Solving Polynomial Inequalities

Using the Graph where does the function have "y" values greater than 0

A polynomial inequality, $x^3 + x^2 - 9x - 9 > 0$, can be solved by examining the graph of the corresponding polynomial function, $y = x^3 + x^2 - 9x - 9$

① x int (y=0)

$0 = (x^3 + x^2)(-9x - 9)$

$0 = x^2(x+1) - 9(x+1)$

$0 = (x^2 - 9)(x+1)$

$0 = (x+3)(x-3)(x+1)$

$\boxed{x = -3, -1, 3}$

② y int (x=0)

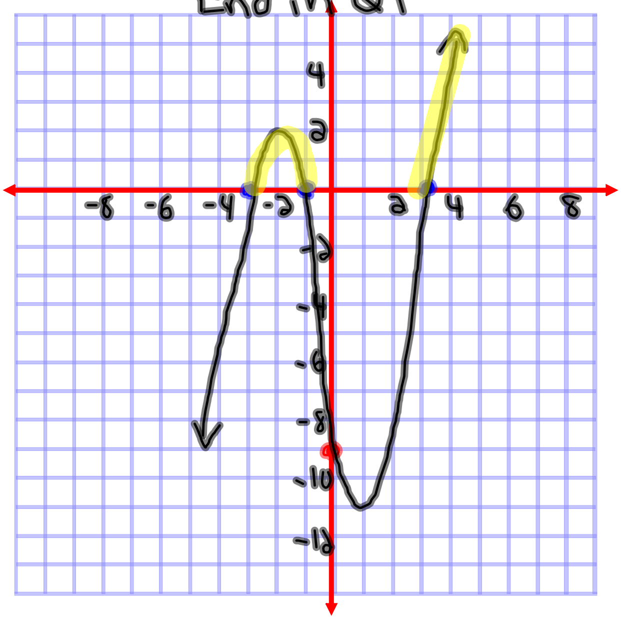
$y = (0)^3 + (0)^2 - 9(0) - 9$

$\boxed{y = -9}$

③ Stretch factor

$a = 1 \rightarrow$ Start in Q3

End in Q1



$-3 < x < -1, \quad x > 3$

$x \in (-3, -1), \quad x \in (3, \infty)$

$\boxed{x \in (-3, -1) \cup (3, \infty)}$

Interval Notation

The statement $-2 < x < 3$ can be written as $x \in (-2, 3)$; that is x belongs to the interval $(-2, 3)$. The round brackets mean that x is not equal to ***-2 or 3.***

The statement $-4 \leq x \leq 2$ can be written as $x \in [-4, 2]$. The square brackets mean that x may be equal to ***-4 or 2.***

Explain the meaning of the following interval notations.

$x \in (-\infty, 2)$ $\qquad -\infty < x < 2 \rightarrow \boxed{x<2}$

$x \in (-\infty, 2]$ $\qquad -\infty < x \leq 2 \rightarrow \boxed{x\leq 2}$

$x \in (3, \infty)$ $\qquad 3 < x < \infty \rightarrow \boxed{x>3}$

$x \in [3, \infty)$ $\qquad 3 \leq x < \infty \rightarrow \boxed{x\geq 3}$

Note: Infinity cannot be inclusive

Solving Polynomial Inequalities

Using the Number Line

Example: $x^3 + x^2 > 6x$

Step 1: State the Roots of the function

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into three intervals. → Quadratic

Quadratic

$x \in (-\infty\,,\ small\ x\text{-}int)$

$x \in (small\ x\text{-}int,\ large\ x\text{-}int)$

$x \in (large\ x\text{-}int,\ \infty)$

4 " → Cubic

5 " → Quartic

Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of x* in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

Using the Number Line

Example: $x^3 + x^2 > 6x$

$x^3+x^2-6x>0$

Step 1: State the Roots of the function

$y=x^3+x^2-6x$

$0=x(x^2+x-6)$

$0=(x)(x+3)(x-2)$

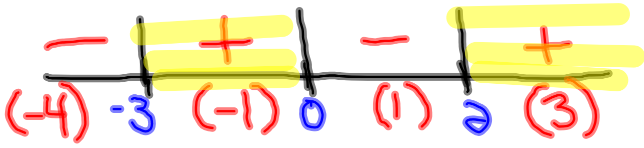
$x=-3,0,2$

Step 2: Draw a number line and mark the roots of the equation. These roots separate the rest of the number line into ~~three~~ 4 intervals.

x $\in$ ***(-∞ , small x-int)***

x $\in$ ***(small x-int, large x-int)***

x $\in$ ***(large x-int, ∞)***



Step 3: The value of the expression $x^3 + x^2 - 6x$ has the same sign throughout each interval in step 2 **because a function can only change signs at a root.** Therefore, choose a *test value of* x in each interval and evaluate the expression. Write a *plus or a minus* over that interval on the number line to indicate whether the expression is positive or negative.

