Questions From Homework

② b) $\lim\limits_{x \to -1^-} g(x) = 2$

c) $\lim\limits_{x \to -1^+} g(x) = 1$

d) $\lim\limits_{x \to -1} g(x) = DNE$

Recall from our previous discussions that ...

1 Theorem $\quad \lim_{x \to a} f(x) = L \quad$ if and only if $\quad \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Left Hand Limit = Right Hand Limit
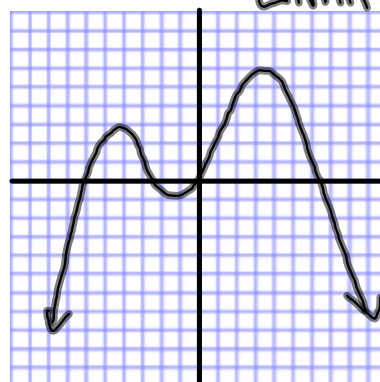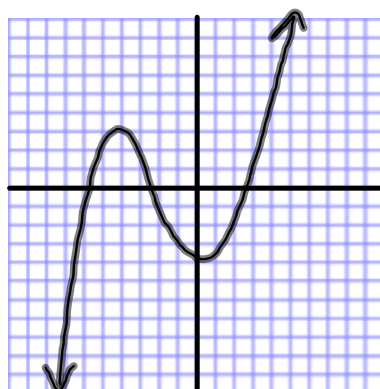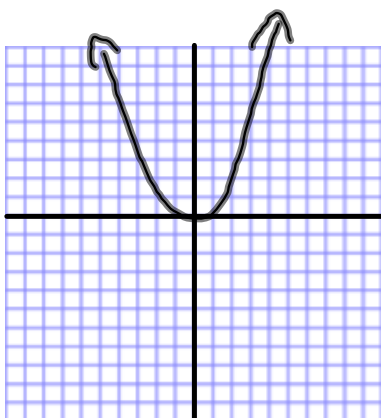
These graphs have limits that exist at every x value and are what we call ***continuous***
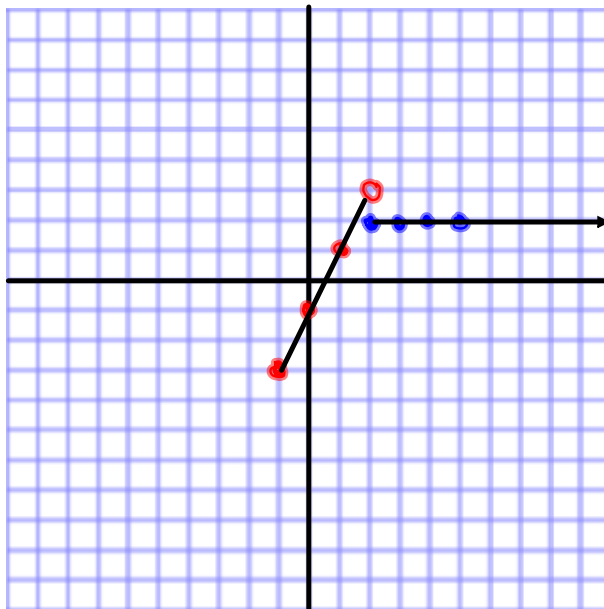
We also want to be able to check limits of piecewise defined functions...

Example:

$$f(x) = \begin{cases} 2x - 1 & \text{if } -1 \leq x < 2 \\ 2 & \text{if } x \geq 2 \end{cases}$$

← Domain

Graph this function:

$2x - 1$

x	$f(x)$
-1	-3
0	-1
1	1
2	3

2

x	$f(x)$
2	2
3	2
4	2
5	2

discontinuous at
x=2

Evaluate the following limits:

$$\lim_{x \to 2^-} f(x) = 3 \qquad \lim_{x \to 2^+} f(x) = 2 \qquad \lim_{x \to 2} f(x) = DNE$$

$$f(2) = 2$$

Continuity

Definition

- We noticed in the preceding section that...
  - the limit of a function as x <u>approaches</u> a
  can often be found simply by...
    - calculating the value of the function <u>at</u> a.
- Functions with this property are called *continuous* at a:

> **1** **Definition** A function f is **continuous at a number a** if
> $$\lim_{x \to a} f(x) = f(a)$$

- This definition implicitly requires three things if f is continuous at a:
  1. $f(a)$ is <u>defined</u>
     - That is, a is in the domain of f
  2. $f(x)$ <u>has</u> a limit as x approaches a
  3. This limit is actually <u>equal to</u> $f(a)$.

