Solutions => Probability Review

1.

a) $P(\text{ace}) = \frac{4}{52}$
$= \frac{1}{13}$

b) $P(\text{not ace})$
$= 1 - P(\text{ace})$
$= 1 - \frac{4}{52}$
$= \frac{52}{52} - \frac{4}{52}$
$= \frac{48}{52}$
$= \frac{12}{13}$

c) $P(\text{diamond}) = \frac{13}{52}$
$= \frac{1}{4}$

d) $P(\text{red card}) = \frac{26}{52}$
$= \frac{1}{2}$

e) $P(4 \text{ of spades}) = \frac{1}{52}$

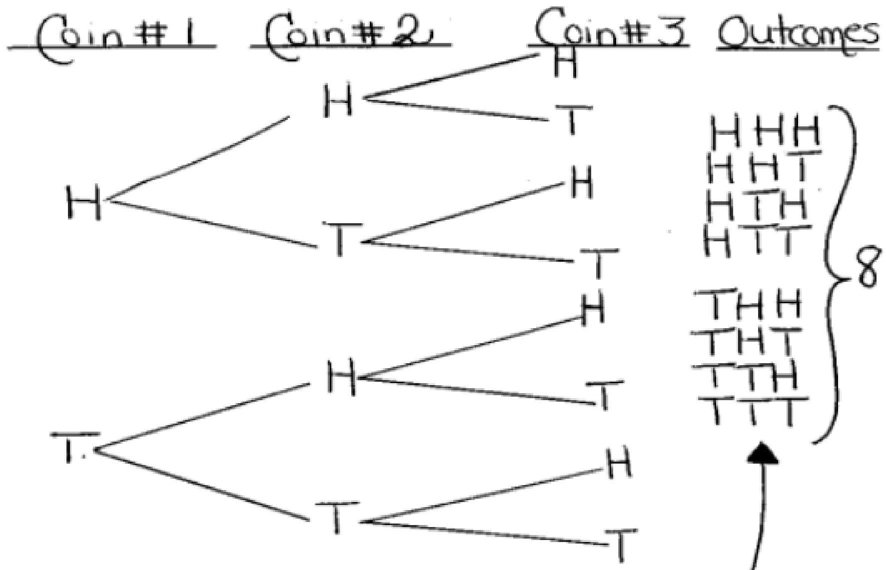
2 a) $P(\text{blue}) = \frac{3}{18}$
$= \frac{1}{6}$

b) $P(\text{blue or green})$
$= P(\text{blue}) + P(\text{green})$
$= \frac{3}{18} + \frac{6}{18}$
$= \frac{9}{18}$
$= \frac{1}{2}$

c) $P(\text{green or red})$
$= P(\text{green}) + P(\text{red})$
$= \frac{6}{18} + \frac{4}{18}$
$= \frac{10}{18}$
$= \frac{5}{9}$

d) $P(\text{blue or black})$
$= P(\text{blue}) + P(\text{black})$
$= \frac{3}{18} + \frac{5}{18}$
$= \frac{8}{18}$
$= \frac{4}{9}$

3.



a) The possible outcomes are

b) P(2 heads followed by 1 tail) = $\frac{1}{8}$

c) P(2 heads and a tail) = $\frac{3}{8}$

d) P(3 coins the same) = $\frac{2}{8}$

$= \frac{1}{4}$

4a) Throwing a 4 with the first die and a 6 with the second ⇒ independent

b) Picking a heart or a diamond from a deck of cards => mutually exclusive.

c) Drawing a spade and a heart from the same deck without replacing the card first drawn. => dependent

d) Picking the winner of the Grey Cup and Stanley Cup => independent

e) Rolling a 3 or an odd number on a die
↳ mutually inclusive.

f) Choosing a black marble and a red marble if the marbles are put back in the bag each time ⇒ independent.

g) Choosing a diamond or a red card
↳ mutually inclusive.

