

SOLUTIONS => EQUATIONS OF CIRCLES/ELLIPSES  
REVIEW

1  $\{x^2 + y^2 = r^2\}$

a)  $r = 6$   
 $\hookrightarrow x^2 + y^2 = (6)^2$   
 $x^2 + y^2 = 36$

c)  $r = 4\sqrt{7}$   
 $\hookrightarrow x^2 + y^2 = (4\sqrt{7})^2$   
 $x^2 + y^2 = (16)(7)$   
 $x^2 + y^2 = 112$

b)  $r = \sqrt{5}$   
 $\hookrightarrow x^2 + y^2 = (\sqrt{5})^2$   
 $x^2 + y^2 = 5$

d)  $r = 3\sqrt{2}$   
 $x^2 + y^2 = (3\sqrt{2})^2$   
 $x^2 + y^2 = (9)(2)$   
 $x^2 + y^2 = 18$

2. A:  $x^2 + y^2 = 81$

a) radius  
 $\hookrightarrow r^2 = 81$   
 $r = \sqrt{81}$   
 $r = 9$  units

b)  $x$ -intercepts  
 $\hookrightarrow -9$  and  $+9$

c)  $y$ -intercepts  
 $\hookrightarrow -9$  and  $9$

d) domain

$$\{x | -9 \leq x \leq 9, x \in \mathbb{R}\}$$

e) range

$$\{y | -9 \leq y \leq 9, y \in \mathbb{R}\}$$

$$2. \text{ B: } x^2 + y^2 = 48$$

a) radius

$$\hookrightarrow r^2 = 48$$

$$r = \sqrt{48}$$

$$r = \sqrt{16 \times 3}$$

$$r = 4\sqrt{3} \text{ units}$$

b)  $x$ -intercepts

$$\hookrightarrow -4\sqrt{3} \text{ and } 4\sqrt{3}$$

c)  $y$ -intercepts

$$\hookrightarrow -4\sqrt{3} \text{ and } 4\sqrt{3}$$

d) domain

$$\{x \mid -4\sqrt{3} \leq x \leq 4\sqrt{3}, x \in \mathbb{R}\}$$

e) range

$$\{y \mid -4\sqrt{3} \leq y \leq 4\sqrt{3}, y \in \mathbb{R}\}$$

$$3. \quad x^2 + y^2 = 25$$

$$\text{a) } (-4, ?)$$

If  $x = -4$ :

$$\begin{aligned} (-4)^2 + y^2 &= 25 \\ 16 + y^2 &= 25 \\ y^2 &= 25 - 16 \\ y^2 &= 9 \\ y &= \pm\sqrt{9} \end{aligned}$$

Coordinate  $\Rightarrow (-4, -3)$   
or  
 $(-4, 3)$

$$b) x^2 + y^2 = 25$$

$$(\overset{\circ}{?}, \overset{\circ}{3})$$

$$\text{If } y = 3^\circ$$

$$x^2 + (3)^2 = 25$$

$$x^2 + 9 = 25$$

$$x^2 = 25 - 9$$

$$x^2 = 16$$

$$x = \pm \sqrt{16}$$

$$x = \pm 4$$

Coordinate:  $(-4, 3)$

or  
 $(4, 3)$

c)  $(5, ?)$

If  $x=5$ :

$$\begin{aligned}(5)^2 + y^2 &= 25 \\ 25 + y^2 &= 25 \\ y^2 &= 25 - 25 \\ y^2 &= 0 \\ y &= \pm\sqrt{0} \\ y &= 0\end{aligned}$$

Coordinate:  $(5, 0)$

d)  $(2\sqrt{3}, ?)$

If  $x = 2\sqrt{3}$ :

$$\begin{aligned}(2\sqrt{3})^2 + y^2 &= 25 \\(4)(3) + y^2 &= 25 \\12 + y^2 &= 25 \\y^2 &= 25 - 12 \\y^2 &= 13 \\y &= \sqrt{13}\end{aligned}$$

Coordinate:  $(2\sqrt{3}, \sqrt{13})$

4.

