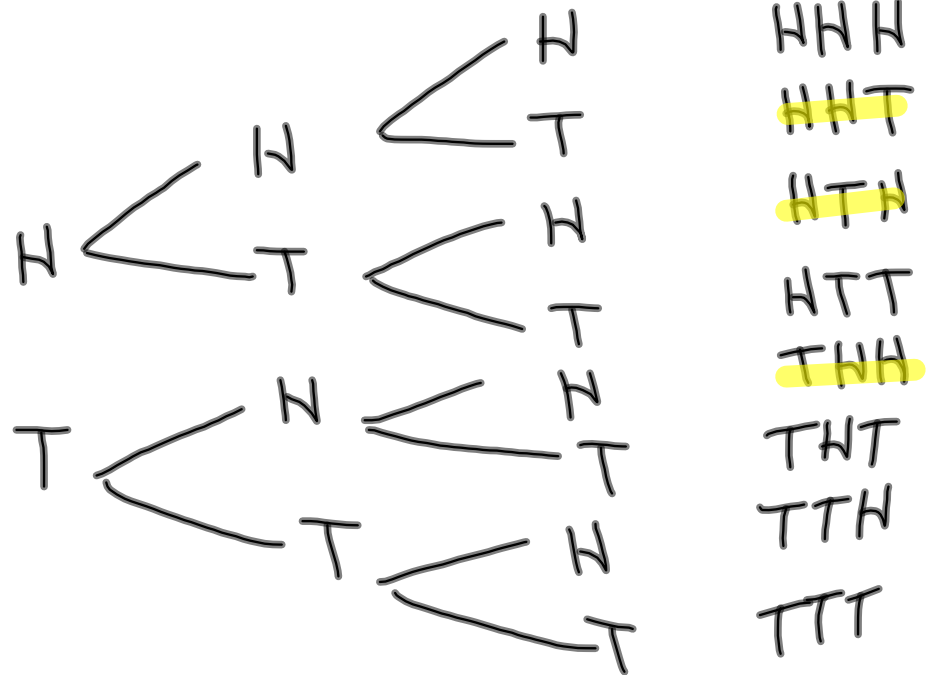


③ 1<sup>st</sup> Toss    2<sup>nd</sup> Toss    3<sup>rd</sup> Toss    Outcomes

a)



b)  $\frac{1}{8}$

c)  $\frac{3}{8}$

d)  $\frac{2}{8} = \frac{1}{4}$

## Solutions $\Rightarrow$ Probability Review

$$\begin{aligned} \text{1. a) } P(\text{ace}) &= \frac{4}{52} \\ &= \frac{1}{13} \end{aligned}$$

$$\begin{aligned} \text{b) } P(\text{not ace}) &= 1 - P(\text{ace}) \\ &= 1 - \frac{4}{52} \\ &= \frac{52}{52} - \frac{4}{52} \\ &= \frac{48}{52} \\ &= \frac{12}{13} \end{aligned}$$

$$\begin{aligned} \text{c) } P(\text{diamond}) &= \frac{13}{52} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{d) } P(\text{red card}) &= \frac{26}{52} \\ &= \frac{1}{2} \end{aligned}$$

