Making a Complete Sketch

Examine the function $f(x) = x^4 - 4x^3$ with respect to...

- Intercepts ✓
- Symmetry ✓
- Asymptotes ✓
- Intervals of Increase or Decrease ✓
- Local Maximum and Minimum values ✓
- Concavity and Points of Inflection ✓
- Sketch the Curve

$$f(x) = x^4 - 4x^3 \qquad f'(x) = 4x^3 - 12x^2 \qquad f''(x) = 12x^2 - 24x$$
$$f(x) = x^3(x-4) \qquad f'(x) = 4x^2(x-3) \qquad f''(x) = 12x(x-2)$$

① Intercepts:

x int (y=0) | y int (x=0)

$0 = x^3(x-4)$ | $f(0) = (0)^4 - 4(0)^3$

$(0,0)$ $x = 0, 4$ $(4,0)$ | $(0,0)$

② Symmetry:

$f(-x) = (-x)^4 - 4(-x)^3$

$= x^4 + 4x^3$

No Symmetry

③ Asymptotes: None

④ Intervals of Inc/Dec:

neither min

$f'(x) = 4x^2(x-3)$

$- - +$

(-1) 0 (1) 3 (4)

Increasing on $(3, \infty)$
Decreasing on $(-\infty, 3)$

CV: x = 0, 3

⑤ Max/Mins:

$f(3) = (3)^4 - 4(3)^3$

$= 81 - 108$

$= -27$

$(3, -27)$ local min

⑥ Concavity:

I.P.   I.P.

$f''(x) = 12x(x-2)$

$+ - +$

(-1) 0 (1) 2 (3)

CU: on $(-\infty, 0) + (2, \infty)$
CD: on $(0, 2)$

CV: x = 0, 2

⑦ Inflection Points:

$f(0) = (0)^4 - 4(0)^3$

$= 0$

$(0,0)$

$f(2) = (2)^4 - 4(2)^3$

$= 16 - 32$

$= -16$

$(2, -16)$

$y = x^4 - 4x^3$

$(0, 0)$

inflection points

$(2, -16)$
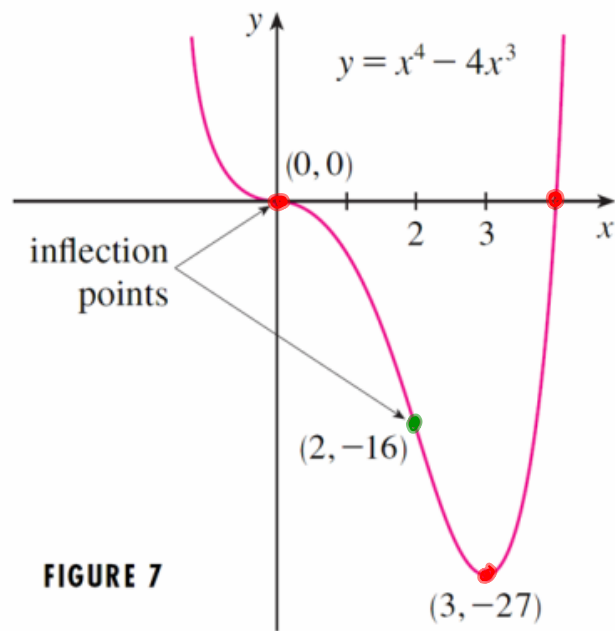
FIGURE 7

$(3, -27)$

homework

Examine the function $f(x) = \dfrac{x^2}{1-x^2}$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

Examine the function $f(x) = \dfrac{x^2}{x-7}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve