# SOLUTIONS => EQUATIONS OF CIRCLES/ELLIPSES REVIEW

1 $\{x^2+y^2=r^2\}$

a) $r=6$
↳ $x^2+y^2=(6)^2$
$x^2+y^2=36$

b) $r=\sqrt{5}$
↳ $x^2+y^2=(\sqrt{5})^2$
$x^2+y^2=5$

c) $r=4\sqrt{7}$
↳ $x^2+y^2=(4\sqrt{7})^2$
$x^2+y^2=(16)(7)$
$x^2+y^2=112$

d) $r=3\sqrt{2}$
$x^2+y^2=(3\sqrt{2})^2$
$x^2+y^2=(9)(2)$
$x^2+y^2=18$

2. A: $x^2+y^2=81$

a) radius
↳ $r^2=81$
$r=\sqrt{81}$
$r=9$ units

b) $x$-intercepts
↳ $-9$ and $+9$

c) $y$-intercepts
↳ $-9$ and $9$

d) domain
$\{x \mid -9 \leq x \leq 9, x \in R\}$

e) range
$\{y \mid -9 \leq x \leq 9, y \in R\}$

2. B: $x^2+y^2=48$

a) radius
↳ $r^2=48$
$r=\sqrt{48}$
$r=\sqrt{16\times3}$
$r=4\sqrt{3}$ units

b) x-intercepts
↳ $-4\sqrt{3}$ and $4\sqrt{3}$

c) y-intercepts
↳ $-4\sqrt{3}$ and $4\sqrt{3}$

d) domain
$\{x \mid -4\sqrt{3} \le x \le 4\sqrt{3}, x \in R\}$

e) range
$\{y \mid -4\sqrt{3} \le y \le 4\sqrt{3}, y \in R\}$

3. $x^2+y^2=25$

a) $(-4, ?)$

If $x=-4$:

$(-4)^2+y^2=25$
$16+y^2=25$
$y^2=25-16$
$y^2=9$
$y=\pm\sqrt{9}$ $y=\pm3$

Coordinate => $(-4,-3)$
or
$(-4,3)$

b) $x^2+y^2=25$

$(?,3)$

If $y=3$:

$x^2+(3)^2=25$

$x^2+9=25$

$x^2=25-9$

$x^2=16$

$x=\pm\sqrt{16}$

$x=\pm 4$

Coordinate: $(-4,3)$ or $(4,3)$

c) $(5,?)$

If $x=5$:

$(5)^2+y^2=25$

$25+y^2=25$

$y^2=25-25$

$y^2=0$

$y=\pm\sqrt{0}$

$y=0$

Coordinate: $(5,0)$

d) $(2\sqrt{3}, ?)$

If $x = 2\sqrt{3}$:

$$(2\sqrt{3})^2 + y^2 = 25$$
$$(4)(3) + y^2 = 25$$
$$12 + y^2 = 25$$
$$y^2 = 25 - 12$$
$$y^2 = 13$$
$$y = \sqrt{13}$$

Coordinate: $(2\sqrt{3}, \sqrt{13})$

4.

| EQUATION | CENTER | DOMAIN | RANGE | X-INTERCEPTS | Y-INTERCEPTS |
|---|---|---|---|---|---|
| $x^2+y^2=9$ | $(0,0)$ | $\{x \mid -3 \le x \le 3, x \in R\}$ | $\{y \mid -3 \le y \le 3, y \in R\}$ | -3 and 3 | -3 and 3 |
| $x^2+y^2=36$ | $(0,0)$ | $\{x \mid -6 \le x \le 6, x \in R\}$ | $\{y \mid -6 \le y \le 6, y \in R\}$ | -6 and 6 | -6 and 6 |
| $x^2+y^2=121$ | $(0,0)$ | $\{x \mid -11 \le x \le 11, x \in R\}$ | $\{y \mid -11 \le y \le 11, y \in R\}$ | -11 and 11 | -11 and 11 |

