

# **Specialized Factoring Techniques**

- **Common Factors**
- **The Sum and Difference of Cubes**
- **Grouping to Find a Common Factor**
- **Quartic Expressions Factored as Trinomials**
- **Grouping to get the Difference of Squares**

Let's Start with a quick refresher!

### Common Factor

$$12x^7y^8 + 24x^9y^4$$

## **Simple Trinomial**

$$x^2 - 5x + 6$$

## **Trinomial Decomposition**

$$4x^2 + 5x - 6$$

## **Difference of Squares**

$$a^2 - b^2 = (a - b)(a + b)$$

$$81x^2 - 49b^2$$

## Common Factor

$$x^3 - x^2 - 12x$$

## **Difference of Cubes**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$x^3 - 8$$

$$27x^3 - 64$$

## Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$x^3 + 27$$

$$64x^3 + 125$$



## Grouping to Find a Common Factor

A common factor can sometimes be found for specific groups of terms in a polynomial expression. The expression is written in the necessary order and each group of terms is then factored, leaving a common factor in brackets, which in turn is factored.

$$x^3 - 2x^2 - 16x + 32$$

$$x^3 + 4x^2 - 4x - 16$$

$$8x^5 - 40x^4 + 32x^3 - x^2 + 5x - 4$$

## Questions From Homework

$$\textcircled{2} \text{ d) } (x^5 - 5x^4)(10x^3 + 50x^2) + 9x - 45$$
$$x^4(x-5) - 10x^2(x-5) + 9(x-5)$$

$$(x-5)(x^4 - 10x^2 + 9)$$

$$(x-5)(x^2 - 1)(x^2 - 9)$$

$$(x-5)(x-1)(x+1)(x-3)(x+3)$$

## Quartic Expressions Factored as Trinomials

$$x^4 - 5x^2 + 4$$

$$(x^2 - 1)(x^2 - 4)$$

$$(x+1)(x-1)(x-2)(x+2)$$

$$\textcircled{4}x^4 - \underline{37}x^2 + 9 \quad \begin{array}{l} -\underline{36}x - \underline{1} = 36 \\ -\underline{36} + \underline{-1} = -37 \end{array}$$

$$(4x^4 - x^2)(36x^2 + 9)$$

$$x^2(4x^2 - 1) - 9(4x^2 - 1)$$

$$\boxed{(4x^2 - 1)} \boxed{(x^2 - 9)}$$

$$(2x-1)(2x+1)(x-3)(x+3)$$

### Grouping to Get the Difference of Squares

If a polynomial expression can be grouped in the form  $(x+m)^2-n^2$ , then it can be factored as the difference of squares.

$$(x^2 - 6x + 9) - b^2$$

$$(x-3)(x-3) - b^2$$

$$\boxed{(x-3)^2} - \boxed{b^2}$$

$$\boxed{(x-3) - b} \boxed{(x-3) + b}$$

$$(x-3-b)(x-3+b)$$

$$x^4 + 8x^2 - a^2 + 16$$

$$(x^4 + 8x^2 + 16) - a^2$$

$$(x^2+4)(x^2+4) - a^2$$

$$\boxed{(x^2+4)^2} - \boxed{a^2}$$

$$\boxed{(x^2+4) - a} \boxed{(x^2+4) + a}$$

$$(x^2+4-a)(x^2+4+a)$$

$$16x^2 + 40x - a^2 + 25$$

$$(16x^2 + 40x + 25) - a^2$$

$$(4x+5)(4x+5) - a^2$$

$$\boxed{(4x+5)^2} - \boxed{a^2}$$

$$\boxed{(4x+5) + a} \boxed{(4x+5) - a}$$

$$(4x+5+a)(4x+5-a)$$

Find the Missing Term First

$$\sqrt{9} = 3 \cdot 2 = 6$$

$$x^4 + \underline{5}x^2 + 9$$

$$(x^4 + 6x^2 + 9) - x^2$$

$$(x^2 + 3)^2 - x^2$$

$$(x^2 + 3) - x)(x^2 + 3) + x$$

$$(x^2 - x + 3)(x^2 + x + 3)$$

$$\sqrt{1} = 1 \cdot 2 = 2$$

$$x^4 - \underline{6}x^2 + 1$$

$$(x^4 - 2x^2 + 1) - 4x^2$$

$$(x^2 - 1)^2 - 4x^2$$

$$(x^2 - 1) - 2x)(x^2 - 1) + 2x$$

$$(x^2 - 2x - 1)(x^2 + 2x - 1)$$

