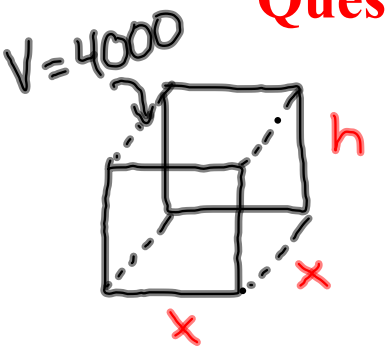
Questions From Homework



Let x = the length & width

Let h = the height

$A = x^2 + 4xh$ ← Express as a single variable

$V = x^2h$

$4000 = x^2h$

$\frac{4000}{x^2} = h$

$A = x^2 + 4x\left[\frac{4000}{x^2}\right]$

$A = x^2 + 16000x^{-1}$

$A' = 2x - \frac{16000}{x^2}$

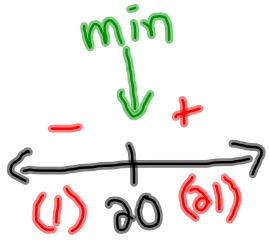
$0 = 2x - \frac{16000}{x^2}$

$\frac{16000}{x^2} = 2x$

$2x^3 = 16000$

$x^3 = 8000$

$x = 20\text{ cm}$



$h = \frac{4000}{(20)^2}$

$h = \frac{4000}{400}$

$h = 10\text{cm}$

∴ The dimensions should be 20cm X 20cm X 10cm.

Questions From Homework

5. Find the point on the parabola $2y = x^2$ that is closest to the point (-4, 1)

$y = \frac{x^2}{2}$ ← minimize the distance.

Let $x = x$

Let $y = y$

$$d = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$d = \sqrt{(x+4)^2+(y-1)^2}$$ ← Express as a single variable

$$d = \sqrt{(x+4)^2+\left(\frac{x^2}{2}-1\right)^2}$$

$$d = \sqrt{x^2+8x+16+\frac{x^4}{4}-x^2+1}$$

$$d = \sqrt{\frac{x^4}{4}+8x+17} = \left[\frac{x^4}{4}+8x+17\right]^{\frac{1}{2}}$$

$$f(x) = \frac{x^4}{4}+8x+17$$

$$f'(x) = x^3+8$$

$$0 = x^3+8$$

$$-8 = x^3$$

$$-2 = x$$

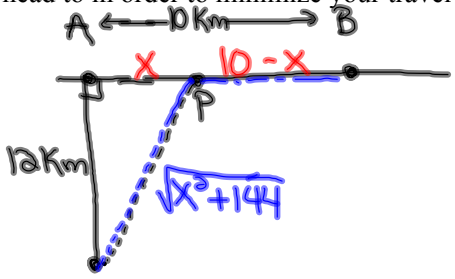
$$y = \frac{x^2}{2}$$

$$y = \frac{(-2)^2}{2}$$

$$y = 2$$

$\therefore$ $(-2, 2)$ is the closest point

You are in a dune buggy in the desert 12km due south of the nearest point A on a straight east-west road. You wish to get to point B on the road 10km east of point A. If your dune buggy can average 15km/h travelling over the desert, and 39km/h travelling on the road, toward what point on the road should you head to in order to minimize your travel time from A to B?



Let x = distance from A to P

$$T = \frac{d}{s}$$

$$T = \frac{\sqrt{x^2+144}}{15} + \frac{10-x}{39}$$

$$T = \frac{1}{15}(x^2+144)^{\frac{1}{2}} + \frac{10}{39} - \frac{1}{39}x$$

$$T' = \frac{1}{30}(x^2+144)^{-\frac{1}{2}}(2x) - \frac{1}{39}$$

$$T' = \frac{x}{15\sqrt{x^2+144}} - \frac{1}{39}$$

$$0 = \frac{x}{15\sqrt{x^2+144}} - \frac{1}{39}$$

$$\frac{1}{39} = \frac{x}{15\sqrt{x^2+144}}$$

$$(39x)^2 = (15\sqrt{x^2+144})^2$$ ← square both sides

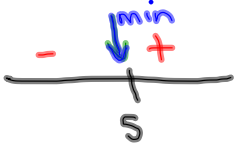
$$1521x^2 = 225(x^2+144)$$

$$1521x^2 = 225x^2 + 32400$$

$$1296x^2 = 32400$$

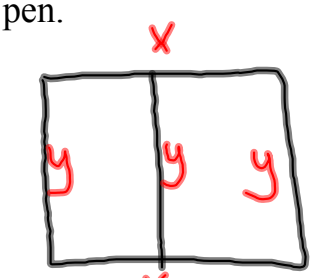
$$x^2 = 25$$

$$x = \pm 5 \text{ km}$$



∴ Head to a point that is 5km east of A.

You have 400 m of fencing to construct a rectangular pen that will be divided into 2 sections of equal size. Find the dimensions that would maximize the area of the whole pen.



Let x = length

Let y = width

$P = 2x + 3y$

$400 = 2x + 3y$

$400 - 2x = 3y$

$$\frac{400-2x}{3} = y$$

$A = xy$

$$A = x\left[\frac{400-2x}{3}\right]$$

$$A = \frac{400x - 2x^2}{3}$$

$$A = \frac{400}{3}x - \frac{2}{3}x^2$$

$$A' = \frac{400}{3} - \frac{4}{3}x$$

$$0 = \frac{400}{3} - \frac{4}{3}x$$

$$\frac{4x}{3} = \frac{400}{3}$$

$12x = 1200$

$x = 100\text{ m}$

$$y = \frac{400 - 2(100)}{3}$$

$$y = \frac{200}{3}$$

$y = 66.\overline{6}\text{ m}$

Find the points on the parabola $y = 6 - x^2$ that are closest to the point (0, 3)

Homework