5. a) passing through $(2,-4)$

↳ If $x=2$ and $y=-4$:

$$x^2+y^2=r^2$$
$$(2)^2+(-4)^2=r^2$$
$$4+16=r^2$$
$$20=r^2$$

↳ Therefore, the equation would be:

$$x^2+y^2=r^2$$
$$x^2+y^2=20$$

b) passing through $(4\sqrt{5},\sqrt{2})$

↳ If $x=4\sqrt{5}$ and $y=\sqrt{2}$:

$$x^2+y^2=r^2$$
$$(4\sqrt{5})^2+(\sqrt{2})^2=r^2$$
$$(16)(5)+2=r^2$$
$$80+2=r^2$$
$$82=r^2$$

↳ Therefore, the equation would be:

$$x^2+y^2=r^2$$
$$x^2+y^2=82$$

c) With an $x$-intercept of $-12$:
↳ Point $(-12,0)$

If $x=-12$ and $y=0$:

$$x^2+y^2=r^2$$
$$(-12)^2+(0)^2=r^2$$
$$144+0=r^2$$
$$144=r^2$$

↳ Therefore, the equation would be:

$$x^2+y^2=r^2$$
$$x^2+y^2=144$$

6. a) $x^2+(y-4)^2=64$
$$(x-0)^2+(y-4)^2=64$$

Center $(0,4)$

Radius: $r^2=64$
$r=\sqrt{64}$
$r=8$ units

b) $(x+2)^2+y^2=49$
$$(x+2)^2+(y+0)^2=49$$

Center $(-2,0)$

Radius: $r^2=49$
$r=\sqrt{49}$
$r=7$ units

c) $(x+1)^2+(y-11)^2=100$

Center $(-1,11)$

Radius: $r^2=100$
$r=\sqrt{100}$
$r=10$ units

d) $(x-16)^2+(y-3)^2=144$

Center $(16,3)$

Radius: $r^2=144$
$r=\sqrt{144}$
$r=12$ units

7.
a) $C(-11,6)$; $r=\sqrt{7}$

$(x-h)^2+(y-k)^2=r^2$
$(x--11)^2+(y-6)^2=(\sqrt{7})^2$
$(x+11)^2+(y-6)^2=7$

b) $C(0,3)$; $r=5$

$(x-h)^2+(y-k)^2=r^2$
$(x-0)^2+(y-3)^2=(5)^2$
$x^2+(y-3)^2=25$

c) $C(4,-4)$; $r=13$

$(x-h)^2+(y-k)^2=r^2$
$(x-4)^2+(y--4)^2=(13)^2$
$(x-4)^2+(y+4)^2=169$

d) $C(-9, 14)$ ; $r = 2$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x - -9)^2 + (y - 14)^2 = (2)^2$$
$$(x+9)^2 + (y-14)^2 = 4$$

8.a) $C(2,-2)$ and passing through $J(8,4)$

Method 1:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8-2)^2 + (4 - -2)^2}$$
$$= \sqrt{(6)^2 + (6)^2}$$
$$= \sqrt{36+36}$$
$$= \sqrt{72}$$
$$= \sqrt{36 \times 2}$$
$$= 6\sqrt{2}$$

$r = 6\sqrt{2}$ ; $C(2,-2)$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y - -2)^2 = (6\sqrt{2})^2$$
$$(x-2)^2 + (y+2)^2 = (36)(2)$$
$$(x-2)^2 + (y+2)^2 = 72$$

Method 2:

$C(2,-2)$ $J(8,4)$

$h$ $k$ $x$ $y$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(8-2)^2 + (4 - -2)^2 = r^2$$
$$(6)^2 + (6)^2 = r^2$$
$$36 + 36 = r^2$$
$$72 = r^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-2)^2 + (y - -2)^2 = 72$$
$$(x-2)^2 + (y+2)^2 = 72$$

b) $C(10,0)$ and passing through $K(1,-3)$

Method 1:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(1-10)^2+(-3-0)^2}$$
$$=\sqrt{(-9)^2+(-3)^2}$$
$$=\sqrt{81+9}$$
$$=\sqrt{90}$$
$$=\sqrt{9\times 10}$$
$$=3\sqrt{10}$$

