Questions From Homework

④ d) $y = \sqrt[3]{x}$, $\overset{x_1}{(-8}, \overset{y_1}{-2)}$

$y = x^{1/3}$

① Find derivative:

$y' = \frac{1}{3}x^{-2/3}$

$$\boxed{y' = \frac{1}{3x^{2/3}}}$$

② Sub in x-value (x = -8)

$y' = \frac{1}{3(-8)^{2/3}}$

$y' = \frac{1}{3(4)}$

$$\boxed{y' = \frac{1}{12}}$$

③ Find the equation:

$y - y_1 = m(x - x_1)$

$12 \cdot y + 12 \cdot 2 = 12 \cdot \frac{1}{12}(x+8)$

$12y + 24 = 1(x+8)$

$12y + 24 = x + 8$

$-x + 12y + 16 = 0$

$$\boxed{x - 12y - 16 = 0}$$

Questions From Homework

⑤ $f(x) = \boxed{\frac{1}{x}}$ | $f(x+h) = \boxed{\frac{1}{x+h}}$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

Multiply by:
$(x)(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{x - (x+h)}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x)(x+h)} = \boxed{\frac{-1}{x^2}}$$

Example:

Find the slope of the tangent line to the graph of the given function at the given x value.

$$g(x) = \sqrt[5]{x} \qquad x = 32$$

$$g(x) = x^{1/5}$$

$$g'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}} = \frac{1}{5\sqrt[5]{x^4}}$$

$$g'(32) = \frac{1}{5\sqrt[5]{(32)^4}} = \frac{1}{5(16)} = \boxed{\frac{1}{80}}$$

Example:

Find the equation of the tangent line to the curve $f(x) = x^6$ at the point (-2, 64) ← y value

↑ x value

Remember that the equation of a line is found by using the point-slope formula... $y - y_1 = m(x - x_1)$

The curve is the graph of the function f(x) = x^6 and we know that the slope of the tangent line at (-2, 64) is the derivative f '(-2)

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

① Find derivative:

$F(x) = x^6$

$F'(x) = 6x^5$

② Fill in your x value: to find the slope of your tangent line:

$f'(-2) = 6(-2)^5$

$= 6(-32)$

$\boxed{= -192} \rightarrow m$

③ Use: $y - y_1 = m(x - x_1)$

$y - 64 = -192(x + 2)$

$y - 64 = -192x - 384$

$\boxed{192x + y + 320 = 0}$

$y = 2\sqrt{x} = 2x^{1/2}$ $\quad (9, 6)$

$x_1 \quad y_1$

① $y' = x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$

② if $x = 9$

$y' = \frac{1}{\sqrt{9}} = \frac{1}{3} = m$

③ $y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{3}(x - 9)$

$y - 6 = \frac{1}{3}x - 3$

$-\frac{1}{3}x + y - 3 = 0$

$\boxed{\frac{1}{3}x - y + 3 = 0}$ or $\boxed{x - 3y + 9 = 0}$

Sums and Differences

- These next rules say that <u>the derivative of a sum (difference) of functions is the sum (difference) of the derivatives</u>:

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x) = 2x^4 + \sqrt{x} = 2x^4 + x^{1/2}$

$F'(x) = 8x^3 + \frac{1}{2}x^{-1/2} = \boxed{8x^3 + \frac{1}{2x^{1/2}}}$

2. $f(x) = 6x^4 - 5x^3 - 2x + 17$

$F'(x) = 24x^3 - 15x^2 - 2$

3. $f(x) = (2x^3 - 5)^2$

$F(x) = (2x^3-5)(2x^3-5)$

$F(x) = 4x^6 - 20x^3 + 25$

$F'(x) = 24x^5 - 60x^2$

Homework