4 a) $f(x) = \frac{x^2-5x-6}{x+1} = \frac{(x-6)\cancel{(x+1)}}{\cancel{(x+1)}} = x-6$

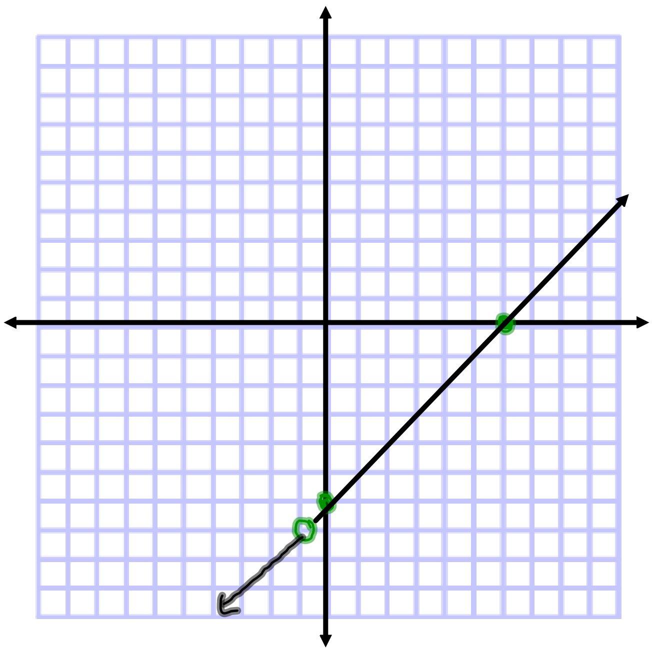
1 Roots: $x=6$

2 V.A. None

3 O.A. $y=x-6$

4 Holes: $x=-1$

5 y int $y=-6$



① d) $\dfrac{\dfrac{2}{x}+\dfrac{3}{xy}}{\dfrac{2}{xy}+\dfrac{3}{y}} \rightarrow \dfrac{\dfrac{2y+3}{xy}}{\dfrac{2+3x}{xy}}$

$\rightarrow \dfrac{2y+3}{\cancel{xy}} \cdot \dfrac{\cancel{xy}}{2+3x} \rightarrow \boxed{\dfrac{2y+3}{2+3x}}$ $x \neq 0, -2/3$
$y \neq 0$

$2+3x=0$
$3x=-2$
$x=-\dfrac{2}{3}$

$\dfrac{\cancel{xy}\,\dfrac{2}{\cancel{x}}+\dfrac{3}{\cancel{xy}}\,\cancel{xy}}{\cancel{xy}\,\dfrac{2}{\cancel{xy}}+\dfrac{3}{\cancel{y}}\,\cancel{xy}} \rightarrow \boxed{\dfrac{2y+3}{2+3x}}$ $x \neq 0, -2/3$
$y \neq 0$

② c) $(\sqrt{3x+15})^2=(1+\sqrt{18+x})^2$

$3x+15=1+2\sqrt{18+x}+18+x$

$3x+15=19+x+2\sqrt{18+x}$

$2x-4=2\sqrt{18+x}$

$2(x-2)=2\sqrt{18+x}$

$(x-2)^2=(\sqrt{18+x})^2$

$x^2-4x+4=18+x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$\boxed{x=7}$ | $x=-2$