$r=3\sqrt{10}$ ; $C(10,0)$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-10)^2+(y-0)^2=(3\sqrt{10})^2$$
$$(x-10)^2+y^2=(9)(10)$$
$$(x-10)^2+y^2=90$$

Method 2:

$C(10,0)$ $K(1,-3)$
$\quad h\ k \qquad\quad x\ y$

$$(x-h)^2+(y-k)^2=r^2$$
$$(1-10)^2+(-3-0)^2=r^2$$
$$(-9)^2+(-3)^2=r^2$$
$$81+9=r^2$$
$$90=r^2$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-10)^2+(y-0)^2=90$$
$$(x-10)^2+y^2=90$$

9a) $x^2+y^2-12y-5=0$

Step 1: $x^2 \quad +y^2-12y=5$

Step 2: $x^2 \quad +y^2-12y+36=5+36$

Step 3: $(x-0)^2+(y-6)^2=41$

Center $(0,6)$ ; $r=\sqrt{41}$ units

b) $x^2+y^2-4x-1=0$

Step 1: $x^2-4x \quad +y^2=1$

Step 2: $x^2-4x+4 \quad +y^2=1+4$

Step 3: $(x-2)^2+(y-0)^2=5$

Center $(2,0)$ ; $r=\sqrt{5}$ units

c) $x^2+y^2-6x-8y-1=0$

Step 1: $x^2-6x \quad +y^2-8y=1$

Step 2: $x^2-6x+9+y^2-8y+16=1+9+16$

Step 3: $(x-3)^2+(y-4)^2=26$

Center $(3,4)$ ; $r=\sqrt{26}$ units

d) $2x^2+2y^2+16x-8y+19=0$

Extra Step: Divide each term by 2

$$x^2+y^2+8x-4y+\frac{19}{2}=0$$

Step 1: $x^2+8x \quad + y^2-4y = -\frac{19}{2}$

Step 2: $x^2+8x+16+y^2-4y+4 = -\frac{19}{2}+16+4$

Step 3: $(x+4)^2+(y-2)^2 = -\frac{19}{2}+\frac{20}{1}$

$$(x+4)^2+(y-2)^2 = -\frac{19}{2}+\frac{40}{2}$$

$$(x+4)^2+(y-2)^2 = \frac{21}{2}$$

Center $(-4,2)$ ; $r^2=\frac{21}{2}$

$r=\sqrt{\frac{21}{2}}$ units

e) $3x^2+3y^2-36x+48y+100=0$

Extra Step: Divide each term by 3.