$y=x^3+x^2-6x$

$y=(-4)^3+(-4)^2-6(-4)$

$y=-64+16+24$

$y=-24$

$y=(-1)^3+(-1)^2-6(-1)$

$y=-1+1+6$

$y=6$

$y=(1)^3+(1)^2-6(1)$

$y=1+1-6$

$y=-4$

Step 4: State the intervals for which $x^3 + x^2 - 6x > 0$

$x \in (-3,0) \cup (2,\infty)$

Homework

$$x^3 - x^2 - 12x \le 0$$

Solve using a graph then using a number line. Express answer in Interval Notation

$\boxed{\text{Graphing}}$

$$x^3 - x^2 - 12x \leq 0$$

where are the y values less than or equal to 0. (Negative)

$$y = x^3 - x^2 - 12x$$

① xint (y=0)

$0 = x(x^2 - x - 12)$

$0 = x(x-4)(x+3)$

$x = -3, 0, 4$

② yint (x=0)

$y = x^3 - x^2 - 12x$

$y = (0)^3 - (0)^2 - 12(0)$

$y = 0$

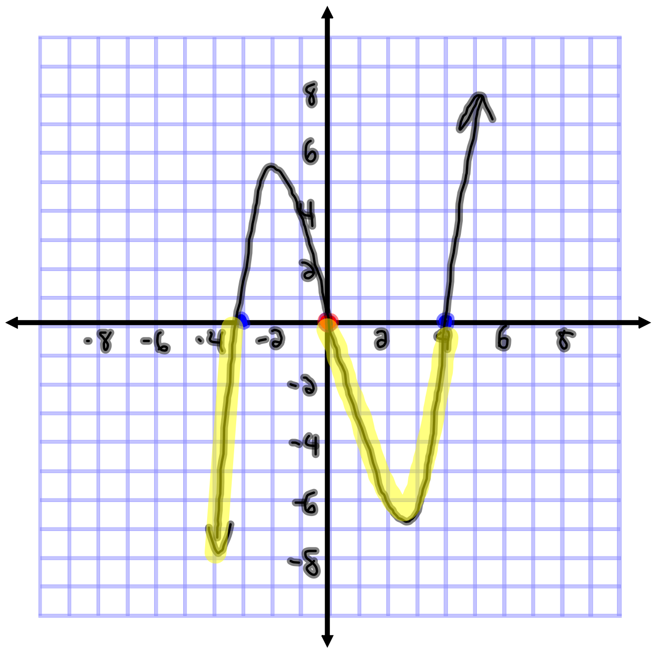
③ Stretch factor

$y = x^3 - x^2 - 12x$

$a = 1$

Starts in Q3

Ends in Q1



$x \in (-\infty, -3] \cup [0, 4]$

$\boxed{\text{Number Line}}$

$$x^3 - x^2 - 12x \le 0$$

We are looking for negative y values (see number line)

$$y = x^3 - x^2 - 12x$$

$\boxed{\text{① Roots } (y=0)}$

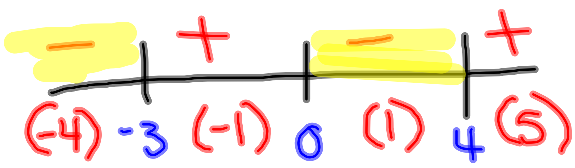
$$0 = x^3 - x^2 - 12x$$

$$0 = x(x^2 - x - 12)$$

$$0 = x(x-4)(x+3)$$

$$x = -3, 0, 4$$

$\boxed{\text{②+③ Number Line / Test Values}}$



$\boxed{\text{④ State the Intervals}}$

$$x \in (-\infty, -3] \cup [0, 4]$$

Homework

Do #7 using a graph then using a number line

7 a) $x^3+5x^2-9x > 45$

$x^3+5x^2-9x-45 > 0$

$y = x^3+5x^2-9x-45$

$\boxed{\text{1 Roots } (y=0)}$

$0 = (x^3+5x^2)(-9x-45)$

$0 = x^2(x+5)-9(x+5)$

$0 = (x+5)(x^2-9)$

$0 = (x+5)(x+3)(x-3)$

$x = -5, -3, 3$