Warm Up

Evaluate the following limits, if they exist:

1. $\lim\limits_{x\to 2} \dfrac{x-2}{x^3-8}$

$$\lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$$

$$\lim_{x\to 2} \frac{1}{4+4+4} = \boxed{\frac{1}{12}}$$

2. $\lim\limits_{x\to 7} \dfrac{\left(\sqrt{x+2}-3\right)\left(\sqrt{x+2}+3\right)}{(x-7)\left(\sqrt{x+2}+3\right)}$

$$\lim_{x\to 7} \frac{\left((x+2)-9\right)}{(x-7)\left(\sqrt{x+2}+3\right)}$$

$$\lim_{x\to 7} \frac{\cancel{(x-7)}}{\cancel{(x-7)}\left(\sqrt{x+2}+3\right)}$$

$$\lim_{x\to 7} \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

3. $\lim\limits_{h\to 0} \dfrac{(a+h)^2-a^2}{h}$

$$\lim_{h\to 0} \frac{(a+h-a)(a+h+a)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{(h)}(2a+h)}{\cancel{h}} = \boxed{2a}$$

Questions from Homework

③ j) $\lim_{t\to 3}\left(2t^2+\sqrt{\frac{6+t}{4-t}}\right)$

$\lim_{t\to 3}\left(2(3)^2+\sqrt{\frac{6+3}{4-3}}\right)$

$\lim_{t\to 3}\left(2(9)+\sqrt{\frac{9}{1}}\right) = 18+3 = \boxed{21}$

④ h) $\lim_{x\to 2} \frac{2x\;\frac{1}{x}-\frac{1}{2}\;2x}{(x-2)(2x)}$

$\lim_{x\to 2} \frac{\overset{-1}{\cancel{(2-x)}}}{2x\cancel{(x-2)}} = \boxed{\frac{-1}{4}}$

⑤ a) $\lim_{h\to 0} \frac{(4+h)^3-64}{h}$

$\lim_{h\to 0} \frac{((4+h)-4)((4+h)^2+4(4+h)+16)}{h}$

$\lim_{h\to 0} \frac{\cancel{h}((4+h)^2+4(4+h)+16)}{\cancel{h}}$

$= 16+16+16$

$\boxed{= 48}$

Questions from Homework

⑤ f) $\lim_{h\to 0} \dfrac{\dfrac{1}{(2+h)^2} - \dfrac{1}{4}}{h}$ $4(2+h)^2$ C.D. = $\boxed{4(2+h)^2}$

$\lim_{h\to 0} \dfrac{4-(2+h)^2}{4h(2+h)^2}$

$\lim_{h\to 0} \dfrac{(2+(2+h))(2-(2+h))}{4h(2+h)^2}$

$\lim_{h\to 0} \dfrac{(4+h)(-\cancel{h})}{4\cancel{h}(2+h)^2}$ -1

$\lim_{h\to 0} \dfrac{(4+0)(-1)}{4(2+0)^2} = \dfrac{-4}{16} = \boxed{\dfrac{-1}{4}}$

⑥ $\lim_{x\to 3} \dfrac{1}{(x-3)^2}$

$\lim_{x\to 3} \dfrac{1}{(3-3)^2} = \boxed{\dfrac{1}{0}}$ → The Limit does not exist

DNE

⑥ e) $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-2x+1}$

$\lim_{x\to 1} \dfrac{(x+2)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}} = \boxed{\dfrac{3}{0}}$ → DNE

⑤ e) $\lim_{h\to 0} \dfrac{(\sqrt{9+h}-3)}{h} \cdot \dfrac{(\sqrt{9+h}+3)}{(\sqrt{9+h}+3)}$

$\lim_{h\to 0} \dfrac{(9+h)-9}{(h)(\sqrt{9+h}+3)}$

$\lim_{h\to 0} \dfrac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)} = \boxed{\dfrac{1}{6}}$

Questions from Homework

⑤ c) $\lim_{h\to 0} \dfrac{\cancel{(1+h)}\dfrac{1}{\cancel{1+h}} - \dfrac{1}{1}(1+h)}{h(1+h)}$ CD $=(1+h)$

$\lim_{h\to 0} \dfrac{1-(1+h)}{h(1+h)}$

$\lim_{h\to 0} \dfrac{1-1-h}{h(1+h)}$

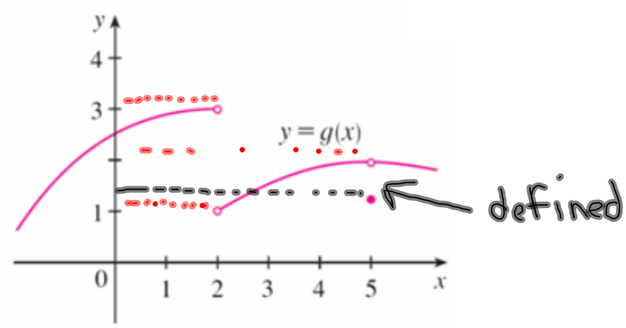
$\lim_{h\to 0} \dfrac{-\cancel{h}}{\cancel{h}(1+h)} = \dfrac{-1}{1} = \boxed{-1}$

The common sense definition of a limit...

When does a limit exist?

One-sided limits

Use the graph shown below to evaluate the following limits:



1. $\lim\limits_{x\to 2^-} g(x) = \boxed{3}$

"as x approaches 2 from the left"

2. $\lim\limits_{x\to 2^+} g(x) = \boxed{1}$

"as x approaches 2 from the right"

3. $\lim\limits_{x\to 2} g(x) = \boxed{\text{DNE}}$

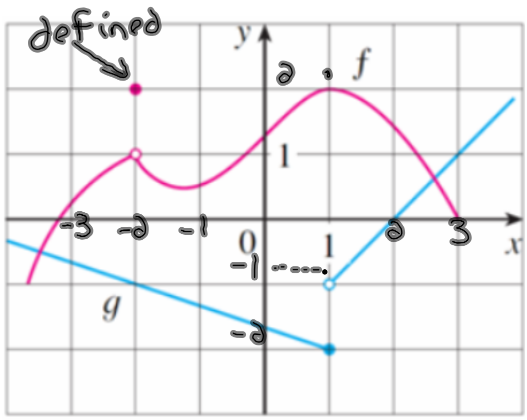
4. $\lim\limits_{x\to 5^-} g(x) = \boxed{2}$

5. $\lim\limits_{x\to 5^+} g(x) = \boxed{2}$

6. $\lim\limits_{x\to 5} g(x) = \boxed{2}$

Notice... $g(5) =$ 1.2 pick closed dot

Example:



Evaluate each of the following:

$f(-2) = 2$ $\quad\lim\limits_{x\to 1^-} g(x) = -2$ $\quad g(1) = -2$

$\lim\limits_{x\to 1^+} g(x) = -1$ $\quad\lim\limits_{x\to 1} g(x) = \text{DNE}$ $\quad\lim\limits_{x\to 1} f(x) = 2$

$\lim\limits_{x\to -2} f(x) = 1$

Homework

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