$$x^2+y^2-12x+16y+\frac{100}{3}=0$$

Step 1: $x^2-12x \quad +y^2+16y = -\frac{100}{3}$

Step 2: $x^2-12x+36+y^2+16y+64=-\frac{100}{3}+36+64$

Step 3: $(x-6)^2+(y+8)^2 = -\frac{100}{3}+\frac{100}{1}$

$$(x-6)^2+(y+8)^2 = -\frac{100}{3}+\frac{300}{3}$$

$$(x-6)^2+(y+8)^2 = \frac{200}{3}$$

Center $(6,-8)$ ; $r^2=\frac{200}{3}$

$r=\sqrt{\frac{200}{3}}$

$r=\frac{\sqrt{100\times 2}}{\sqrt{3}}$

$r=\frac{10\sqrt{2}}{\sqrt{3}}$ units

10. $x^2+y^2-6x-8y-39=0$

Step 1: $x^2-6x \quad +y^2-8y=39$

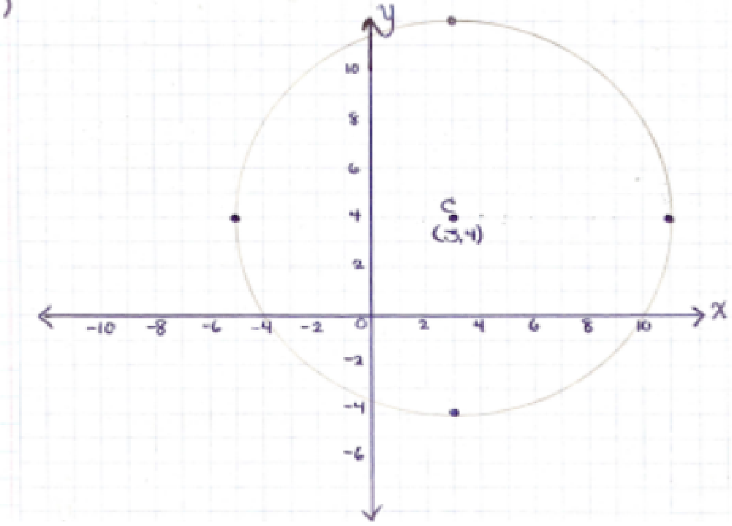
Step 2: $x^2-6x+9+y^2-8y+16=39+9+16$

Step 3: $(x-3)^2+(y-4)^2=64$

a) Center $(3,4)$

b) radius: $r^2=64$
$r=\sqrt{64}$
$r=8$ units

c)




d) Domain: $\{x \mid -5 \leq x \leq 11, x \in R\}$

e) Range: $\{y \mid -4 \leq y \leq 12, y \in R\}$

11. a) $\dfrac{x^2}{36}+\dfrac{y^2}{1}=1$

$\dfrac{x^2}{(6)^2}+\dfrac{y^2}{(1)^2}=1$

↳ Horizontal Ellipse

$a=6$
$b=1$

i) Major Axis $=2a$
$=2(6)$
$=12$ units

Minor Axis $=2b$
$=2(1)$
$=2$ units

ii) Vertices.
$(-6,0)$ and $(6,0)$

iii) x-ints ⇒ $-6$ and $6$
y-ints ⇒ $-1$ and $1$

b) $\dfrac{x^2}{16}+\dfrac{y^2}{49}=1$

$\dfrac{x^2}{(4)^2}+\dfrac{y^2}{(7)^2}=1$

↳ Vertical Ellipse

$a=7$
$b=4$

i) Major Axis $=2a$
$=2(7)$
$=14$ units

Minor Axis $=2b$
$=2(4)$
$=8$ units

ii) vertices.
$(0,-7)$ and $(0,7)$

iii) x-ints ⇒ $-4$ and $4$
y-ints ⇒ $-7$ and $7$

c) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$

↳ Vertical Ellipse.

$a = 3$

$b = 2$

i) Major Axis $= 2a$
$= 2(3)$
$= 6$ units

Minor Axis $= 2b$
$= 2(2)$
$= 4$ units

ii) vertices
$(0,-3)$ and $(0,3)$

iii) x-ints => $-2$ and $2$
y-ints => $-3$ and $3$

12.
a) Major Axis is 12 => $2a = 12$
$a = 6$

Minor Axis is 5 => $2b = 5$
$b = \frac{5}{2}$

*Vertical.

Equation: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{(5/2)^2} + \frac{y^2}{(6)^2} = 1$

$\frac{x^2}{25/4} + \frac{y^2}{36} = 1$

↳ $\frac{4x^2}{25} + \frac{y^2}{36} = 1$

b) → $x$-ints ⇒ $\pm 7$

→ $y$-ints ⇒ $\pm 9$.

Equation: $\frac{x^2}{(7)^2} + \frac{y^2}{(9)^2} = 1$

$\frac{x^2}{49} + \frac{y^2}{81} = 1$

c) → One vertex ⇒ $(10,0)$

↳ Remaining vertex must be $(-10,0)$

* Therefore the major axis is 20 units

↳ $2a = 20$

$a = 10$.

