Solving Polynomial Inequalities

Using the Graph

where does the function have "y" values greater than 0

A polynomial inequality, $x^3 + x^2 - 9x - 9 > 0$, can be solved by examining the graph of the corresponding polynomial function, $y = x^3 + x^2 - 9x - 9$

① x int (y=0)

$0 = (x^3 + x^2)(-9x - 9)$

$0 = x^2(x+1) - 9(x+1)$

$0 = (x^2 - 9)(x+1)$

$0 = (x+3)(x-3)(x+1)$

$\boxed{x = -3, -1, 3}$

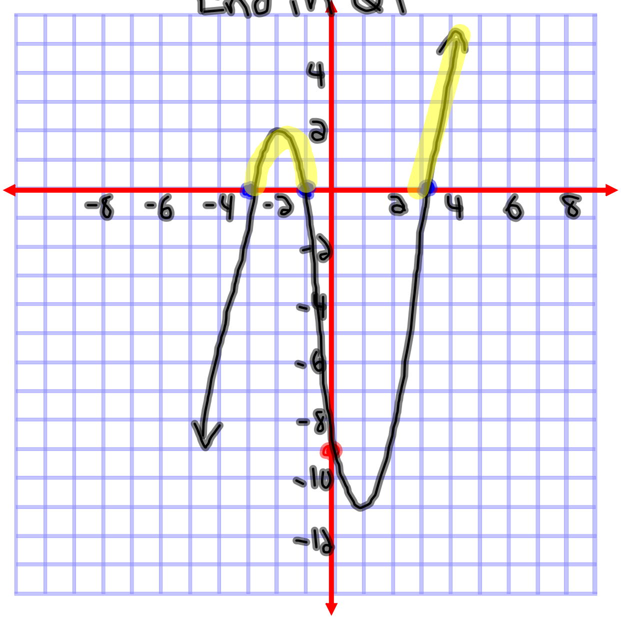
② y int (x=0)

$y = (0)^3 + (0)^2 - 9(0) - 9$

$\boxed{y = -9}$

③ Stretch factor

$a = 1 \rightarrow$ Start in Q3
End in Q1



$-3 < x < -1$, $x > 3$

$x \in (-3, -1)$, $x \in (3, \infty)$

$\boxed{x \in (-3, -1) \cup (3, \infty)}$

Interval Notation

The statement ***-2 < x < 3*** can be written as ***x*** $\in$ ***(-2, 3)***; that is *x* belongs to the interval ***(-2, 3).*** The round brackets mean that *x* is not equal to ***-2 or 3.***

The statement -4 $\le$ ***x*** $\le$ ***2*** can be written as ***x*** $\in$ **[*-4, 2*].** The square brackets mean that *x* may be equal to ***-4 or 2.***

Explain the meaning of the following interval notations.

$x \in (-\infty, 2)$ $\qquad -\infty < x < 2 \rightarrow \boxed{x<2}$

$x \in (-\infty, 2]$ $\qquad -\infty < x \le 2 \rightarrow \boxed{x\le 2}$

$x \in (3, \infty)$ $\qquad 3 < x < \infty \rightarrow \boxed{x>3}$

$x \in [3, \infty)$ $\qquad 3 \le x < \infty \rightarrow \boxed{x\ge 3}$

Note: Infinity cannot be inclusive

Solving Polynomial Inequalities

Using the Number Line

Example: $x^3 + x^2 > 6x$

Step 1: State the Roots of the function

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals. → Quadratic

Quadratic
$$\begin{cases} x \in (-\infty, \text{ small } x\text{-int}) \\ x \in (\text{small } x\text{-int}, \text{ large } x\text{-int}) \\ x \in (\text{large } x\text{-int}, \infty) \end{cases}$$

4 " → Cubic

5 " → Quartic

Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

Using the Number Line

Example: $x^3 + x^2 > 6x$

$x^3 + x^2 - 6x > 0$

Step 1: State the Roots of the function

$y = x^3 + x^2 - 6x$

$0 = x(x^2 + x - 6)$

$0 = (x)(x+3)(x-2)$

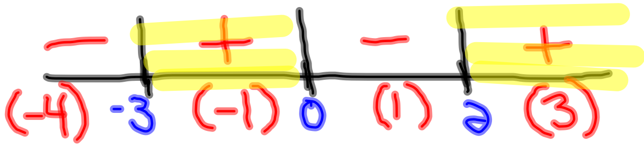
$x = -3, 0, 2$

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into ~~three~~ 4 intervals.

***x* ∈ (-∞ , small x-int)**

***x* ∈ (small x-int, large x-int)**

***x* ∈ (large x-int, ∞)**



Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

$y = x^3 + x^2 - 6x$

$y = (-4)^3 + (-4)^2 - 6(-4)$

$y = -64 + 16 + 24$

$y = -24$

$y = (-1)^3 + (-1)^2 - 6(-1)$

$y = -1 + 1 + 6$

$y = 6$

$y = (1)^3 + (1)^2 - 6(1)$

$y = 1 + 1 - 6$

$y = -4$

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

$x \in (-3, 0) \cup (2, \infty)$

Homework

$$x^3 - x^2 - 12x \leq 0$$

Solve using a graph then using a number line. Express answer in Interval Notation