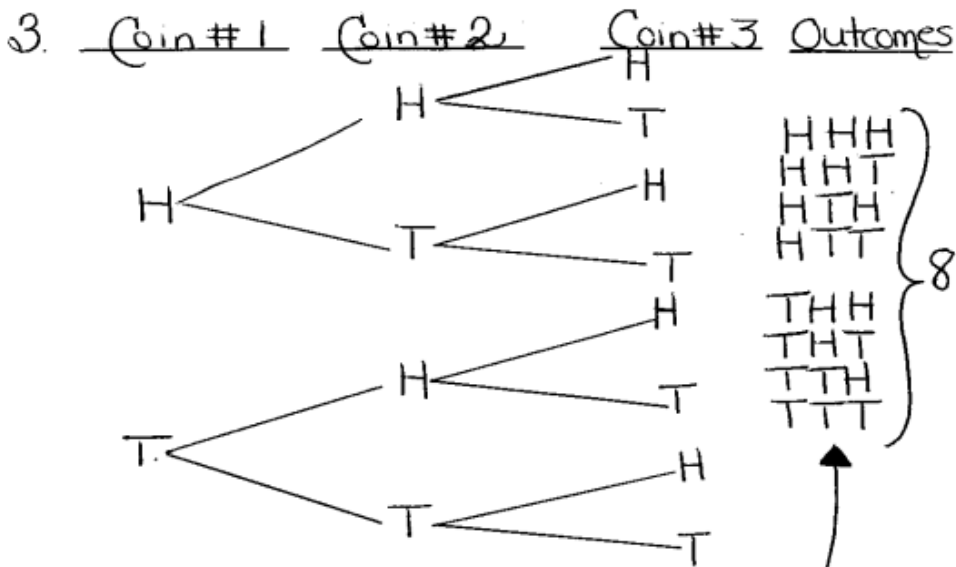
$$\text{e) } P(4 \text{ of spades}) = \frac{1}{52}$$

$$\begin{aligned} 2a) P(\text{blue}) &= \frac{3}{18} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} b) P(\text{blue or green}) &= P(\text{blue}) + P(\text{green}) \\ &= \frac{3}{18} + \frac{6}{18} \\ &= \frac{9}{18} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} c) P(\text{green or red}) &= P(\text{green}) + P(\text{red}) \\ &= \frac{6}{18} + \frac{4}{18} \\ &= \frac{10}{18} \\ &= \frac{5}{9} \end{aligned}$$

$$\begin{aligned} d) P(\text{blue or black}) &= P(\text{blue}) + P(\text{black}) \\ &= \frac{3}{18} + \frac{5}{18} \\ &= \frac{8}{18} \\ &= \frac{4}{9} \end{aligned}$$



a) The possible outcomes are

b)  $P(2 \text{ heads followed by 1 tail}) = \frac{1}{8}$

c)  $P(2 \text{ heads and a tail}) = \frac{3}{8}$

d)  $P(3 \text{ coins the same}) = \frac{2}{8}$   
 $= \frac{1}{4}$

- 4a) Throwing a 4 with the first die and a 6 with the second  $\Rightarrow$  independent
- b) Picking a heart or a diamond from a deck of cards  $\Rightarrow$  mutually exclusive.
- c) Drawing a spade and a heart from the same deck without replacing the card first drawn.  $\Rightarrow$  dependent
- d) Picking the winner of the Grey Cup and Stanley Cup  $\Rightarrow$  independent
- e) Rolling a 3 or an odd number on a die  
 $\hookrightarrow$  mutually inclusive.
- f) Choosing a black marble and a red marble if the marbles are put back in the bag each time  $\Rightarrow$  independent.
- g) Choosing a diamond or a red card  
 $\hookrightarrow$  mutually inclusive.

5.

$$\begin{aligned}
 \text{a) } P(\text{red and rolling 6}) &= P(\text{red}) \times P(\text{rolling 6}) \\
 &= \frac{26}{52} \times \frac{1}{6} \\
 &= \frac{1}{2} \times \frac{1}{6} \\
 &= \frac{1}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } P(\text{spade and rolling 4}) &= P(\text{spade}) \times P(\text{rolling 4}) \\
 &= \frac{13}{52} \times \frac{1}{6} \\
 &= \frac{1}{4} \times \frac{1}{6} \\
 &= \frac{1}{24}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } P(\text{face card and rolling less than 3}) &= P(\text{f.c.}) \times P(<3) \\
 &= \frac{12}{52} \times \frac{2}{6} \\
 &= \frac{3}{13} \times \frac{1}{3} \\
 &= \frac{3}{39} \\
 &= \frac{1}{13}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } P(\text{not black and not rolling a 4}) &= P(\text{not black}) \times P(\text{not rolling 4}) \\
 &= \frac{26}{52} \times \frac{5}{6} \\
 &= \frac{1}{2} \times \frac{5}{6} \\
 &= \frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 6) \quad a) & P(\text{green and green}) \\
 &= P(\text{green}) \times P(\text{green} | \text{green}) \\
 &= \frac{7}{15} \times \frac{6}{14} \\
 &= \frac{7}{15} \times \frac{3}{7} \\
 &= \frac{21}{105} \\
 &= \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 b) & P(\text{green and blue and red}) \\
 &= P(\text{green}) \times P(\text{blue} | \text{green}) \times P(\text{red} | \text{green \& blue}) \\
 &= \frac{7}{15} \times \frac{5}{14} \times \frac{3}{13} \\
 &= \frac{105}{2730} \\
 &= \frac{1}{26}
 \end{aligned}$$