→ Minor Axis $= 6$

↳ $2b = 6$

$b = 3$

Equation: $\frac{x^2}{(10)^2} + \frac{y^2}{(3)^2} = 1$

$\frac{x^2}{100} + \frac{y^2}{9} = 1$

13a) $16(x-1)^2 + 12(y+3)^2 = 48$

$\frac{16(x-1)^2}{48} + \frac{12(y+3)^2}{48} = \frac{48}{48}$

$\frac{(x-1)^2}{3} + \frac{(y+3)^2}{4} = 1$

Center $(1,-3)$

Since this ellipse is vertical, it is parallel to the $y$-axis.

b) $3(x+4)^2 + 9(y-2)^2 = 27$

$\frac{3(x+4)^2}{27} + \frac{9(y-2)^2}{27} = \frac{27}{27}$

$\frac{(x+4)^2}{9} + \frac{(y-2)^2}{3} = 1$

Center $(-4,2)$

Since this ellipse is horizonal, it is parallel to the $x$-axis.

14. a) $9x^2+25y^2-36x-100y-89=0$ x-axis.

Step 1: $9x^2-36x \ +25y^2-100y=89$

Extra Step: $9(x^2-4x)+25(y^2-4y)=89$

Step 2: $9(x^2-4x+4)+25(y^2-4y+4)=89+36+100$

Step 3: $9(x-2)^2+25(y-2)^2=225$

$$\frac{9(x-2)^2}{225}+\frac{25(y-2)^2}{225}=\frac{225}{225}$$

$$\frac{(x-2)^2}{25}+\frac{(y-2)^2}{9}=1$$

b) $2x^2+5y^2+20x-30y+75=0$

Step 1: $2x^2+20x \ +5y^2-30y=-75$

Extra Step: $2(x^2+10x)+5(y^2-6y)=-75$

Step 2: $2(x^2+10x+25)+5(y^2-6y+9)=-75+50+45$

Step 3: $2(x+5)^2+5(y-3)^2=20$

$$\frac{2(x+5)^2}{20}+\frac{5(y-3)^2}{20}=\frac{20}{20}$$

$$\frac{(x+5)^2}{10}+\frac{(y-3)^2}{4}=1$$

15. $25x^2+16y^2+150x+32y-159=0$

Step 1: $25x^2+150x \quad +16y^2+32y=159$

Extra Step: $25(x^2+6x)+16(y^2+2y)=159$

Step 2: $25(x^2+6x+9)+16(y^2+2y+1)=159+225+16$

Step 3: $25(x+3)^2+16(y+1)^2=400$

$$\frac{25(x+3)^2}{400}+\frac{16(y+1)^2}{400}=\frac{400}{400}$$

$$\frac{(x+3)^2}{16}+\frac{(y+1)^2}{25}=1$$

a) Center $(-3,-1)$

b) $\frac{(x+3)^2}{(4)^2}+\frac{(y+1)^2}{(5)^2}=1$ $\quad C(-3,-1)$

vertices

$(-3,-1-5)$ and $(-3,-1+5)$

$(-3,-6)$ and $(-3,4)$

c) Major Axis $=2a$
$=2(5)$
$=10$ units

Minor Axis $=2b$
$=2(4)$
$=8$ units

16. $4x^2+9y^2-8x-54y+49=0$

Step 1: $4x^2-8x \quad +9y^2-54y=-49$

Extra Step: $4(x^2-2x)+9(y^2-6y)=-49$

Step 2: $4(x^2-2x+1)+9(y^2-6y+9)=-49+4+81$

Step 3: $4(x-1)^2+9(y-3)^2=36$

$$\frac{4(x-1)^2}{36}+\frac{9(y-3)^2}{36}=\frac{36}{36}$$

$$\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4}=1$$

a) Center $(1,3)$

b) $\frac{(x-1)^2}{(3)^2}+\frac{(y-3)^2}{(2)^2}=1$

vertices $(1-3,3)$ and $(1+3,3)$
$(-2,3)$ and $(4,3)$

d) Major Axis $=2a$
$=2(3)$
$=6$ units

Minor Axis $=2b$
$=2(2)$
$=4$ units