The Fundamental Counting Principle

The principle which states that all possible outcomes in a *sample space* can be found by multiplying the number of ways each event can occur.

Example 1: A deli has a lunch special which consists of a sandwich, soup, dessert and drink for $4.99.

They offer the following choices:

4 **Sandwich**: chicken salad, ham, and tuna, and roast beef
3 **Soup**: tomato, chicken noodle, vegetable
2 **Dessert**: cookie and pie
5 **Drink**: tea, coffee, coke, diet coke and sprite

How many lunch specials are there?

Let's use the basic counting principle:

There are 4 stages or events: choosing a sandwich, choosing a soup, choosing a dessert and choosing a drink.

There are **4 choices for the sandwich, 3 choices for the soup, 2 choices for the dessert and 5 choices for the drink**.

Putting that all together we get:

Sand.		Soup		Dessert		Drink		# of lunch specials
4	**x**	**3**	**x**	**2**	**x**	**5**	**=**	**120**

So there are 120 lunch specials possible

Example 2: You are taking a test that has five True/False questions.
If you answer each question with True or False and leave none of them blank, in how many ways can you answer the whole test?

Let's use the basic counting principle:
There are 5 events: question 1, question 2, question 3, question 4, and question 5.
There are **2 choices for each question.**

Putting that all together we get:

quest. 1		quest. 2		quest. 3		quest. 4		quest. 5		# of ways to answer test
2	**x**	**2**	**x**	**2**	**x**	**2**	**x**	**2**	**=**	**32**

So there are 32 different ways to answer the whole test

Example 3: A company places a 6-symbol code on each unit of product.
The code consists of 4 digits, the first of which is the number 5, followed by 2 letters, the first of which is NOT a vowel.
How many different codes are possible?

Let's use the basic counting principle:

There are 6 stages or events: digit 1, digit 2, digit 3, digit 4, letter 1, and letter 2.

In general there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The first digit is limited to being the number 5, so there is only one possibility for that one. There are no restriction on digits 2 - 4, so each one of those has 10 possibilities.
In general, there are 26 letters in the alphabet. The first letter, cannot be a vowel (a, e, i, o, u), so that means there are 21 possible letters that could go there. The second letter has no restriction, so there are 26 possibilities for that one.

Putting that all together we get:

digit 1		digit 2		digit 3		digit 4		letter 1		letter 2		# of codes
1	x	10	x	10	x	10	x	21	x	26	=	546000

So there are 546000 different 6-symbol codes possible

Factorial Notation

The product of consecutive natural numbers, in decreasing order to the number 1, can be represented using ***factorial notation.***

The symbol for factorial is the exclamation mark, !.

Example 1

Calculate the following:

a) 5!

b) 3!

c) 10!

Solution

a) 5! is read as "five factorial" b) 3! is read as "three factorial"

5! = 5 x 4 x 3 x 2 x 1
   = 120

3! = 3 x 2 x 1
   = 6

c) 10! is read as "ten factorial"

10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
    = 3 628 800

Fortunately, all scientific calculators have a factoral key so we don't have to manually do the calculations.

Example 2

Simplify the following:

a) $\frac{14!}{8!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$

NOTE: The factors 8, 7, 6, ... , 1 in the denominator of the fraction can cancel with the like factors in the numerator of the fraction. So all we are left with is:

$14 \times 13 \times 12 \times 11 \times 10 \times 9 = 2\ 162\ 160$

b) $\frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5$

$= 15\ 120$

Example 3

Write 12 x 11 x 10 x 9 as a ratio/quotient of factorials.

Solution

The expression contains the first four terms of 12! We can write an expression that would divide the remaining terms.

$$\frac{12 \times 11 \times 10 \times 9 \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$$

$$= \frac{12!}{8!}$$

Example 4

Write 10 x 9 x 8 x 3 x 2 x 1 as a ratio of factorials.

Solution

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3 \times 2 \times 1$$

$= \frac{10!}{7!} \times 3!$ **or** $\mathbf{\frac{10!\,3!}{7!}}$

NOTE: By definition $0! = 1$