In English!
- **Graph must be defined at that point**
- **Limit from left and right must be equal**
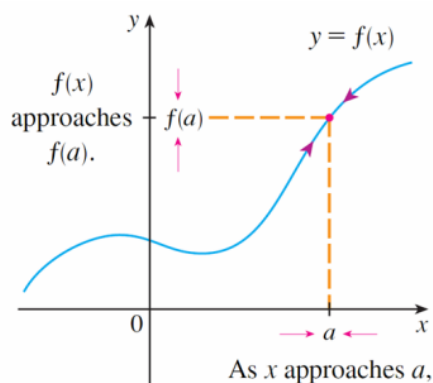- **Limit must be the same as the y value**

$f(x)$ approaches $f(a)$.

$y = f(x)$

As x approaches a,

FIGURE 1

Examine the graph shown below
for points of discontinuity...

FIGURE 2

not defined at x=1

no limit exists at x=3

the limit does not equal the defined height at x=5
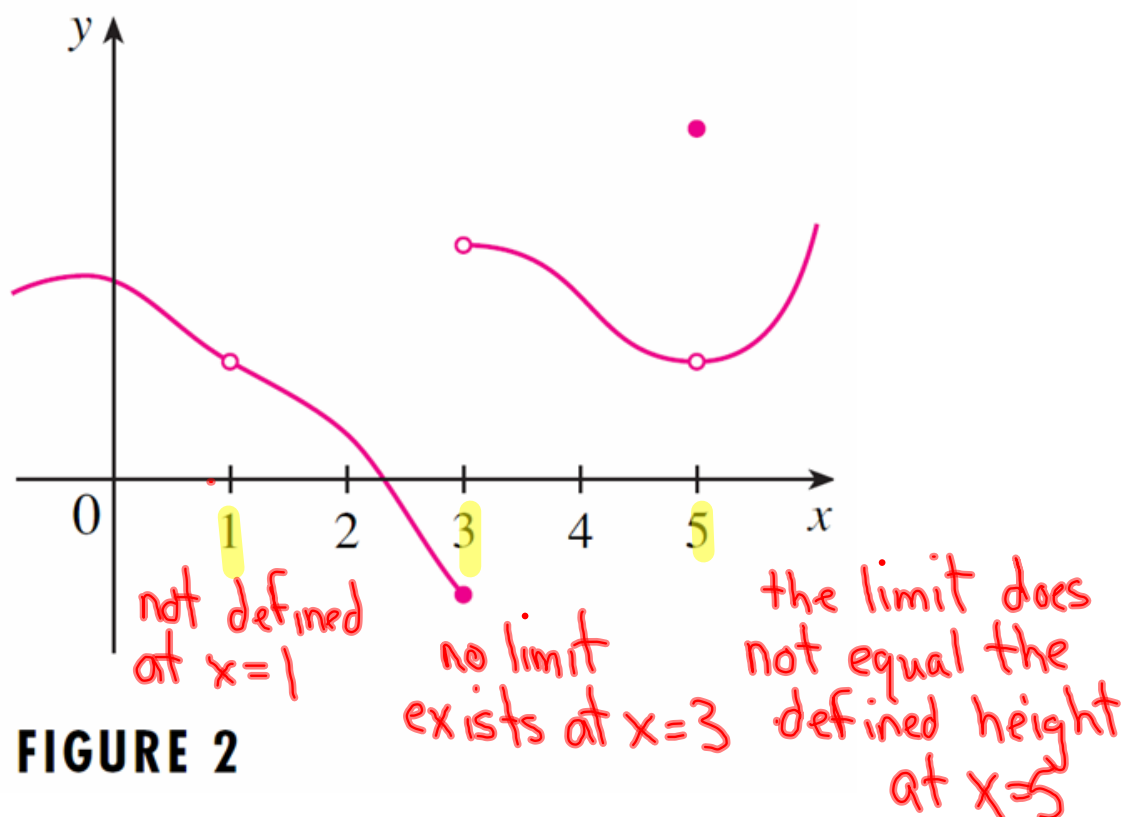
- *f* is discontinuous at 1 because *f*(1) is not defined…
  - …despite the fact that *f* has a limit at *a* = 1
- *f* is also discontinuous at 3 , but for a different reason:
  - *f*(3) is defined, but *f* has no limit at *a* = 3 .
- *f* has both a value and a limit at 5 , but they are different; thus *f* is discontinuous at 5 .

