Warm Up

Solve the following system of equations and identify the type of system... $\longrightarrow$ Inconsistent

1. $3x + 2y + z = 3$
2. $x - 3y + z = 4$
3. $-6x - 4y - 2z = 1$

$$\begin{array}{r} 6x+4y+2z=6 \\ (+)\ \underline{-6x-4y-2z=1} \\ 0=7 \end{array}$$

No Solution

Questions from homework

③ f) $\begin{pmatrix} 5 & 9 \\ 10 & -8 \end{pmatrix}$

Determinant Method

① $\text{Det} = 5(-8) - (9)(10)$

$= -40 - 90$

$= -130$

② New Matrix:

$\begin{pmatrix} -8 & -9 \\ -10 & 5 \end{pmatrix}$

③ $-\frac{1}{130}\begin{pmatrix} -8 & -9 \\ -10 & 5 \end{pmatrix}$

$\begin{pmatrix} 8/130 & 9/130 \\ 10/130 & -5/130 \end{pmatrix}$

Inverse $\boxed{\begin{pmatrix} 4/65 & 9/130 \\ 1/13 & -1/26 \end{pmatrix}}$

Identity Matrix Method

$\left(\begin{array}{cc|cc} 5 & \underline{9} & 1 & 0 \\ \underline{10} & -8 & 0 & 1 \end{array}\right)$ $\begin{array}{l} 8R1 + 9R2 \\ R2 - 2R1 \end{array}$ $\left(\begin{array}{cc|cc} \underline{130} & 0 & 8 & 9 \\ 0 & \underline{-26} & -2 & 1 \end{array}\right) \begin{array}{l} \div 130 \\ \div -26 \end{array}$

$\left(\begin{array}{cc|cc} 1 & 0 & 8/130 & 9/130 \\ 0 & 1 & -2/-26 & -\frac{1}{26} \end{array}\right) \longrightarrow \left(\begin{array}{cc|cc} 1 & 0 & 4/65 & 9/130 \\ 0 & 1 & 1/13 & -1/26 \end{array}\right)$

Solving Equations Using Matrices

or 3X3

Matrix Elimination involves taking the coefficients from a 2X2 system, placing them in a matrix, and working to make a new matrix by multiplying, dividing, and combining rows.

The combination of the coefficients from a system of equations and their solutions in an equivalent form is called an **augmented matrix**.

Ex. $$\begin{aligned} 2x + y + 3z &= 0 \\ x + y - 2z &= -1 \\ x - 2y - z &= 3 \end{aligned} \longrightarrow \left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & -2 & -1 \\ 1 & -2 & -1 & 3 \end{array}\right)$$

$$\begin{aligned} x + 3y &= 4 \\ 3x + 4y &= 2 \end{aligned} \longrightarrow \left(\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array}\right)$$

Row Reduced Echelon Form

The goal in solving a system of equation using matrices is to obtain a new matrix - row reduced echelon form of a matrix. It takes the form:

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \end{pmatrix} \quad \textbf{Or} \quad \begin{pmatrix} 1 & \# & \# & x \\ 0 & 1 & \# & y \\ 0 & 0 & 1 & z \end{pmatrix}$$

To reduce a matrix to its row echelon form, we can:

a) Multiply or divide a row by a constant.

b) Add or subtract one row from another.

c) Interchange rows.

Solve the following system of equations using an augmented matrix reduced to its row echelon form...

$$x + 3y = 4$$

$$3x + 4y = 2$$

$$3①\ \left(\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array}\right) \xrightarrow{②-①} \left(\begin{array}{cc|c} 3 & 9 & 12 \\ 3 & 4 & 2 \end{array}\right) \xrightarrow{②-3①} \left(\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & -5 & -10 \end{array}\right)$$

$-5y = -10$

$\boxed{y = 2}$

$x + 3(2) = 4$

$x + 6 = 4$

$\boxed{x = -2}$

$(-2, 2)$

Try this one on your own...

$$3x + 2y = 12$$
$$2x + 3y = 13$$

$$2(1) - 3(2) \quad \left(\begin{array}{cc|c} 3 & 2 & 12 \\ 2 & 3 & 13 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 3 & 2 & 12 \\ 0 & -5 & -15 \end{array}\right)$$

$-5y = -15$

$\boxed{y = 3}$

$3x + 2(3) = 12$

$3x + 6 = 12$

$3x = 6$

$\boxed{x = 2}$

1. **Express system in the form of an augmented matrix**
2. **Eliminate *"x"* in equation 2 and 3.**
3. **Eliminate *"y"* in equation 3 (must add/subtract 2 and 3)**
4. **Create triangle of zeroes and solve.**

Ex. $2x + y - z = -1$
$3x - y + 2z = 8$
$2x + 2y - 3z = -6$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 3 & -1 & 2 & 8 \\ 2 & 2 & -3 & -6 \end{array}\right) \xrightarrow[3-1]{2(2)-3(1)}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -5 & 7 & 19 \\ 0 & 1 & -2 & -5 \end{array}\right) \xrightarrow[5(3)+(2)]{} \left(\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -5 & 7 & 19 \\ 0 & 0 & -3 & -6 \end{array}\right)$$

$-3z = -6$
$\boxed{z = 2}$

$-5y + 7(2) = 19$
$-5y + 14 = 19$
$-5y = 5$
$\boxed{y = -1}$

$2x + (-1) - (2) = -1$
$2x - 3 = -1$
$2x = 2$
$\boxed{x = 1}$

Homework

2 a, c, e, g