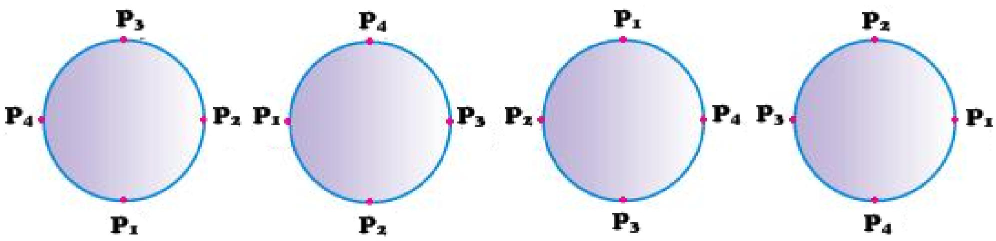
Permutations - Special Cases

Circular Arrangements

When objects are arranged along a line with first and last place, they form a linear permutation. So far we have dealt only with linear permutations. When objects are arranged along a closed curve or a circle, in which any place may be regarded as the first or last place, they form a **circular permutation**.

The permutation in a row or along a line has a beginning and an end, but there is nothing like beginning or end or first and last in a circular permutation. In circular permutations, we consider one of the objects as fixed and the remaining objects are arranged as in linear permutation.

For example, the following arrangements of 4 people, P_1, P_2, P_3, P_4, in a circle would be considered as the same arrangement.



Here we see that when n = 4, there will be 4 repetitions.

In general, to calculate the number of ways that "n" items can be arranged in a circular fashion, the following formula is used:

$$\frac{{}_nP_n}{n} \quad \textbf{OR} \quad (n-1)!$$

Example

The 12 members of the student council are to be seated at a round table. In how many ways can they be arranged?

Solution

Since n = 12,

$\frac{{}_nP_n}{n}$ **OR** $(n-1)!$

$= \frac{{}_{12}P_{12}}{12}$ $= (12-1)!$

$= \frac{479\,001\,600}{12}$ $= 11!$

$= 39\,916\,800$ $= 39\,916\,800$

There are 39 916 800 ways that the members of the student council can sit around the round table.

Repetitions

To calculate the number of ways that "**n**" items can be arranged if one or more of the items repeat, the following formula is used:

${}_nP_{r\,(\text{repeats})} = \dfrac{n!}{p!q!s!}$, where **n** = # if items and **p**, **q**, **s** = the number of times each group of items repeat.

Example 1

Find the total number of arrangements of the word "JILL".

Solution

There is a total of 4 letters, therefore **n** = 4.
Since "**L**" repeats twice, **p** = 2. Each of the remaining letters do not repeat, therefore **q** = 0 and **s** = 0.

$$
\begin{aligned}
{}_{n}P_{r\text{ (repeats)}} &= \frac{n!}{p!q!s!} \\
&= \frac{4!}{2!0!0!} \\
&= \frac{24}{2(1)(1)} \\
&= \frac{24}{2} \\
&= 12
\end{aligned}
$$

There are 12 different ways to arrange the word "JILL".

Example 2

Find the total number of arrangements of MIRAMICHI.

Solution

There is a total of 9 letters, therefore **n** = 9.
"**M**" repeats twice, **p** = 2
"**I**" repeats three times, **q** = 3

$$
\begin{aligned}
{}_{n}P_{r\ (\text{repeats})} &= \frac{n!}{p!q!s!} \\
&= \frac{9!}{2!3!0!} \\
&= \frac{362\,880}{(2)(6)(1)} \\
&= \frac{362\,880}{12} \\
&= 30\,240
\end{aligned}
$$

There are 30 240 ways to arrange the word "MIRAMICHI".

① a) $n = 6$ $\quad {}_6P_6 = 720$

$r = 6$

b) <u>round table</u>

$n = 6$

$$(n-1)!$$
$$= (6-1)!$$
$$= 5!$$
$$= 120$$

Winning team	Losing Team
$n = 18$ $r = 2$ ${}_{18}P_2 = 306$	$n = 15$ $r = 1$ ${}_{15}P_1 = 15$

306×15

$= 4590$

③ a) SILK

$n=4$

$r=4$

${}_4P_4 = 24$

b) SILL

$n=4$

$p=2$

$q=0$

$s=0$

$$\frac{n!}{p!q!s!} = \frac{4!}{2!0!0!} = \frac{24}{2(1)(1)} = \frac{24}{2} = 12$$

④ MISSISSIPPI

$n = 11$

$p = 4$ (letter I)

$q = 4$ (letter S)

$s = 2$ (letter P)

$$\frac{n!}{p!\,q!\,s!}$$

$$= \frac{11!}{4!\,4!\,2!}$$