4 = \dfrac{100!}{4!(100-4)!}$

$_{100}C_4 = 3\,921\,225$

P(guessing the winning number)

$= \dfrac{1}{3\,921\,225}$

8. a) Row 12 is shown.
The second number is always the same as the row number.

b) Row 13.

$${}_{13}C_0 \ {}_{13}C_1 \ {}_{13}C_2 \ {}_{13}C_3 \ {}_{13}C_4 \ {}_{13}C_5 \ {}_{13}C_6 \ {}_{13}C_7 \ {}_{13}C_8 \ {}_{13}C_9 \ {}_{13}C_{10} \ {}_{13}C_{11} \ {}_{13}C_{12} \ {}_{13}C_{13}$$

$$= 1 \quad 13 \quad 78 \quad 286 \quad 715 \quad 1287 \quad 1716 \quad 1716 \quad 1287 \quad 715 \quad 286 \quad 78 \quad 13 \quad 1$$

c) Row 11

$${}_{11}C_0 \ {}_{11}C_1 \ {}_{11}C_2 \ {}_{11}C_3 \ {}_{11}C_4 \ {}_{11}C_5 \ {}_{11}C_6 \ {}_{11}C_7 \ {}_{11}C_8 \ {}_{11}C_9 \ {}_{11}C_{10} \ {}_{11}C_{11}$$

$$= 1 \quad 11 \quad 55 \quad 169 \quad 330 \quad 462 \quad 462 \quad 330 \quad 169 \quad 55 \quad 11 \quad 1$$

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a) $n=4$, $j=1$

nC_{j-1}
$= 4C_{1-1}$
$= 4C_0$
$= 1$

$n=4$, $j=2$

nC_{j-1}
$= 4C_{2-1}$
$= 4C_1$
$= 4$

b) $n=12$, $j=1$

nC_{j-1}
$= 12C_{1-1}$
$= 12C_0$
$= 1$

$n=12$, $j=2$

nC_{j-1}
$= 12C_{2-1}$
$= 12C_1$
$= 12$

$n=12$, $j=3$

nC_{j-1}
$= 12C_{3-1}$
$= 12C_2$
$= 66$

c) $n = 18$
$r = 7$

$_nC_{r-1}$
$= {}_{18}C_{7-1}$
$= {}_{18}C_6$
$= 18\,564$

d) $n = 15$
$r = 8$

$_nC_{r-1}$
$= {}_{15}C_{8-1}$
$= {}_{15}C_7$
$= 6435$

10. a) $(x+2)^6$

$= {}_6C_0(x)^6(2)^0 + {}_6C_1(x)^5(2)^1 + {}_6C_2(x)^4(2)^2 + {}_6C_3(x)^3(2)^3 + {}_6C_4(x)^2(2)^4 + {}_6C_5(x)^1(2)^5 + {}_6C_6(x)^0(2)^6$

$=(1)(x^6)(1)+(6)(x^5)(2)+(15)(x^4)(4)+(20)(x^3)(8)+(15)(x^2)(16)+(6)(x^1)(32)+(1)(1)(64)$

$=1x^6+12x^5+60x^4+160x^3+240x^2+192x^1+64$

b) $(x+3)^4$.

$= {}_4C_0(x)^4(3)^0 + {}_4C_1(x)^3(3)^1 + {}_4C_2(x)^2(3)^2 + {}_4C_3(x)^1(3)^3 + {}_4C_4(x)^0(3)^4$

$=(1)(x^4)(1)+(4)(x^3)(3)+(6)(x^2)(9)+(4)(x^1)(27)+(1)(1)(81)$

$=1x^4+12x^3+54x^2+108x^1+81$

c) $(x-4)^5$

$\hookrightarrow (x+(-4))^5$

$= {}_5C_0(x)^5(-4)^0 + {}_5C_1(x)^4(-4)^1 + {}_5C_2(x)^3(-4)^2 + {}_5C_3(x)^2(-4)^3 + {}_5C_4(x)^1(-4)^4 + {}_5C_5(x)^0(-4)^5$

$= (1)(x^5)(1) + (5)(x^4)(-4) + (10)(x^3)(16) + (10)(x^2)(-64) + (5)(x^1)(256) + (1)(1)(-1024)$

$= 1x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x^1 - 1024$

d) $(x-6)^3$

$\hookrightarrow (x+(-6))^3$

$= {}_3C_0(x)^3(-6)^0 + {}_3C_1(x)^2(-6)^1 + {}_3C_2(x)^1(-6)^2 + {}_3C_3(x)^0(-6)^3$

$= (1)(x^3)(1) + (3)(x^2)(-6) + (3)(x^1)(36) + (1)(1)(-216)$

$= 1x^3 - 18x^2 + 108x^1 - 216$

11. $(x+3)^{20}$

$= {}_{20}C_0(x)^{20}(3)^0 + {}_{20}C_1(x)^{19}(3)^1 + {}_{20}C_2(x)^{18}(3)^2 \circ\circ\circ$

$= (1)(x^{20})(1) + (20)(x^{19})(3) + (190)(x^{18})(9) \circ\circ\circ$

$= 1x^{20} + 60x^{19} + 1710x^{18} \circ\circ\circ$