EQUATION	CENTER	DOMAIN	RANGE	X-INTERCEPTS	Y-INTERCEPTS
$x^2 + y^2 = 9$	$(0, 0)$	$\{x   -3 \leq x \leq 3, x \in \mathbb{R}\}$	$\{y   -3 \leq y \leq 3, y \in \mathbb{R}\}$	-3 and 3	-3 and 3
$x^2 + y^2 = 36$	$(0, 0)$	$\{x   -6 \leq x \leq 6, x \in \mathbb{R}\}$	$\{y   -6 \leq y \leq 6, y \in \mathbb{R}\}$	-6 and 6	-6 and 6
$x^2 + y^2 = 121$	$(0, 0)$	$\{x   -11 \leq x \leq 11, x \in \mathbb{R}\}$	$\{y   -11 \leq y \leq 11, y \in \mathbb{R}\}$	-11 and 11	-11 and 11

5. a) passing through  $(2, -4)$

$\hookrightarrow$  If  $x=2$  and  $y=-4$ :

$$\begin{aligned}x^2 + y^2 &= r^2 \\(2)^2 + (-4)^2 &= r^2 \\4 + 16 &= r^2 \\20 &= r^2\end{aligned}$$

$\hookrightarrow$  Therefore, the equation would be:

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 20\end{aligned}$$

b) passing through  $(4\sqrt{5}, \sqrt{2})$

$\hookrightarrow$  If  $x = 4\sqrt{5}$  and  $y = \sqrt{2}$ :

$$\begin{aligned}x^2 + y^2 &= r^2 \\(4\sqrt{5})^2 + (\sqrt{2})^2 &= r^2 \\(16)(5) + 2 &= r^2 \\80 + 2 &= r^2 \\82 &= r^2\end{aligned}$$

$\hookrightarrow$  Therefore, the equation would be:

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 82\end{aligned}$$

c) with an  $x$ -intercept of -12:  
↳ Point (-12, 0)

If  $x = -12$  and  $y = 0$ :

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-12)^2 + (0)^2 &= r^2 \\144 + 0 &= r^2 \\144 &= r^2\end{aligned}$$

↳ Therefore, the equation would be:

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 144\end{aligned}$$

$$6. \text{ a) } x^2 + (y-4)^2 = 64$$
$$(x-0)^2 + (y-4)^2 = 64$$

Center  $(0, 4)$

Radius :  $r^2 = 64$   
 $r = \sqrt{64}$   
 $r = 8$  units

$$\text{b) } (x+2)^2 + y^2 = 49$$
$$(x+2)^2 + (y+0)^2 = 49$$

Center  $(-2, 0)$

Radius :  $r^2 = 49$   
 $r = \sqrt{49}$   
 $r = 7$  units

$$c) (x+1)^2 + (y-11)^2 = 100$$

Center  $(-1, 11)$

Radius:  $r^2 = 100$   
 $r = \sqrt{100}$   
 $r = 10$  units

$$d) (x-16)^2 + (y-3)^2 = 144$$

Center  $(16, 3)$

Radius:  $r^2 = 144$   
 $r = \sqrt{144}$   
 $r = 12$  units

$$a) C(-11, 6); r = \sqrt{7}$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\(x-(-11))^2 + (y-6)^2 &= (\sqrt{7})^2 \\(x+11)^2 + (y-6)^2 &= 7\end{aligned}$$

$$b) C(0, 3); r = 5$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\(x-0)^2 + (y-3)^2 &= (5)^2 \\x^2 + (y-3)^2 &= 25\end{aligned}$$

$$c) C(4, -4); r = 13$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\(x-4)^2 + (y-(-4))^2 &= (13)^2 \\(x-4)^2 + (y+4)^2 &= 169\end{aligned}$$

d)  $C(-9, 14); r=2$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-9)^2 + (y-14)^2 = 2^2$$

$$(x+9)^2 + (y-14)^2 = 4$$

8.a) C(2,-2) and passing through T(8,4)

