

## Permutations: (Order Matters)

Key words:

- arranged
- line
- President, VP, Secretary
- Gold, Silver, Bronze

## Writing an Expression for Permutations

If we were asked to write  $7 \times 6 \times 5 \times 4$  as a ratio of factorials, we would write the following:

$$7 \times 6 \times 5 \times 4 = \frac{7!}{3!}$$

Now consider the following. Twenty-five students are randomly selected from a class of 75 to go to a concert. The students are assigned to specific seats at the concert, therefore the order they are chosen is important. Obviously this results in a large number of possible outcomes.

There are 75 choices for the first student, 74 for the second, 73 for the third, etc. So the total number of possible outcomes is given by:

$$75 \times 74 \times 73 \times 72 \times \dots \times 53 \times 52 \times 51 \text{ (We stop here since there are only 25 students chosen)}$$

This would take a while to calculate even using a calculator and the chances of hitting an incorrect key on the calculator are high. But if we write:

$$75 \times 74 \times 73 \times 72 \times \dots \times 53 \times 52 \times 51 = \frac{75!}{50!},$$

we can use the factorial key on your calculator to evaluate this and find the result to be  $8.2 \times 10^{44}$ .

If we look at the fraction that we used above, we can see that the numerator (75) is the total number of students in the class and the denominator (50) is the total number of students minus the number of students who are "picked" to go. This is called a Permutation!!!

In general, a **permutation** is an *arrangement* of objects in different orders, where the **order** of the arrangement is **important!!!**

If "**n**" is the size of the sample space, and "**r**" is the number of items chosen on each trial, then the total number of **permutations** is written as:

$${}_n P_r \text{ and is calculated as } {}_n P_r = \frac{n!}{(n-r)!}$$

$$\begin{aligned} n &= 75 \\ r &= 25 \end{aligned}$$

$${}_n P_r = \frac{75!}{(75-25)!} = \frac{75!}{50!}$$

$$= 8.2 \times 10^{44}$$

↙ Permutation

**Example 1**

Five different books are on a shelf. In how many ways could you **arrange** them?

**Solution**

The term “**arrange**” tells us that this is a **permutation**, where  $n = 5$  and  $r = 5$ .

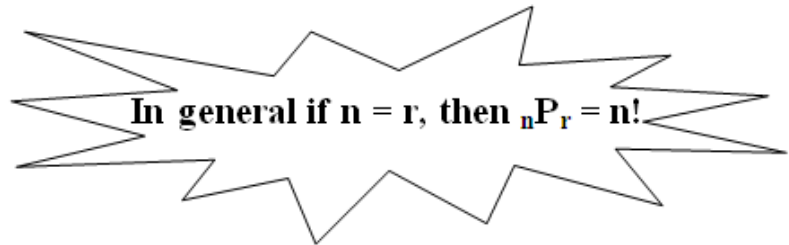
Therefore:  ${}_n P_r = \frac{n!}{(n-r)!}$

$${}_5 P_5 = \frac{5!}{(5-5)!}$$

$$= \frac{5!}{0!}$$

$$= \frac{120}{1} \text{ (Recall that } 0! = 1\text{)}$$

$$= 120$$



**Example 2**

How many different 3-digit numerals can be formed from the digits 4, 5, 6, 7, 8 if a digit can appear just once in a numeral?

$r=3$

$n=5$

**Solution**

Since the order of the digits shows place value in a numeral, this is a **permutation** with  $n=5$  and  $r=3$ .

Therefore:  ${}_n P_r = \frac{n!}{(n-r)!}$

$${}_5 P_3 = \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{120}{2}$$

$$= 60$$



A diagram showing the final result of the calculation. It consists of a rectangular box containing the expression  ${}_5 P_3 = 60$ .

Homework:

$$\textcircled{1} \quad n=3 \\ r=3$$

$${}_3P_3$$

$$\textcircled{2} \quad n=5 \\ r=5$$

$${}_5P_5$$

$$\textcircled{4} \quad n=17 \\ r=2$$

$${}_{17}P_2$$