Permutations: (Order Matters)

Key words:

- arranged
- line
- President, VP, Secretary
- Gold, Silver, Bronze

Writing an Expression for Permutations

If we were asked to write **7 x 6 x 5 x 4** as a ratio of factorials, we would write the following:

$$\mathbf{7 \times 6 \times 5 \times 4} = \frac{7!}{3!}$$

Now consider the following. Twenty-five students are randomly selected from a class of 75 to go to a concert. The students are assigned to specific seats at the concert, therefore the order they are chosen is important. Obviously this results in a large number of possible outcomes.

There are **75** choices for the first student, **74** for the second, **73** for the third, etc. So the total number of possible outcomes is given by:

75 x 74 x 73 x 72 x … x 53 x 52 x 51 (We stop here since there are only 25 students chosen)

This would take a while to calculate even using a calculator and the chances of hitting an incorrect key on the calculator are high. But it we write:

75 x 74 x 73 x 72 x … x 53 x 52 x 51 = $\frac{75!}{50!}$, we can use the factorial key on your calculator to evaluate this and find the result to be 8.2×10^{44}.

If we look at the fraction that we used above, we can see that the numerator (**75**) is the total number of students in the class and the denominator (**50**) is the total number of students minus the number of students who are "**picked**" to go. This is called a **<u>Permutation</u>**!!!

In general, a **permutation** is an ***arrangement*** of objects in different orders, where the **order** of the arrangement **is important**!!!

If "**n**" is the size of the sample space, and "**r**" is the number of items chosen on each trial, then the total number of **permutations** is written as:

$${}_{n}P_{r} \text{ and is calculated as } {}_{n}P_{r} = \frac{n!}{(n-r)!}$$

$n = 75$

$r = 25$

$${}_{n}P_{r} = \frac{75!}{(75-25)!} = \frac{75!}{50!}$$

$$= 8.2 \times 10^{44}$$

Permutation

Example 1

Five different books are on a shelf. In how many ways could you arrange them?

Solution

The term "**arrange**" tells us that this is a **permutation**, where n = 5 and r = 5.

Therefore: $_nP_r = \frac{n!}{(n-r)!}$

$$
\begin{aligned}
{}_5P_5 &= \frac{5!}{(5-5)!} \\
&= \frac{5!}{0!} \\
&= \frac{120}{1} \quad \text{(Recall that } 0! = 1) \\
&= \mathbf{120}
\end{aligned}
$$

In general if n = r, then $_nP_r = n!$

Example 2

$r = 3$ $n = 5$

How many different 3-digit numerals can be formed from the digits $\underbrace{4, 5, 6, 7, 8}_{n=5}$ if a digit can appear just once in a numeral?

Solution

Since the order of the digits shows place value in a numeral, this is a **permutation** with $n = 5$ and $r = 3$.

Therefore: $_nP_r = \dfrac{n!}{(n-r)!}$

$$_5P_3 = \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{120}{2}$$

$$= \mathbf{60}$$

$\rightarrow \boxed{5}\boxed{_nP_r}\boxed{3} \rightarrow$ Enter

$$\boxed{_5P_3 = 60}$$

Homework:

① $n=3$
$r=3$

${}_3P_3$

② $n=5$
$r=5$

${}_5P_5$

④ $n=17$
$r=2$

${}_{17}P_2$