$$\begin{aligned}
 c) & P(\text{blue and blue and blue and blue and blue}) \\
 &= P(b) \times P(b|b) \times P(b|2b) \times P(b|3b) \times P(b|4b) \\
 &= \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \times \frac{2}{12} \times \frac{1}{11} \\
 &= \frac{1}{3} \times \frac{2}{7} \times \frac{3}{13} \times \frac{1}{6} \times \frac{1}{11} \\
 &= \frac{6}{18018} \\
 &= \frac{1}{3003}
 \end{aligned}$$

7.

$$\begin{aligned} \text{a) } P(\text{odd or } 3) &= P(\text{odd}) + P(3) - P(\text{odd and } 3) \\ &= \frac{3}{6} + \frac{1}{6} - \frac{1}{6} \\ &= \frac{4}{6} - \frac{1}{6} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{b) } P(\text{even or number greater than } 4) &= P(\text{even}) + P(\# > 4) - P(\text{even and } \# > 4) \\ &= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} \\ &= \frac{5}{6} - \frac{1}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{c) } P(6 \text{ or } 4) &= P(6) + P(4) \\ &= \frac{1}{6} + \frac{1}{6} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$



$$\begin{aligned} \text{d) } P(\text{greater than 3 or less than 4}) &= P(\text{greater than 3}) + P(\text{less than 4}) \\ &= \frac{3}{6} + \frac{3}{6} \\ &= \frac{6}{6} \\ &= 1 \leftarrow \text{CERTAIN!} \end{aligned}$$

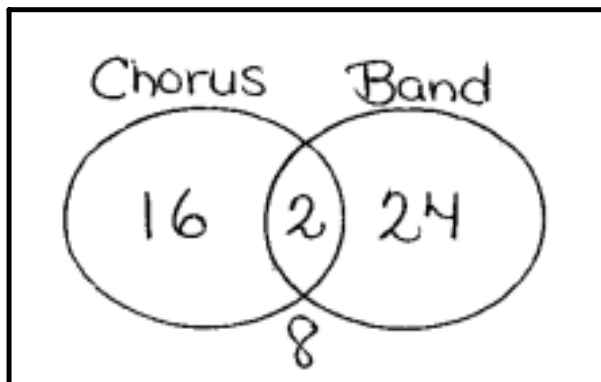
$$\begin{aligned} \text{e) } P(\text{even or odd}) &= P(\text{even}) + P(\text{odd}) \\ &= \frac{3}{6} + \frac{3}{6} \\ &= \frac{6}{6} \\ &= 1 \leftarrow \text{CERTAIN!} \end{aligned}$$

8. 50 total students
- 18 take Chorus
  - 26 take Band
  - 2 take both Chorus and Band.

Conclusion:  $18 - 2 = 16$  take only Chorus  
 $26 - 2 = 24$  take only Band.