5.

a) P(red and rolling 6)

$= P(\text{red}) \times P(\text{rolling } 6)$

$= \frac{26}{52} \times \frac{1}{6}$

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

b) P(spade and rolling 4)

$= P(\text{spade}) \times P(\text{rolling } 4)$

$= \frac{13}{52} \times \frac{1}{6}$

$= \frac{1}{4} \times \frac{1}{6}$

$= \frac{1}{24}$

c) P(face card and rolling less than 3)

$= P(\text{f.c.}) \times P(<3)$

$= \frac{12}{52} \times \frac{2}{6}$

$= \frac{3}{13} \times \frac{1}{3}$

$= \frac{3}{39}$

$= \frac{1}{13}$

d) P(not black and not rolling a 4)

$= P(\text{not black}) \times P(\text{not rolling } 4)$

$= \frac{26}{52} \times \frac{5}{6}$

$= \frac{1}{2} \times \frac{5}{6}$

$= \frac{5}{12}$

6.

a) P(green and green)

$= P(\text{green}) \times P(\text{green} \mid \text{green})$

$= \frac{7}{15} \times \frac{6}{14}$

$= \frac{7}{15} \times \frac{3}{7}$

$= \frac{21}{105}$

$= \frac{1}{5}$

b) P(green and blue and red)

$= P(\text{green}) \times P(\text{blue} \mid \text{green}) \times P(\text{red} \mid \text{green \& blue})$

$= \frac{7}{15} \times \frac{5}{14} \times \frac{3}{13}$

$= \frac{105}{2730}$

$= \frac{1}{26}$

c) P(blue and blue and blue and blue and blue)

$= P(b) \times P(b \mid b) \times P(b \mid 2b) \times P(b \mid 3b) \times P(b \mid 4b)$

$= \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \times \frac{2}{12} \times \frac{1}{11}$

$= \frac{1}{3} \times \frac{2}{7} \times \frac{3}{13} \times \frac{1}{6} \times \frac{1}{11}$

$= \frac{6}{18018}$

$= \frac{1}{3003}$

7.

a) P(odd or 3)

$= P(\text{odd}) + P(3) - P(\text{odd and } 3)$

$= \frac{3}{6} + \frac{1}{6} - \frac{1}{6}$

$= \frac{4}{6} - \frac{1}{6}$

$= \frac{3}{6}$

$= \frac{1}{2}$

b) P(even or number greater than 4)

$= P(\text{even}) + P(\# > 4) - P(\text{even and } \# > 4)$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$

$= \frac{5}{6} - \frac{1}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

c) P(6 or 4)

$= P(6) + P(4)$

$= \frac{1}{6} + \frac{1}{6}$

$= \frac{2}{6}$

$= \frac{1}{3}$

d) P(greater than 3 or less than 4)

$= P(\text{greater than } 3) + P(\text{less than } 4)$

$= \frac{3}{6} + \frac{3}{6}$

$= \frac{6}{6}$

$= 1$ ← CERTAIN!

e) P(even or odd)

$= P(\text{even}) + P(\text{odd})$

$= \frac{3}{6} + \frac{3}{6}$

$= \frac{6}{6}$

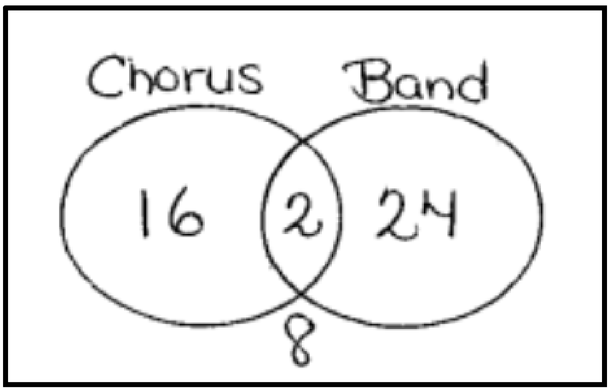
$= 1$ ← CERTAIN!

8. 50 total students
- 18 take Chorus
- 26 take Band
- 2 take both Chorus and Band

Conclusion: 18 − 2 = 16 take only Chorus
26 − 2 = 24 take only Band.

