$f(x) = x^2 + 3x - 2$ $\qquad$ $g(x) = 2x - 3$

① b) $g(f(x))$

$g(x^2+3x-2) = 2(x^2+3x-2) - 3$

$= 2x^2 + 6x - 4 - 3$

$= 2x^2 + 6x - 7$

② a) $y = x^2 + 8x + 3$

$y - 3 \overset{+16}{=} (x^2 + 8x + 16)$

$y + 13 = (x+4)^2$

$y = (x+4)^2 - 13$

Vertex: $(-4, -13)$

③ a) $y = (x+2)(x+3)(x-2)$

(i) Roots → xint ($y=0$)

$0 = (x+2)(x+3)(x-2)$

$x = -3, -2, 2$

$a = 1$

Starts in Q3

Ends in Q1

(ii) y int ($x=0$)

$y = (0+2)(0+3)(0-2)$

$y = (2)(3)(-2)$

$y = -12$

(iii) Approx Local Max

($x = -2.5$)

$y = (-2.5+2)(-2.5+3)(-2.5-2)$

$y = (-0.5)(0.5)(-4.5)$

$y = 1.125$

$(-2.5, 1.125)$

Approx Local Min

($x=0$)

$y = (0+2)(0+3)(0-2)$

$y = (2)(3)(-2)$

$y = -12$

$(0, -12)$ ✱ Approxim

⑦ a) $x^3+5x^2-9x>45$

$x^3+5x^2-9x-45>0$ ← positive "y" values

$y=x^3+5x^2-9x-45$

① Roots ($y=0$)

$0=(x^3+5x^2)(9x-45)$

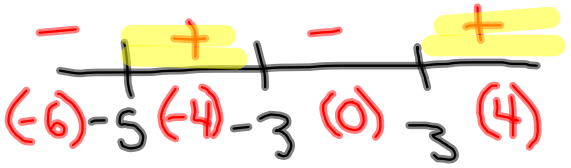
$0=x^2(x+5)-9(x+5)$

$0=(x^2-9)(x+5)$

$0=(x+3)(x-3)(x+5)$

$x=-5,-3,3$

②/③ Number Line / Test Value



$x\in(-5,-3)\cup(3,\infty)$

Test Values

$y=(-6+3)(-6-3)(-6+5)$

$y=(-3)(-9)(-1)$

$y=-27$

(-4)

$y=(-1)(-7)(1)$

$y=7$

(0)

$y=(3)(-3)(5)$

$y=-45$

(4)

$y=(7)(1)(9)$

$y=63$

7a) $x^3+5x^2-9x \leq 45$

Changed from original

$x^3+5x^2-9x-45 \leq 0$

$y = x^3+5x^2-9x-45$

y values that less than or equal to 0

① Roots ($y=0$)

$0 = x^3+5x^2-9x-45$

$x = -5, -3, 3$

(Factored on previous page)

② y int ($x=0$)

