Questions from homework

③ (1) $y = 4 - 13x - 6x^2 - x^3$

$y' = -13 - 12x - 3x^2$

$y'' = -12 - 6x$

a) $y' = -(13 + 12x + 3x^2)$ ← Always Negative

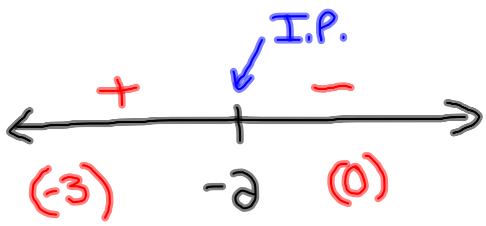
Decreasing on $(-\infty, \infty)$

b) None → There are no critical #'s

c) $y'' = -12 - 6x$

$y'' = -6(2 + x)$

CV: $x = -2$



CU on $(-\infty, -2)$

CD on $(-2, \infty)$

d) Find y when $x = -2$ IP: $(-2, 14)$

$y = 4 - 13x - 6x^2 - x^3$

$y = 4 - 13(-2) - 6(-2)^2 - (-2)^3$

$y = 4 + 26 - 24 + 8$

$y = 14$

② k) $y = x^{2/3}(5+x) = 5x^{2/3} + x^{5/3}$

$y' = \frac{10}{3}x^{-1/3} + \frac{5}{3}x^{2/3}$

$y'' = -\frac{10}{9}x^{-4/3} + \frac{10}{9}x^{-1/3}$

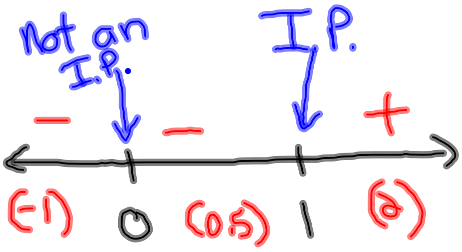
$y'' = -\frac{10}{9}x^{-4/3}\left[1 - x\right]$

$y'' = \frac{-10(1-x)}{9\sqrt[3]{x^4}}$

CV: $1 - x = 0$ | $9x^{4/3} = 0$

$-x = -1$ | $x^{4/3} = 0$

$x = 1$ | $x = 0$



CD on $(-\infty, 1)$

CU on $(1, \infty)$

Inflection Point $(x=1)$

$y = x^{2/3}(5+x)$

$y = (1)^{2/3}(5+(1))$

$y = 6$

$\boxed{(1, 6)}$

② b) $y = \dfrac{x^2}{\sqrt{x+1}} = \dfrac{x^2}{(x+1)^{1/2}}$

$$y' = \frac{(x+1)^{1/2}(2x) - x^2\left(\frac{1}{2}\right)(x+1)^{-1/2}(1)}{x+1}$$

$$y' = \frac{2\cdot\ 2x(x+1)^{1/2} - \frac{\cancel{2}\cdot\ x^2}{\cancel{2}}(x+1)^{-1/2}}{2\cdot\ x+1}$$

$$y' = \frac{4x(x+1)^{1/2} - x^2(x+1)^{-1/2}}{2(x+1)}$$

$$y' = \frac{x\cancel{(x+1)^{-1/2}}\left[4\overset{4x+4}{(x+1)} \overset{-x}{- x}\right]}{2\cancel{(x+1)}} = \frac{x\overset{3x^2+4x}{(3x+4)}}{2(x+1)^{3/2}}$$

$$y'' = \frac{2(x+1)^{3/2}(6x+4) - (3x^2+4x)(3)(x+1)^{1/2}(1)}{4(x+1)^3}$$

$$y'' = \frac{\cancel{(x+1)^{1/2}}\left[2(x+1)(6x+4) - 3(3x^2+4x)\right]}{4\cancel{(x+1)}^3}$$

$$y'' = \frac{12x^2+20x+8-9x^2-12x}{4(x+1)^{5/2}}$$

$$y'' = \frac{3x^2+8x+8}{4(x+1)^{5/2}}$$ ← Always Positive

CV: $3x^2+8x+8=0$

Doesn't factor

$4(x+1)^{5/2} = 0$

$4\sqrt{(x+1)^5} = 0$

$\sqrt{(x+1)^5} = 0$

$(x+1)^5 = 0$

$x+1 = 0$

$x = -1$

Undef. +

(-2) -1 (0)

CU: $(-1,\infty)$

Second Derivative Test for Local Extrema

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

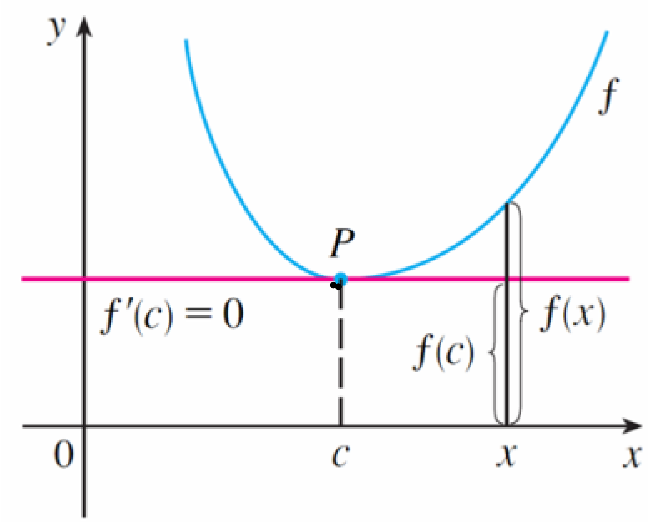


FIGURE 6

$f''(c) > 0$, f is concave upward

Use the second derivative test to find the local maximum and minimum values of

$$f(x) = x^3 - 12x + 5$$

$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x-2)(x+2)$

CV: $x = \pm 2$

max   min

$+$ $-$ $+$

(-3) -2 (0) $+2$ (3)

$f''(x) = 6x$

Second Derivative test:

$f''(2) = 6(2)$	$f''(-2) = 6(-2)$
$= 12$	$= -12$
since $f'(2) = 0$ and $f''(2)$ is a positive, $f(2)$ is a min	since $f'(-2) = 0$ and $f''(-2)$ is a negative, $f(-2)$ is a max.

min	max
$f(2) = (2)^3 - 12(2) + 5$	$f(-2) = (-2)^3 - 12(-2) + 5$
$= 8 - 24 + 5$	$= -8 + 24 + 5$
$= -11$	$= 21$
$(2, -11)$	$(-2, 21)$

Find the maximum and minimum values of $f(x) = x^4 - 8x^3$

Use this together with concavity and points of inflection, to sketch the curve.

Homework