Warm Up

1. Simplify: $\dfrac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$

$$\frac{\frac{9}{9x^2}-\frac{x^2}{9x^2}}{x-3}$$

$$\frac{9-x^2}{9x^2}\cdot\frac{1}{x-3}$$

$$\frac{9-x^2}{9x^2(x-3)}$$

$$\frac{-1(x^2-9)}{9x^2(x-3)}$$

$$\frac{(-1)(x+3)\cancel{(x-3)}}{9x^2\cancel{(x-3)}}$$

$$\frac{-x-3}{9x^2}$$

3. Rationalize the denominator:

$$\frac{x+2}{\sqrt{x-4}-\sqrt{x-6}}\cdot\frac{(\sqrt{x-4}+\sqrt{x-6})}{(\sqrt{x-4}+\sqrt{x-6})}$$

$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{(x-4)-(x-6)}$$

$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{x-4-x+6}$$

$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{\;}$$

Limit of a Function → The intended height of a function.

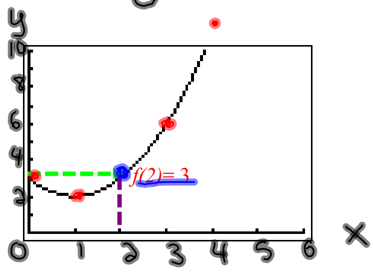
Limit of a Function

Let's examine the function $f(x)=x^2-2x+3$ or $y=x^2-2x+3$

Plot2 Plot3
\Y1 = X²−2X+3
\Y2 =
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =

X	Y1
0	3
1	2
2	3
3	6
4	11
5	18
6	27

X=0



We can see that $f(2)=3$...let's check the behaviour of f as we get closer and closer to $x = 2$.

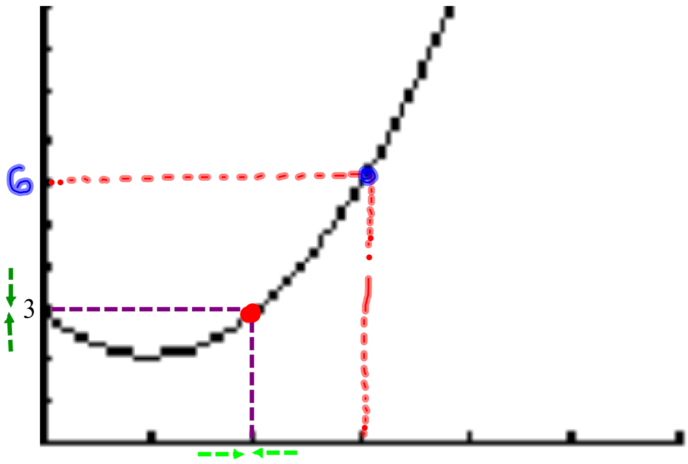
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As x gets closer to 2 from the left y is getting closer to 3.

As x gets closer to 2 from the right y is getting closer to 3.

From the above, the notion of the limit of a function arises...



As x approaches 2

Notation: $\lim_{x\to 2} f(x)=3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

$$\lim_{x\to 3} f(x) = 6$$

The common sense definition of a limit...

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A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ <u>arbitrarily close to L</u>

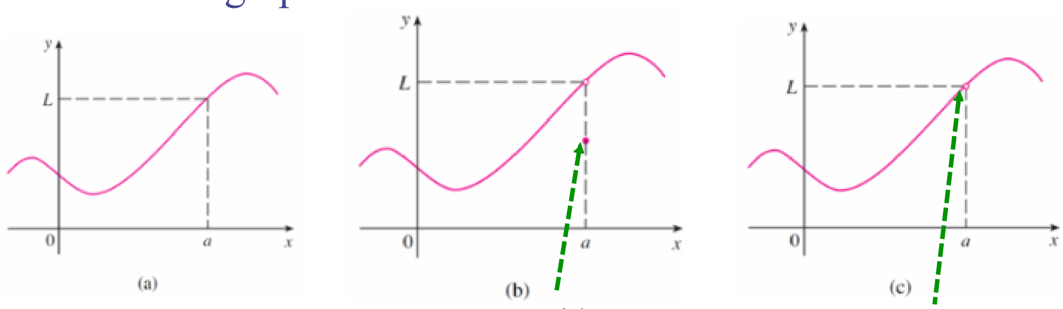
- (as close to L as we like)

by taking x to be <u>sufficiently close to a</u>

- (on either side of a)

but <u>not</u> equal to a .

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at a , it is true that

$$\lim_{x \to a} f(x) = L$$

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2}\frac{x^2-2x+1}{x+3}$$

$$\lim_{x\to -2}\frac{(-2)^2-2(-2)+1}{(-2)+3}$$

$$\lim_{x\to -2}\frac{4+4+1}{1}=9$$

$$\lim_{x\to 3}(16-x^2)$$

$$\lim_{x\to 3} 16-(3)^2$$

$$\lim_{x\to 3} 16-9=7$$

- Indeterminate limits... $\boxed{\Rightarrow \text{Direct substitution leads to } \frac{0}{0}}$ *

⇒ Factor
⇒ Rationalize
⇒ Expand
⇒ Find Common Denominators

Examples:

$$\lim_{x\to 4}\frac{x^2-16}{x-4}$$

$$\lim_{x\to 4}\frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}}$$

$$\lim_{x\to 4}(4)+4=\boxed{8}$$

$$\lim_{h\to 0}\frac{\left(\sqrt{4+h}-2\right)}{h}\frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{(4+h)-4}{h\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0}\frac{1}{\sqrt{4+0}+2}=\boxed{\frac{1}{4}}$$

Try these...remember to use your algebra skills
to try and eliminate the **indeterminate form**.

$$\lim_{x \to 0} \frac{x^2 + 3x}{(x+2)^2 - (x-2)^2}$$

$$\lim_{x \to -2} \frac{x^4 - 16}{x^3 + 8}$$

.

$$\lim_{x \to 2} \frac{(x+2)^2 - 16}{x^2 - 4}$$

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$$

Homework