$y = (0)^3+5(0)^2-9(0)-45$

$y = -45$

app. local max

③ Degree

3rd Degree

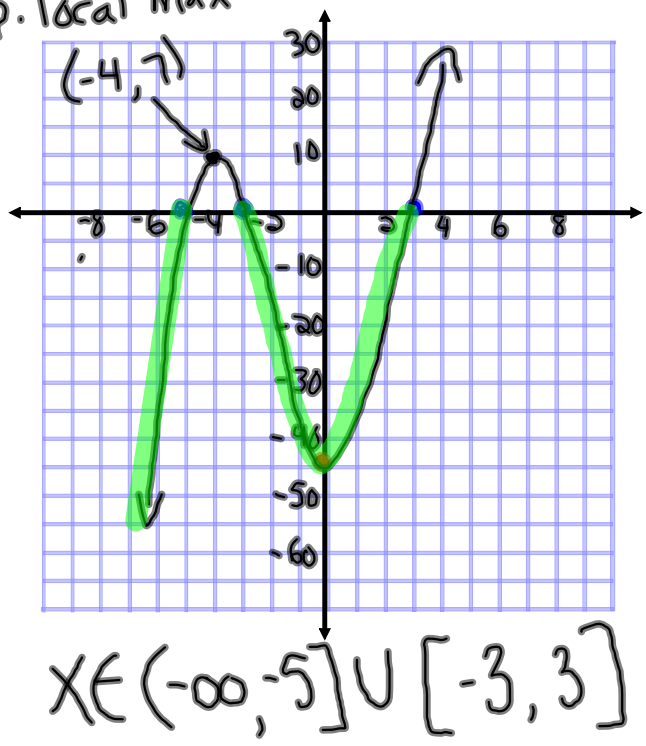
Cubic

④ Stretch Factor

$a = 1$

Starts in Q3

Ends in Q1



$x \in (-\infty, -5] \cup [-3, 3]$

① i) $4x^2 - c^2 - 12x + 9$

$(4x^2 - 12x + 9) - c^2$

$(2x-3)(2x-3) - c^2$

$(2x-3)^2 - c^2$

$(2x-3-c)(2x-3+c)$

⑥ $x^3+x^2 < 16x+16$

$x^3+x^2-16x-16 < 0$ ← "y" values that are less than 0

$y = x^3+x^2-16x-16$

① Roots ($y=0$)

$y = x^3+x^2-16x-16$

$0 = (x^3+x^2)(-16x-16)$

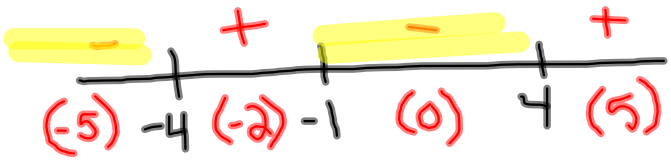
$0 = x^2(x+1)-16(x+1)$

$0 = (x^2-16)(x+1)$

$0 = (x-4)(x+4)(x+1)$

$x = -4, -1, 4$

②/③ Number Line / Test Value



④ State Intervals

$x \in (-\infty, -4) \cup (-1, 4)$

Answer

* $y=(x-4)(x+4)(x+1)$ Not necessary to show

$y=(-5-4)(-5+4)(-5+1)$ ← this

$y=(-9)(-1)(-4)$

$y=-36$

① fs $4x^4 - 33x^2 + 36$ $\sqrt{4 \cdot 36} = 12 \cdot 2 = 24$

$(4x^4 - 24x^2 + 36) - 9x^2$

$(2x^2 - 6)^2 - 9x^2$

$(2x^2 - 6 - 3x)(2x^2 - 6 + 3x)$

① e) $x^4 + x^2 - 20$

$5 \cdot x^{-4} = -20$

$5 + -4 = 1$

$(x^2+5)(x^2-4)$

$(x^2+5)(x+2)(x-2)$

When factoring

① Try Common factor

② Count the terms

① b) $8x^6+27$

$(2x^2+3)(4x^4-6x^2+9)$

c) $3x^4+7x^3+2x^2$

$x^2(3x^2+7x+2)$ Decomp.

$1 \times 6 = 6$

$1+6=7$

$x^2(3x^2+6x+x+2)$

$x^2(3x(x+2)+1(x+2))$

$x^2(3x+1)(x+2)$

f) $4x^4 - 33x^2 + 36$ $\sqrt{4 \cdot 36} = 12 \cdot 2 = 24$

$(4x^4 - 24x^2 + 36) - 9x^2$

$(2x^2 - 6)^2 - 9x^2$

$(2x^2 - 6 - 3x)(2x^2 - 6 + 3x)$

g) $x^4 + 14x^2 + 81$ $\sqrt{81} = 9 \cdot 2 = 18$

$(x^4 + 18x^2 + 81) - 4x^2$

$(x^2 + 9)^2 - 4x^2$

$(x^2 + 9 - 2x)(x^2 + 9 + 2x)$

③ $y = (x+3)^2(x-2)(x-3)$

a) **Roots ($y=0$)**

$0 = (x+3)^2(x-2)(x-3)$

$x = -3, -3, 2, 3$

b) **y int ($x=0$)**

$y = (x+3)^2(x-2)(x-3)$

$y = (3)^2(-2)(-3)$

$y = 54$

Degree: 4th

Quartic

Stretch Factor:

$a = 1$ Starts in Q2

Ends in Q1

c) Approx Local Max ($x = -0.5$)

$y = (-0.5+3)^2(-0.5-2)(0.5-3)$

$y = (2.5)^2(-2.5)(-3.5)$

$y = 54.7$

$(-0.5, 54.7)$

② $y = 2x^3 - 5x^2 - 4x + 3$

$y = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$

$y = -2 - 5 + 4 + 3$

$y = 0$

$(x+1)$ is a factor

$$\begin{array}{r|l} 2x^2 - 7x + 3 & \text{Decomp.} \\ x+1 \;\overline{)\; 2x^3 - 5x^2 - 4x + 3} & (x+1)(2x^2 - 7x + 3) \\ -(2x^3 + 2x^2) & (x+1)(2x^2 - x - 6x + 3) \\ \hline -7x^2 - 4x & (x+1)(x(2x-1) - 3(2x-1)) \\ -(-7x^2 - 7x) & (x+1)(x-3)(2x-1) \\ \hline 3x + 3 & \\ -(3x+3) & \\ \hline 0 & \end{array}$$