50 − 16 − 24 − 2 = 8 do not take either Chorus or Band.

a) Venn Diagram



b) 8 students are notenrolled in either Chorus or Band.

c) i) $P(\text{band}) = \frac{26}{50}$
$= \frac{13}{25}$

ii) $P(\text{not band})$
$= 1 - P(\text{band})$
$= 1 - \frac{26}{50}$
$= \frac{50}{50} - \frac{26}{50}$
$= \frac{24}{50}$
$= \frac{12}{25}$

iii) $P(\text{band or chorus})$
$= \frac{18}{50} + \frac{26}{50} - \frac{2}{50}$
$= \frac{44}{50} - \frac{2}{50}$
$= \frac{42}{50}$ or $\frac{21}{25}$

iv) $P(\text{band and chorus})$
$= \frac{2}{50}$
$= \frac{1}{25}$

9\. a) P(goalie and goalie)

$$= P(\text{goalie}) \times P(\text{goalie} \mid \text{goalie})$$

$$= \frac{6}{30} \times \frac{5}{29}$$

$$= \frac{1}{5} \times \frac{5}{29}$$

$$= \frac{5}{145}$$

$$= \frac{1}{29}$$

b) P(forward and forward)

$$= P(\text{forward}) \times P(\text{forward} \mid \text{forward})$$

$$= \frac{10}{30} \times \frac{9}{29}$$

$$= \frac{1}{3} \times \frac{9}{29}$$

$$= \frac{9}{87}$$

$$= \frac{3}{29}.$$

c) P(forward and defenseman)

$= P(\text{forward}) \times P(\text{defenseman} \mid \text{forward})$

$= \frac{10}{30} \times \frac{14}{29}$

$= \frac{1}{3} \times \frac{14}{29}$

$= \frac{14}{87}$

d) P(goalie and forward)

$= P(\text{goalie}) \times P(\text{forward} \mid \text{goalie})$

$= \frac{6}{30} \times \frac{10}{29}$

$= \frac{1}{5} \times \frac{10}{29}$

$= \frac{10}{145}$

$= \frac{2}{29}$

e) P(defensemen and goalie)

$= P(\text{defensemen}) \times P(\text{goalie} \mid \text{defenseman})$

$= \frac{14}{30} \times \frac{6}{29}$

$= \frac{7}{15} \times \frac{6}{29}$

$= \frac{42}{435}$

$= \frac{14}{145}$

10.

a) P(black or even)

$= P(\text{black}) + P(\text{even}) - P(\text{black and even})$

$= \frac{6}{18} + \frac{9}{18} - \frac{3}{18}$

$= \frac{15}{18} - \frac{3}{18}$

$= \frac{12}{18}$

$= \frac{2}{3}$

b) P(green or 4)

$= P(\text{green}) + P(4) - P(\text{green and } 4)$

$= \frac{6}{18} + \frac{3}{18} - \frac{1}{18}$

$= \frac{9}{18} - \frac{1}{18}$

$= \frac{8}{18}$

$= \frac{4}{9}$

c) P(black or blue)

$= P(\text{black}) + P(\text{blue})$

$= \frac{1}{3} + \frac{1}{3}$

$= \frac{2}{3}.$

d) P(blue or black or green)

$= P(\text{blue}) + P(\text{black}) + P(\text{green})$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{3}{3}$

$= 1$ ← CERTAIN!

e) P(black or greater than 3)

$= P(\text{black}) + P(>3) - P(\text{black and} >3)$

$= \frac{6}{18} + \frac{9}{18} - \frac{3}{18}$

$= \frac{15}{18} - \frac{3}{18}$

$= \frac{12}{18}$

$= \frac{2}{3}$

f) P(green or black)

$= P(\text{green}) + P(\text{black})$

$= \frac{1}{3} + \frac{1}{3}$

$= \frac{2}{3}$

g) P(blue or odd)

$= P(\text{blue}) + P(\text{odd}) - P(\text{blue and odd})$

$= \frac{6}{18} + \frac{9}{18} - \frac{3}{18}$

$= \frac{15}{18} - \frac{3}{18}$

$= \frac{12}{18}$

$= \frac{2}{3}$