Binomial Expansion of $(x - y)^n$

To expand $(x - y)^n$, it is helpful to rewrite the expression as $\mathbf{(}x + (-y)\mathbf{)}^n$ and follow the same procedure as you used in the previous lesson—with the addition of BRACKETS!

2, 4, 6, 8

Recall that a **negative** term raised to an **EVEN** exponent will result in a ***positive*** answer.
Recall that a **negative** term raised to an **ODD** exponent will result in a ***negative*** answer.

1, 3, 5, 7, 9

If we use the above rules to expand a binomial in the form $(x - y)^n$, the proper "sign" will result in front of each term.

Example 1

Expand $(x-2)^4$ $=$ $(x+(-2))^4$

Row 4

$a = x$

$b = -2$

Solution

Rewrite the expression as: **$(x + (-2))^4$**, and complete as usual…

$= {}_4C_0(x)^4(-2)^0 + {}_4C_1(x)^3(-2)^1 + {}_4C_2(x)^2(-2)^2 + {}_4C_3(x)^1(-2)^3 + {}_4C_4(x)^0(-2)^4$

$= (1)(x^4)(1) + (4)(x^3)(-2) + (6)(x^2)(4) + (4)(x^1)(-8) + (1)(1)(16)$

$= 1x^4 - 8x^3 + 24x^2 - 32x^1 + 16$

Notice how the signs alternate from positive to negative!!!

ASSIGNMENT

Expand each of the following binomials using the procedure indicated above:

a) $(x-3)^5$
b) $(x-4)^6$
c) $(x-2)^4$
d) $(x-y)^3$

a) $(x-3)^5 \rightarrow (\underline{\underline{x}}+\underline{(-3)})^{5}$

Row 5
$a=x$
$b=-3$

$${}_5C_0(x)^5(-3)^0 + {}_5C_1(x)^4(-3)^1 + {}_5C_2(x)^3(-3)^2 + {}_5C_3(x)^2(-3)^3 + {}_5C_4(x)^1(-3)^4 + {}_5C_5(x)^0(-3)^5$$

$$(1)(x^5)(1)+(5)(x^4)(-3)+(10)(x^3)(9)+(10)(x^2)(-27)+(5)(x)(81)+(1)(1)(-243)$$

$$x^5-15x^4+90x^3-270x^2+405x-243$$

d) $(x-y)^3 \rightarrow (\underline{\underline{x}}+\underline{\underline{(-y)}})^{3}$

Row 3
$a=x$
$b=-y$

$${}_3C_0(x)^3(-y)^0 + {}_3C_1(x)^2(-y)^1 + {}_3C_2(x)^1(-y)^2 + {}_3C_3(x)^0(-y)^3$$

$$(1)(x^3)(1)+(3)(x^2)(-y)+(3)(x^1)(y^2)+(1)(1)(-y^3)$$

$$x^3-3x^2y+3xy^2-y^3$$