Questions from Homework

Ex 3

① g) $\log_5 \sqrt[3]{x^2+1}$

$\log_5 (x^2+1)^{1/3}$

$\frac{1}{3} \log_5 (x^2+1)$

③ a) $\log_{10} 12 + \frac{1}{2} \log_{10} 7 - \log_{10} 2$

$\log_{10} 12 + \log_{10} 7^{1/2} - \log_{10} 2$

$\log_{10} 12 + \log_{10} \sqrt{7} - \log_{10} 2$

$\log_{10} 12\sqrt{7} - \log_{10} 2$

$\log_{10} \left(\frac{12\sqrt{7}}{2}\right)$

$\log_{10} 6\sqrt{7}$

$\boxed{\text{Exercise 2}}$

④ g) $\log_2 (\log_3 x) = 4$

Base Answer Exponent

$\rightarrow \quad 2^4 = \log_3 x$

exponent $16 = \log_3 x$ ← answer

Base

$\rightarrow \quad 3^{16} = x$

$43\,046\,721 = x$

$\boxed{\text{Exercise 3}}$

② a) $\log_5 \sqrt{125}$

$5^y = \sqrt{125}$

$5^y = (125)^{1/2}$

$5^y = (5^3)^{1/2}$

$\cancel{5}^y = \cancel{5}^{3/2}$

$\boxed{y = 3/2}$

Exercise 2

④ d) $\log_3(2-x) = 3$

$$3^3 = 2-x$$

$$27 = 2-x$$

$$\boxed{x = -25}$$

Logarithms

exponential form

$$x = a^y$$

Say "the base ***a*** to the exponent ***y*** is ***x***."

logarithmic form

$$y = \log_a x$$

Say "***y*** is the exponent to which you raise base ***a*** to get the answer ***x***."

$$x = a^y \longleftrightarrow y = \log_a x$$

When you work with equations involving logarithms you need to use the laws of logarithms, which are summarized below:

$$\log_a M + \log_a N = \log_a (M \times N)$$

$$\log_a M - \log_a N = \log_a \left(\frac{M}{N} \right)$$

$$\log_a (N^p) = p \log_a N$$

$$\log_a (N^{\frac{p}{q}}) = \frac{p}{q} \log_a N$$

The base of a logarithm can be any real number. However, a logarithm to the base 10 is especially useful because the decimal system, and as a result your calculator, is also based on the number 10. Logarithms to the base 10 are called *common logarithms* and are written as

$$\log_{10} x \quad \overset{=}{\text{or}} \quad \log x$$

Example 1

Find $\log 56 = 1.7481$

$\rightarrow \quad 10^{1.7481} = 56$

Common logarithms appear in many formulas as shown in the following example.

Example 2

The approximate distance above sea level, *d,* in kilometers, is given by the formula:

$$d = \frac{500(\log P - 2)}{27}$$

where *P* is the pressure in kilopascals.

a) If the reading on a barometer is 750 *kPa,* then how far above sea level are you?

b) What is the barometric pressure 1*km* above sea level?

a) $P = 750\text{kPa}$
$d = ?$

$$d = \frac{500(\log 750 - 2)}{27}$$

$$d = \frac{500(2.8751 - 2)}{27}$$

$$d = \frac{500(0.8751)}{27}$$

$$d = \frac{437.55}{27}$$

$$\boxed{d = 16.2\text{ km}}$$

b) $d = 1\text{km}$
$P = ?$

$$1 = \frac{500(\log P - 2)}{27}$$

$$\frac{27}{500} = \frac{\cancel{500}(\log P - 2)}{\cancel{500}}$$

$$0.054 = \log P - 2$$

$$2.054 = \log P$$

$$10^{2.054} = P$$

$$\boxed{113.2\text{ kPa} = P}$$

The irrational number *"e"* which is approximately 2.71828... plays an important role in the development of mathematics. The value of *e* can be approximated by the following expression:

$$\left(1+\frac{1}{n}\right)^{n}$$

As *"n"* gets larger, the expression approaches the number 2.71828... which is an approximation of *e.* This value is called *"Euler's Constant"* named after Leonard Euler.

$$\boxed{e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}}$$

Logarithms with the base of *e* are often used in advanced mathematics are called *natural logarithms.* The notation ln *x* is used to indicate logarithms to the base *e.* Thus,

$$\boxed{\ln x = \log_e x}$$

Example 3

Solve

a) $y = \ln 3$

$y = 1.0986$

$1.0986 = \ln 3$

$2.71828^{1.0986} = 3$

b) $2.685 = \ln x$

$e^{2.685} = x$

$14.66 = x$

Homework