Method 1:

$$\begin{aligned}
 D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(8-2)^2 + (4-2)^2} \\
 &= \sqrt{(6)^2 + (6)^2} \\
 &= \sqrt{36+36} \\
 &= \sqrt{72} \\
 &= \sqrt{36 \times 2} \\
 &= 6\sqrt{2}
 \end{aligned}$$

$$r = 6\sqrt{2}; C(2, -2)$$

$$\begin{aligned}
 (x-h)^2 + (y-k)^2 &= r^2 \\
 (x-2)^2 + (y-2)^2 &= (6\sqrt{2})^2 \\
 (x-2)^2 + (y+2)^2 &= (36)(2) \\
 (x-2)^2 + (y+2)^2 &= 72
 \end{aligned}$$

Method 2:

$$\begin{array}{ll}
 C(2, -2) & T(8, 4) \\
 h & K \\
 x & y
 \end{array}$$

$$\begin{aligned}
 (x-h)^2 + (y-k)^2 &= r^2 \\
 (8-2)^2 + (4-2)^2 &= r^2 \\
 (6)^2 + (6)^2 &= r^2 \\
 36 + 36 &= r^2 \\
 72 &= r^2
 \end{aligned}$$

$$\begin{aligned}
 (x-h)^2 + (y-k)^2 &= r^2 \\
 (x-2)^2 + (y-2)^2 &= 72 \\
 (x-2)^2 + (y+2)^2 &= 72
 \end{aligned}$$

b) C(10,0) and passing through K(1,-3)

Method 1:

$$\begin{aligned}
 D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(1-10)^2 + (-3-0)^2} \\
 &= \sqrt{(-9)^2 + (-3)^2} \\
 &= \sqrt{81+9} \\
 &= \sqrt{90} \\
 &= \sqrt{9 \times 10} \\
 &= 3\sqrt{10}
 \end{aligned}$$

$$r = 3\sqrt{10}; C(10,0)$$

$$\begin{cases}
 (x-h)^2 + (y-K)^2 = r^2 \\
 (x-10)^2 + (y-0)^2 = (3\sqrt{10})^2 \\
 (x-10)^2 + y^2 = 90 \\
 (x-10)^2 + y^2 = 90
 \end{cases}$$

Method 2:

$$\begin{array}{ll}
 C(10,0) & K(1,-3) \\
 h & K \\
 x & y
 \end{array}$$

$$\begin{aligned}
 (x-h)^2 + (y-K)^2 &= r^2 \\
 (1-10)^2 + (-3-0)^2 &= r^2 \\
 (-9)^2 + (-3)^2 &= r^2 \\
 81 + 9 &= r^2 \\
 90 &= r^2
 \end{aligned}$$

$$\begin{aligned}
 (x-h)^2 + (y-K)^2 &= r^2 \\
 (x-10)^2 + (y-0)^2 &= 90 \\
 (x-10)^2 + y^2 &= 90
 \end{aligned}$$

