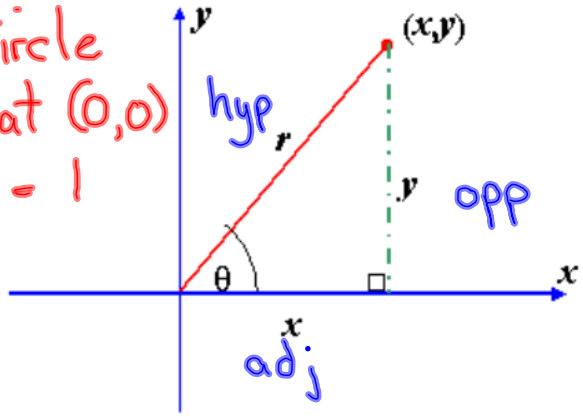
Trig Identities

Reciprocal Identities

Unit Circle
- Center at (0,0)
- Radius = 1



$$\sin\theta = \frac{y}{r} = y \qquad \cos\theta = \frac{x}{r} = x \qquad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} = \frac{1}{y} \qquad \sec\theta = \frac{r}{x} = \frac{1}{x} \qquad \cot\theta = \frac{x}{y}$$

Reciprocal

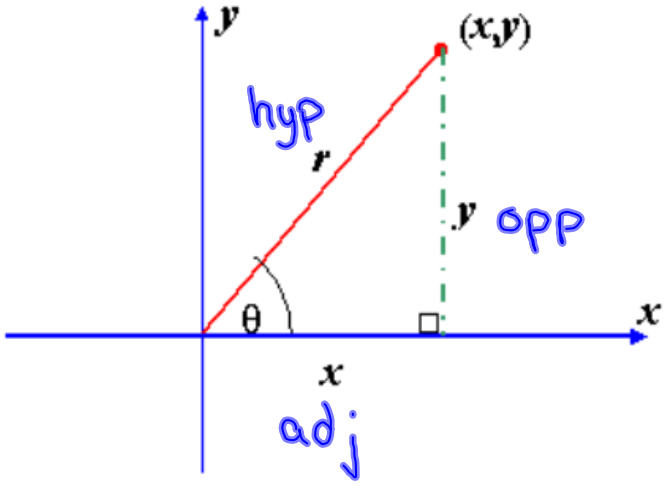
$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta = \frac{1}{\csc^2\theta} \qquad \cos^2\theta = \frac{1}{\sec^2\theta} \qquad \tan^2\theta = \frac{1}{\cot^2\theta}$$

$$\csc^2\theta = \frac{1}{\sin^2\theta} \qquad \sec^2\theta = \frac{1}{\cos^2\theta} \qquad \cot^2\theta = \frac{1}{\tan^2\theta}$$

Quotient Identities



$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

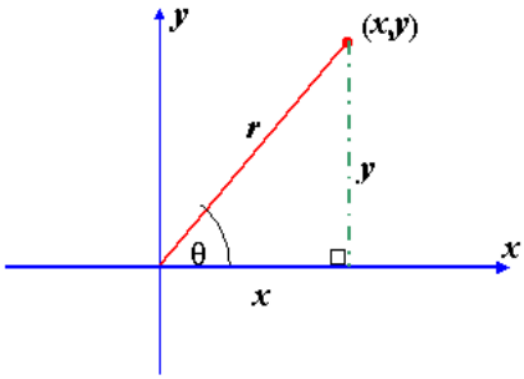
$$\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

Quotient

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ $\quad\quad$ $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$

Pythagorean Identities



$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$\div r^2$

$$x^2 + y^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\frac{x^2}{x^2} + \frac{y^2}{x^2} = \frac{r^2}{x^2}$$

$\div x^2$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\tan^2\theta = \sec^2\theta - 1$$

$$1 = \sec^2\theta - \tan^2\theta$$

$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$$

$\div y^2$

$$\cot^2\theta + 1 = \csc^2\theta$$

$$\cot^2\theta = \csc^2\theta - 1$$

$$1 = \csc^2\theta - \cot^2\theta$$

Trigonometric Identities

You must know these!

Quotient

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Reciprocal

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Strategies for Proving Identities:

- Work on the most complex side and simplify so it has the same form as the simpler side
- Methods used in simplifying: direct substitution, factoring, finding a common denominator, multiplying by the conjugate: $(1-\cos\theta) \rightarrow (1+\cos\theta)$
- $\sin\theta$ and $\cos\theta$ are good guys. (leave them alone as much as possible)

Prove the following:

$$\frac{\tan x}{\sin x} = \sec x$$

$$\frac{\left(\frac{\sin x}{\cos x}\right)}{\sin x} \qquad \frac{1}{\cos x}$$

$$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$$

$$\frac{1}{\cos x}$$

$$\cos\theta \bullet \sec\theta = 1$$

$$\cos\theta \cdot \left(\frac{1}{\cos\theta}\right) \qquad 1$$

$$1$$

Prove the following:

$\cot\theta \bullet \sin\theta = \cos\theta$

$\left(\frac{\cos\theta}{\sin\theta}\right) \bullet \sin\theta$

$\cos\theta$

$\cos\theta$

$\frac{\cos x}{\tan x} = \frac{1-\sin^2 x}{\sin x}$

$\frac{\cos x}{\left(\frac{\sin x}{\cos x}\right)}$

$\cos x \bullet \frac{\cos x}{\sin x}$

$\frac{\cos^2 x}{\sin x}$

$\frac{\cos^2 x}{\sin x}$

Ex. Prove that $\sin y + \sin y \cot^2 y = \csc y$

factor out a $\sin y$ → $\sin y(1+\cot^2 y)$

$\sin y(\csc^2 y)$

$\cancel{\sin y}\left(\frac{1}{\sin^{\cancel{2}} y}\right)$

$\frac{1}{\sin y}$

$\frac{1}{\sin y}$

Homework

1. $\tan\theta\cos\theta = \sin\theta$
2. $\cot\theta\sec\theta = \csc\theta$
3. $\dfrac{1+\cot^2\theta}{\csc^2\theta} = 1$
4. $\dfrac{\tan^2\theta}{1+\tan^2\theta} = \sin^2\theta$
5. $\dfrac{\tan^2\theta}{\sin^2\theta} = 1+\tan^2\theta$