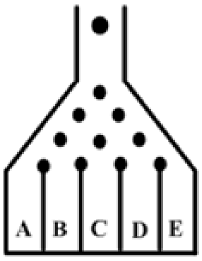
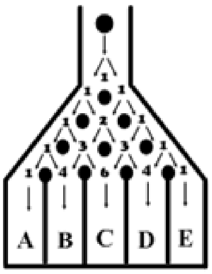
Pascal's Triangle

The popular game show "The Price is Right" has a game called Plinko. In this game, a contestant drops a disk from the top of a board. The board contains a number of posts that the disk hits which deflects its path as it travels to the bottom of the board. A small version of this game is illustrated by the diagrams below:



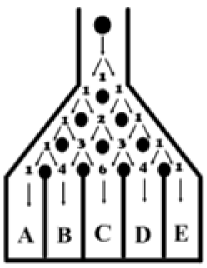


The disks falls and eventually reaches either of the 5 slots, **A**, **B**, **C**, **D**, or **E**. What is the probability that the disk would end up in any particular slot at the bottom? Is it equally likely to end up in any slot, or are some slots more likely than others?

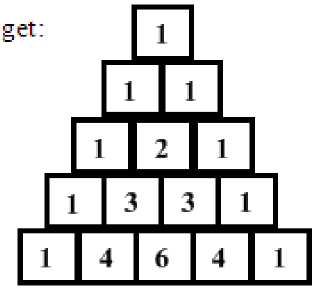
First of all, we assume for this variation of the game that the disk can only move one space to the left or right when it hits a post. In the TV game the disks seem to be able to move any number of spaces when it is deflected.

Suppose we wanted to find out the number of possible paths that a disk could take to reach slot A at the bottom. If you look at the second diagram above, you will see all of the possible paths that the disk could possibly take to reach the bottom, represented by the arrows. For example, as soon as it is dropped, it only has one possible path until it hits the first post. Then it can go either to the left or to the right (2 paths). Whichever way it goes, it will hit another post and either deflect to the left or the right. This continues until the disk reaches the bottom slots.

If you look at the diagram closely, you will see that there is only one possible path for the disk to get to the outside slots **A** and **E**, 4 paths to get to slots **B** and **D**, and 6 paths to get to slot **C**. So, it is more likely that the disk will land in the centre slot **C**.



If we just take the numbers for the possible paths of each row we would get:



This array of numbers is called ***Pascal's Triangle*** and has many applications to mathematics in algebra, probability, and other areas. This pattern was studied extensively by the famous mathematician Blaise Pascal. The triangle has infinitely many rows

Can you figure out how the next row can be generated from the previous row?

It is obvious that the first and last entry of each row is **1**. **Any other number is found by adding the two numbers in the row above the desired number.** For example, the first 4 in the last row is found by adding the 1 and the 3 in the row above it. The 6 is found by adding the 3 and 3. The second 4 is found by adding the 3 and 1. You can see that each row of the triangle is symmetrical about the center.

The single number 1 at the top of the triangle is designated as row 0. The following rows are designated as row 1, 2, 3, etc. **It can also be seen that the second element in any row determines the row number.**

Using the blank template provided to you, fill in the top portion of each block. This will give you the first 10 rows of Pascal's Triangle!

Pascal's Triangle and Combinations

The previous work that we did with **Combinations** can be directly related to the entries of Pascal's Triangle.

Consider the following:

$${}_0C_0=1$$
$${}_1C_0=1 \quad {}_1C_1=1$$
$${}_2C_0=1 \quad {}_2C_1=2 \quad {}_2C_2=1$$
$${}_3C_0=1 \quad {}_3C_1=3 \quad {}_3C_2=3 \quad {}_3C_3=1$$
$${}_4C_0=1 \quad {}_4C_1=4 \quad {}_4C_2=6 \quad {}_4C_3=4 \quad {}_4C_4=1$$

Using this pattern, you can now fill in the bottom portion of each row on your template.

We can now use this pattern to determine that any element in any row of Pascal's Triangle can be found using $\boxed{{}_nC_{i-1}}$, where **n** = row and **i** = element.

Example 1

Find the 4th element in row 12.

i n

Solution

n = 12 and i = 4

$${}_{12}C_{4-1}$$
$${}_{12}C_3$$
$$= \mathbf{220}$$

The 4th element in row 12 is 220.

5^{th} in row 8

by looking at Pascal's triangle we see it equals 70

Using formula:

$n = 8$ $\quad\quad$ ${}_nC_{i-1}$

$i = 5$ $\quad\quad$ $= {}_8C_{5-1}$

$= {}_8C_4$

$= 70$