$$9a) x^2 + y^2 - 12y - 5 = 0$$

$$\text{Step 1: } x^2 + y^2 - 12y = 5$$

$$\text{Step 2: } x^2 + y^2 - 12y + 36 = 5 + 36$$

$$\text{Step 3: } (x-0)^2 + (y-6)^2 = 41$$

Center (0, 6) ;  $r = \sqrt{41}$  units

$$b) x^2 + y^2 - 4x - 1 = 0$$

$$\text{Step 1: } x^2 - 4x + y^2 = 1$$

$$\text{Step 2: } x^2 - 4x + 4 + y^2 = 1 + 4$$

$$\text{Step 3: } (x-2)^2 + (y-0)^2 = 5$$

Center (2, 0) ;  $r = \sqrt{5}$  units

$$c) x^2 + y^2 - 6x - 8y - 1 = 0$$

$$\text{Step 1: } x^2 - 6x + y^2 - 8y = 1$$

$$\text{Step 2: } x^2 - 6x + 9 + y^2 - 8y + 16 = 1 + 9 + 16$$

$$\text{Step 3: } (x-3)^2 + (y-4)^2 = 26$$

Center (3, 4) ;  $r = \sqrt{26}$  units

$$d) 2x^2 + 2y^2 + 16x - 8y + 19 = 0$$

Extra Step: Divide each term by 2

$$x^2 + y^2 + 8x - 4y + \frac{19}{2} = 0$$

$$\text{Step 1: } x^2 + 8x + y^2 - 4y = -\frac{19}{2}$$

$$\text{Step 2: } x^2 + 8x + 16 + y^2 - 4y + 4 = -\frac{19}{2} + 16 + 4$$

$$\text{Step 3: } (x+4)^2 + (y-2)^2 = -\frac{19}{2} + \frac{20}{1}$$

$$(x+4)^2 + (y-2)^2 = -\frac{19}{2} + \frac{40}{2}$$

$$(x+4)^2 + (y-2)^2 = \frac{21}{2}$$

$$\text{Center } (-4, 2); r^2 = \frac{21}{2}$$

$$r = \sqrt{\frac{21}{2}} \text{ units}$$

$$e) 3x^2 + 3y^2 - 36x + 48y + 100 = 0$$

Extra Step: Divide each term by 3.

$$x^2 + y^2 - 12x + 16y + \frac{100}{3} = 0$$

$$\text{Step 1: } x^2 - 12x + y^2 + 16y = -\frac{100}{3}$$

$$\text{Step 2: } x^2 - 12x + 36 + y^2 + 16y + 64 = -\frac{100}{3} + 36 + 64$$

$$\text{Step 3: } (x-6)^2 + (y+8)^2 = -\frac{100}{3} + \frac{100}{1}$$

$$(x-6)^2 + (y+8)^2 = -\frac{100}{3} + \frac{300}{3}$$

$$(x-6)^2 + (y+8)^2 = \frac{200}{3}$$

$$\text{Center } (6, -8); r^2 = \frac{200}{3}$$

$$r = \sqrt{\frac{200}{3}}$$

$$r = \sqrt{\frac{100 \times 2}{3}}$$

$$r = \frac{10\sqrt{2}}{\sqrt{3}} \text{ units}$$

$$10. \ x^2 + y^2 - 6x - 8y - 39 = 0$$

$$\text{Step 1: } x^2 - 6x + y^2 - 8y = 39$$

$$\text{Step 2: } x^2 - 6x + 9 + y^2 - 8y + 16 = 39 + 9 + 16$$

$$\text{Step 3: } (x-3)^2 + (y-4)^2 = 64$$

a) Center  $(3, 4)$

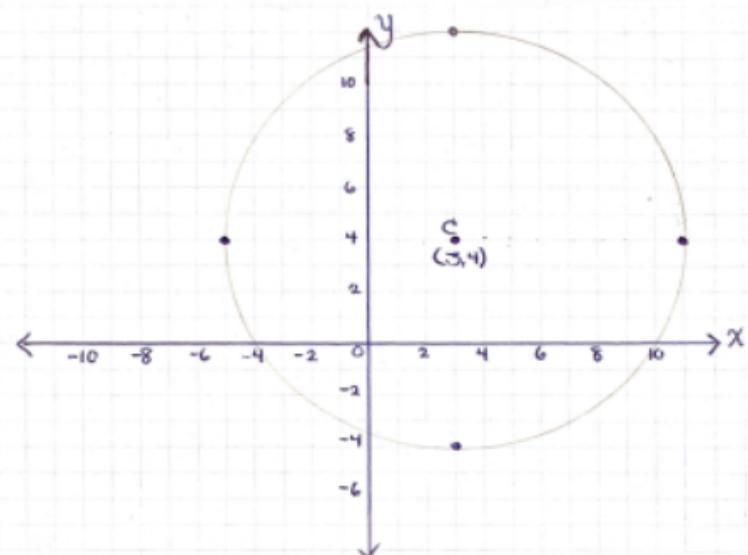
d) Domain:  $\{x | -5 \leq x \leq 11, x \in \mathbb{R}\}$

b) radius:  $r^2 = 64$

$$r = \sqrt{64} \text{ units}$$

e) Range:  $\{y | -4 \leq y \leq 12, y \in \mathbb{R}\}$

c)