$50 - 16 - 24 - 2 = 8$  do not  
take either Chorus or Band.

a) Venn Diagram



b) 8 students are not enrolled in either Chorus or Band.

$$\begin{aligned} \text{c) i) } P(\text{band}) &= \frac{26}{50} \\ &= \frac{13}{25} \end{aligned}$$

$$\begin{aligned} \text{ii) } P(\text{not band}) &= 1 - P(\text{band}) \\ &= 1 - \frac{26}{50} \\ &= \frac{50}{50} - \frac{26}{50} \\ &= \frac{24}{50} \\ &= \frac{12}{25} \end{aligned}$$

$$\begin{aligned} \text{iii) } P(\text{band or chorus}) &= \frac{18}{50} + \frac{26}{50} - \frac{2}{50} \\ &= \frac{44}{50} - \frac{2}{50} \\ &= \frac{42}{50} \text{ or } \frac{21}{25} \end{aligned}$$

$$\begin{aligned} \text{iv) } P(\text{band and chorus}) &= \frac{2}{50} \\ &= \frac{1}{25} \end{aligned}$$

$$\begin{aligned} 9. a) & P(\text{goalie and goalie}) \\ &= P(\text{goalie}) \times P(\text{goalie} | \text{goalie}) \\ &= \frac{6}{30} \times \frac{5}{29} \\ &= \frac{1}{5} \times \frac{5}{29} \\ &= \frac{5}{145} \\ &= \frac{1}{29} \end{aligned}$$

$$\begin{aligned} b) & P(\text{forward and forward}) \\ &= P(\text{forward}) \times P(\text{forward} | \text{forward}) \\ &= \frac{10}{30} \times \frac{9}{29} \\ &= \frac{1}{3} \times \frac{9}{29} \\ &= \frac{9}{87} \\ &= \frac{3}{29} \end{aligned}$$

$$\begin{aligned}
 \text{c) } P(\text{forward and defenseman}) &= P(\text{forward}) \times P(\text{defenseman} | \text{forward}) \\
 &= \frac{10}{30} \times \frac{14}{29} \\
 &= \frac{1}{3} \times \frac{14}{29} \\
 &= \frac{14}{87}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } P(\text{goalie and forward}) &= P(\text{goalie}) \times P(\text{forward} | \text{goalie}) \\
 &= \frac{6}{30} \times \frac{10}{29} \\
 &= \frac{1}{5} \times \frac{10}{29} \\
 &= \frac{10}{145} \\
 &= \frac{2}{29}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } P(\text{defensemen and goalie}) &= P(\text{defensemen}) \times P(\text{goalie} | \text{defenseman}) \\
 &= \frac{14}{30} \times \frac{6}{29} \\
 &= \frac{7}{15} \times \frac{6}{29} \\
 &= \frac{42}{435} \\
 &= \frac{14}{145}
 \end{aligned}$$

$$\begin{aligned} 10. \text{ a) } & P(\text{black or even}) \\ &= P(\text{black}) + P(\text{even}) - P(\text{black and even}) \\ &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\ &= \frac{15}{18} - \frac{3}{18} \\ &= \frac{12}{18} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & P(\text{green or 4}) \\ &= P(\text{green}) + P(4) - P(\text{green and 4}) \\ &= \frac{6}{18} + \frac{3}{18} - \frac{1}{18} \\ &= \frac{9}{18} - \frac{1}{18} \\ &= \frac{8}{18} \\ &= \frac{4}{9} \end{aligned}$$

$$\begin{aligned} \text{c) } & P(\text{black or blue}) \\ &= P(\text{black}) + P(\text{blue}) \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned}
 \text{d) } P(\text{blue or black or green}) &= P(\text{blue}) + P(\text{black}) + P(\text{green}) \\
 &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\
 &= \frac{3}{3} \\
 &= 1 \leftarrow \text{CERTAIN!}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } P(\text{black or greater than 3}) &= P(\text{black}) + P(>3) - P(\text{black and } >3) \\
 &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\
 &= \frac{15}{18} - \frac{3}{18} \\
 &= \frac{12}{18} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned} \text{f) } P(\text{green or black}) &= P(\text{green}) + P(\text{black}) \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{g) } P(\text{blue or odd}) &= P(\text{blue}) + P(\text{odd}) - P(\text{blue and odd}) \\ &= \frac{6}{18} + \frac{9}{18} - \frac{3}{18} \\ &= \frac{15}{18} - \frac{3}{18} \\ &= \frac{12}{18} \\ &= \frac{2}{3} \end{aligned}$$