③ $y = (x+3)^3(x-2)(x-3)$ → 4th Degree

Stretch Factor

$a = 1$ Start in Q2 Ends in Q1

a) Roots ($y=0$)

$y = (x+3)^3(x-2)(x-3)$

$0 = (x+3)^3(x-2)(x-3)$

$x = -3, -3, 2, 3$

b) y int ($x=0$)

$y = (0+3)^3(0-2)(0-3)$

$y = (3)^3(-2)(-3)$

$y = 54$

c) Approx. Local max ($x=-0.5$)

$y = (-0.5+3)^3(-0.5-2)(-0.5-3)$

$y = (2.5)^3(-2.5)(-3.5)$

$y = 54.7$

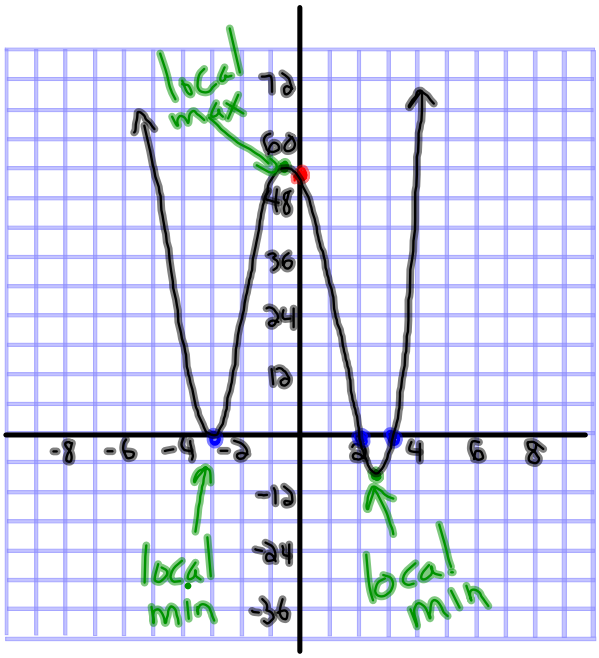
$(-0.5, 54.7)$

d) Local min ($x=2.5$)

$y = (2.5+3)^3(2.5-2)(2.5-3)$

$y = -7.6$

$(2.5, -7.6)$



① c) $3x^4+7x^3+2x^2$

$x^2(3x^2+7x+2)$ Decomposition

$x^2((3x^2+6x)+(x+2))$

$x^2(3x(x+2)+1(x+2))$

$x^2(x+2)(3x+1)$

e) x^4+x^2-20 $\quad 5\cdot x\cdot -4=-20$, $5+-4=1$

$(x^2-4)(x^2+5)$

$(x+2)(x-2)(x^2+5)$

g) x^4+14x^2+81 $\quad \sqrt{81}=9\cdot 2=18$

$(x^4+18x^2+81)-4x^2$

$(x^2+9)^2-4x^2$

$(x^2+9+2x)(x^2+9-2x)$

i) x^2-a^2-6x+9

$(x^2-6x+9)-a^2$

$(x-3)^2-a^2$

$(x-3-a)(x-3+a)$

⑥ $x^3 - x^2 < 12x$

$x^3 - x^2 - 12x \leq 0$ ← less than 0 or equal (Negative)

$y = x^3 - x^2 - 12x$

① Roots (y=0)

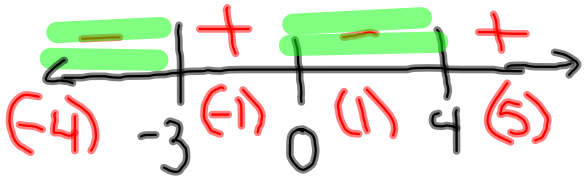
$y = x^3 - x^2 - 12x$

$0 = x(x^2 - x - 12)$

$0 = (x)(x-4)(x+3)$

$x = -3, 0, 4$

②③ Number Line/Test Values



④ State Intervals

$x \in (-\infty, -3] \cup [0, 4]$

Test

- Multiple Choice
- Polynomial Inequalities
- Graphing
- Factoring
- Composite Functions