$$\text{II. a) } \frac{x^2}{36} + \frac{y^2}{1} = 1$$

$$\frac{x^2}{(6)^2} + \frac{y^2}{(1)^2} = 1$$

$\hookrightarrow$  Horizontal Ellipse

$$a=6$$

$$b=1$$

$$\text{i) Major Axis} = 2a \\ = 2(6) \\ = 12 \text{ units}$$

$$\text{Minor Axis} = 2b \\ = 2(1) \\ = 2 \text{ units}$$

$$\text{ii) Vertices.} \\ (-6, 0) \text{ and } (6, 0)$$

$$\text{iii) } x\text{-ints} \Rightarrow -6 \text{ and } 6 \\ y\text{-ints} \Rightarrow -1 \text{ and } 1$$

$$\text{b) } \frac{x^2}{16} + \frac{y^2}{49} = 1$$

$$\frac{x^2}{(4)^2} + \frac{y^2}{(7)^2} = 1$$

$\hookrightarrow$  Vertical Ellipse

$$a=7$$

$$b=4$$

$$\text{i) Major Axis} = 2a \\ = 2(7) \\ = 14 \text{ units}$$

$$\text{Minor Axis} = 2b \\ = 2(4) \\ = 8 \text{ units}$$

$$\text{ii) Vertices.} \\ (0, -7) \text{ and } (0, 7)$$

$$\text{iii) } x\text{-ints} \Rightarrow -4 \text{ and } 4 \\ y\text{-ints} \Rightarrow -7 \text{ and } 7$$

$$c) \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$$

↳ Vertical Ellipse.

$$a=3$$

$$b=2$$

i) Major Axis =  $2a$   
=  $2(3)$   
= 6 units

Minor Axis =  $2b$   
=  $2(2)$   
= 4 units

ii) vertices.  
(0, -3) and (0, 3)

iii) x-ints  $\Rightarrow$  -2 and 2  
y-ints  $\Rightarrow$  -3 and 3

12.

a) Major Axis is 12  $\Rightarrow 2a = 12$   
 $a = 6$

Minor Axis is 5  $\Rightarrow 2b = 5$   
 $b = \frac{5}{2}$

\* Vertical.

Equation:  $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\frac{x^2}{(\frac{5}{2})^2} + \frac{y^2}{(6)^2} = 1$$

$$\frac{x^2}{\frac{25}{4}} + \frac{y^2}{36} = 1$$

$$\hookrightarrow \frac{4x^2}{25} + \frac{y^2}{36} = 1$$

b)  $\rightarrow$  x-ints  $\Rightarrow \pm 7$

$\rightarrow$  y-ints  $\Rightarrow \pm 9$

Equation:  $\frac{x^2}{(7)^2} + \frac{y^2}{(9)^2} = 1$

$$\frac{x^2}{49} + \frac{y^2}{81} = 1$$

c)  $\rightarrow$  One vertex  $\Rightarrow (10, 0)$

$\hookrightarrow$  Remaining vertex must be  $(-10, 0)$

\* Therefore the major axis is 20 units  
 $\hookrightarrow 2a = 20$   
 $a = 10$ .

$\rightarrow$  Minor Axis = 6

$$\hookrightarrow 2b = 6$$
$$b = 3$$

Equation:  $\frac{x^2}{(10)^2} + \frac{y^2}{(3)^2} = 1$

$$\frac{x^2}{100} + \frac{y^2}{9} = 1$$

$$13a) 16(x-1)^2 + 12(y+3)^2 = 48$$

$$\frac{16(x-1)^2}{48} + \frac{12(y+3)^2}{48} = \frac{48}{48}$$
$$\frac{(x-1)^2}{3} + \frac{(y+3)^2}{4} = 1$$

Center  $(1, -3)$

Since this ellipse is vertical, it is parallel to the  $y$ -axis.

$$b) 3(x+4)^2 + 9(y-2)^2 = 27$$

$$\frac{3(x+4)^2}{27} + \frac{9(y-2)^2}{27} = \frac{27}{27}$$
$$\frac{(x+4)^2}{9} + \frac{(y-2)^2}{3} = 1$$