Let's simplify things...

A function whose graph has holes or breaks is considered
discontinuous at these particular points. (state the x values)

If you have to lift your pencil from the page to sketch the graph,
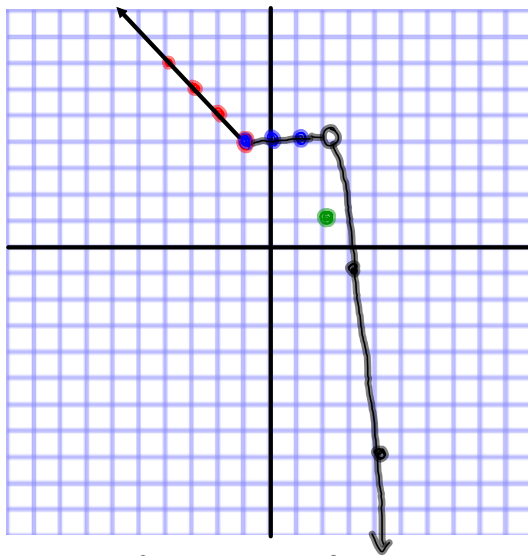it is discontinuous anywhere you lift your pencil

Examples:

Given the function $f(x) = \begin{cases} 3-x & , & \text{if } x < -1 \\ 4 & , & \text{if } -1 \leq x < 2 \\ 1 & , & \text{if } x = 2 \\ 8-x^2 & , & \text{if } x > 2 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical
reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.

$3-x$

x	$f(x)$
-1	4
-2	5
-3	6

4

x	$f(x)$
-1	4
0	4
1	4
2	4

1

x	$f(x)$
2	1

$8-x^2$

x	$f(x)$
2	4
3	-1
4	-8

discontinuous at x=2

$f(2) = 1$

$\lim\limits_{x \to 2} f(x) = 4$

In English!
- Graph must be defined at that point
- Limit from left and right must be equal
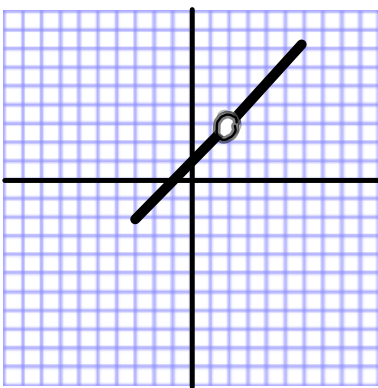- Limit must be the same as the y value

$f(2) \neq \lim\limits_{x \to 2} f(x)$

∴ The limit does not equal the defined height
of the function.

Summary of Continuity:
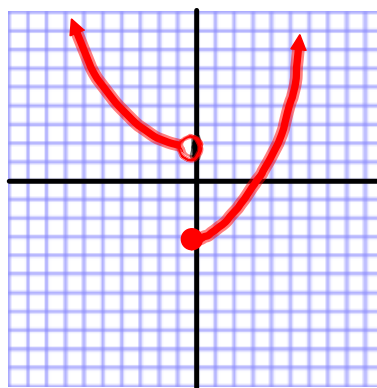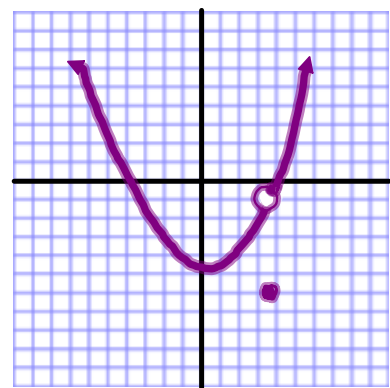
All of these graphs are discontinuous.

| Hole Function | Step Graph | |

Limit exists
but it is not
defined
Not Continuous

Limit does not
exist
Not Continuous

Limit is not the
same as the y
value
Not Continuous

Homework

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5, 6, 7, 8, 9

Try this one...

$$f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \leq 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$$

Evaluate:

$$\lim_{x \to 1} f(x) \qquad\qquad \lim_{x \to 3} f(x)$$

Given the function $f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \le 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.