

Warm Up

Prove the following identity:

$$\frac{\sin x}{(1 - \cos x)} - \frac{\sin x \cos x}{(1 + \cos x)} = \csc x (1 + \cos^2 x)$$

$$\frac{\sin x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)} - \frac{\sin x \cos x (1 - \cos x)}{(1 - \cos x)(1 + \cos x)}$$

$$\boxed{\csc x (1 + \cos^2 x)}$$

$$\frac{\cancel{\sin x} + \cancel{\sin x \cos x} - \cancel{\sin x \cos x} + \sin x \cos^2 x}{(1 - \cos x)(1 + \cos x)}$$

$$\left(\frac{1}{\sin x}\right) (1 + \cos^2 x)$$

$$\frac{\sin x + \sin x \cos^2 x}{(1 - \cos^2 x)}$$

$$\boxed{\frac{1 + \cos^2 x}{\sin x}}$$

$$\frac{(\cancel{\sin x})(1 + \cos^2 x)}{(\cancel{\sin^2 x})}$$

$$\boxed{\frac{1 + \cos^2 x}{\sin x}}$$

Bonus

Prove the following identity:

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$$

Foil \rightarrow $\frac{(1 + \sin x)(1 + \sin x)}{\cos x(1 + \sin x)} + \frac{\cos^2 x}{\cos x(1 + \sin x)}$

Pythagorean = 1

$$\frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)}$$

Factor \rightarrow $\frac{2 + 2\sin x}{\cos x(1 + \sin x)}$

$$\frac{2(1 + \sin x)}{\cos x(1 + \sin x)}$$

$$\frac{2}{\cos x}$$

$$2 \left(\frac{1}{\cos x} \right)$$

$$\frac{2}{\cos x}$$

Questions from Homework

$$\textcircled{11} \quad \frac{1 + 2\sin\theta\cos\theta}{\sin\theta + \cos\theta} = \frac{\sin\theta + \cos\theta}{1}$$

$$1 + 2\sin\theta\cos\theta \stackrel{\text{FOIL}}{=} (\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$$

$$\underbrace{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}_{\text{Pythagorean} = 1}$$

$$\boxed{1 + 2\sin\theta\cos\theta}$$

* Add to Identities

$$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$$

$$\textcircled{1} \quad \boxed{\sec^2\theta} - \boxed{\sin^2\theta} = \cos^2\theta + \tan^2\theta$$

$$\tan^2\theta + 1 - (1 - \cos^2\theta)$$

$$\tan^2\theta + 1 - 1 + \cos^2\theta$$

$$\boxed{\tan^2\theta + \cos^2\theta} \quad \boxed{\cos^2\theta + \tan^2\theta}$$

$$\textcircled{6} \quad \boxed{\sin^2\theta} + 2\cos^2\theta - 1 = \boxed{\cos^2\theta}$$

$$1 - \cos^2\theta + 2\cos^2\theta - 1$$

$$\boxed{\cos^2\theta}$$

Homework