Center  $(-4, 2)$

Since this ellipse is horizontal, it is parallel to the  $x$ -axis.

$$14. \text{ a) } 9x^2 + 25y^2 - 36x - 100y - 89 = 0 \quad x\text{-axis.}$$

$$\text{Step 1: } 9x^2 - 36x + 25y^2 - 100y = 89$$

$$\text{Extra Step: } 9(x^2 - 4x) + 25(y^2 - 4y) = 89$$

$$\text{Step 2: } 9(x^2 - 4x + 4) + 25(y^2 - 4y + 4) = 89 + 36 + 100$$

$$\text{Step 3: } 9(x-2)^2 + 25(y-2)^2 = 225$$

$$\frac{9(x-2)^2}{225} + \frac{25(y-2)^2}{225} = \frac{225}{225}$$

$$\frac{(x-2)^2}{25} + \frac{(y-2)^2}{9} = 1$$

$$b) 2x^2 + 5y^2 + 20x - 30y + 75 = 0$$

$$\text{Step 1: } 2x^2 + 20x + 5y^2 - 30y = -75$$

$$\text{Extra Step: } 2(x^2 + 10x) + 5(y^2 - 6y) = -75$$

$$\text{Step 2: } 2(x^2 + 10x + 25) + 5(y^2 - 6y + 9) = -75 + 50 + 45$$

$$\text{Step 3: } 2(x+5)^2 + 5(y-3)^2 = 20$$

$$\frac{2(x+5)^2}{20} + \frac{5(y-3)^2}{20} = \frac{20}{20}$$
$$\frac{(x+5)^2}{10} + \frac{(y-3)^2}{4} = 1$$

$$15. 25x^2 + 16y^2 + 150x + 32y - 159 = 0$$

$$\text{Step 1: } 25x^2 + 150x + 16y^2 + 32y = 159$$

$$\text{Extra Step: } 25(x^2 + 6x) + 16(y^2 + 2y) = 159$$

$$\text{Step 2: } 25(x^2 + 6x + 9) + 16(y^2 + 2y + 1) = 159 + 225 + 16$$

$$\text{Step 3: } 25(x+3)^2 + 16(y+1)^2 = 400$$

$$\frac{25(x+3)^2}{400} + \frac{16(y+1)^2}{400} = \frac{400}{400}$$

$$\frac{(x+3)^2}{16} + \frac{(y+1)^2}{25} = 1$$

a) Center  $(-3, -1)$

$$b) \frac{(x+3)^2}{(4)^2} + \frac{(y+1)^2}{(5)^2} = 1 \quad C(-3, -1)$$

vertices

(-3, -1-5) and (-3, -1+5)

(-3, -6) and (-3, 4)

$$c) \text{Major Axis} = 2a = 2(5) = 10 \text{ units} \quad \text{Minor Axis} = 2b = 2(4) = 8 \text{ units}$$

$$16. 4x^2 + 9y^2 - 8x - 54y + 49 = 0$$

$$\text{Step 1: } 4x^2 - 8x + 9y^2 - 54y = -49$$

$$\text{Extra Step: } 4(x^2 - 2x) + 9(y^2 - 6y) = -49$$

$$\text{Step 2: } 4(x^2 - 2x + 1) + 9(y^2 - 6y + 9) = -49 + 4 + 81$$

$$\text{Step 3: } 4(x-1)^2 + 9(y-3)^2 = 36$$

$$\frac{4(x-1)^2}{36} + \frac{9(y-3)^2}{36} = \frac{36}{36}$$
$$\frac{(x-1)^2}{9} + \frac{(y-3)^2}{4} = 1$$

a) Center  $(1, 3)$

b)  $\frac{(x-1)^2}{9} + \frac{(y-3)^2}{4} = 1$

vertices  $(1-3, 3)$  and  $(1+3, 3)$   
 $(-2, 3)$  and  $(4, 3)$

c) Major Axis =  $2a$   
=  $2(3)$   
= 6 units

Minor Axis =  $2b$   
=  $2(2)$   
= 4 units