④ b) $f(x) = \dfrac{x^2-2x-3}{x^2+6x+8} = \dfrac{(x-3)(x+1)}{(x+2)(x+4)}$

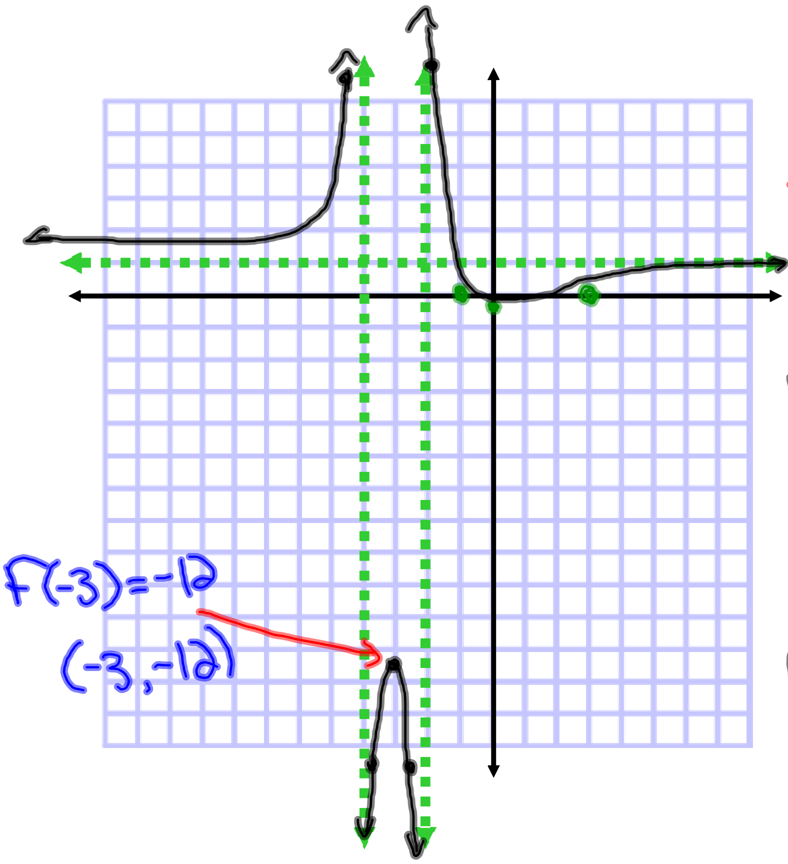
① roots
$x = -1, 3$

② V.A.
$x = -4, -2$

③ H.A.
$y = 1$

④ Holes:
None

⑤ y int
$y = -3/8$



Check Behaviour near V.A.

$x = -4$

$\lim_{x \to -4^-} f(x) = +\infty$

$\lim_{x \to -4^+} f(x) = -\infty$

$x = -2$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = +\infty$

④ c) $f(x) = \frac{x^2-4}{x^2-9} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$

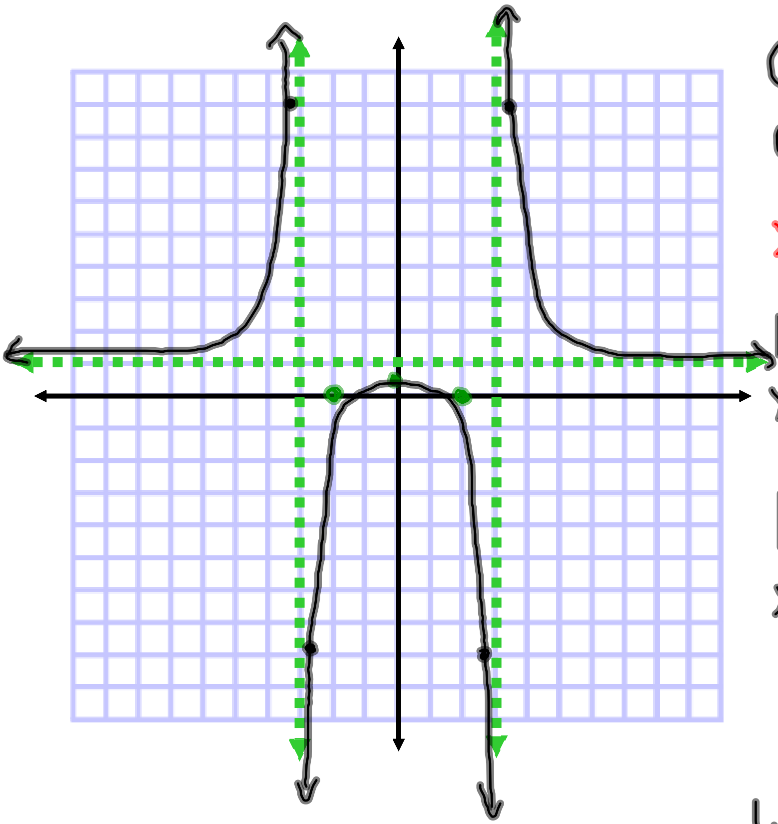
① Roots: $x = \pm 2$

② V.A. $x = \pm 3$

③ H.A. $y = 1$

④ Holes: None

⑤ y int $y = 4/9$



Check the behaviour near the V.A.

$x = -3$

$\lim_{x \to -3^-} f(x) = +\infty$

$\lim_{x \to -3^+} f(x) = -\infty$

$x = 3$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = +\infty$