Correct Homework Sheet

② b) $y = \sqrt[3]{\dfrac{1-x^6}{2+(5x-1)^4}} = \left[\dfrac{1-x^6}{2+(5x-1)^4}\right]^{1/3}$

$$y' = \frac{1}{3}\left[\frac{1-x^6}{2+(5x-1)^4}\right]^{-2/3}\left[\frac{(2+(5x-1)^4)(-6x^5)-(1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2}\right]$$

Correct Homework Sheet

③ b) $$f(x) = \frac{8x^3(12x^2-5x)^8}{2-3(1-32x^{10})^{1/5}}$$

$$\frac{\left[2-3(1-32x^{10})^{1/5}\right]\left[(8x^3)(8)(12x^2-5x)^7(24x-5)+(24x^2)(12x^2-5x)^8\right]-\left[8x^3(12x^2-5x)^8\right]\left[\left(-\frac{3}{5}\right)(1-32x^{10})^{-4/5}(-320x^9)\right]}{\left[2-3(1-32x^{10})^{1/5}\right]^2}$$

③ c) $$f(x) = \frac{\left[x^5-x(4-x^2)^{1/2}\right]^6}{12x^{1/2}(5x^3-8)^7}$$

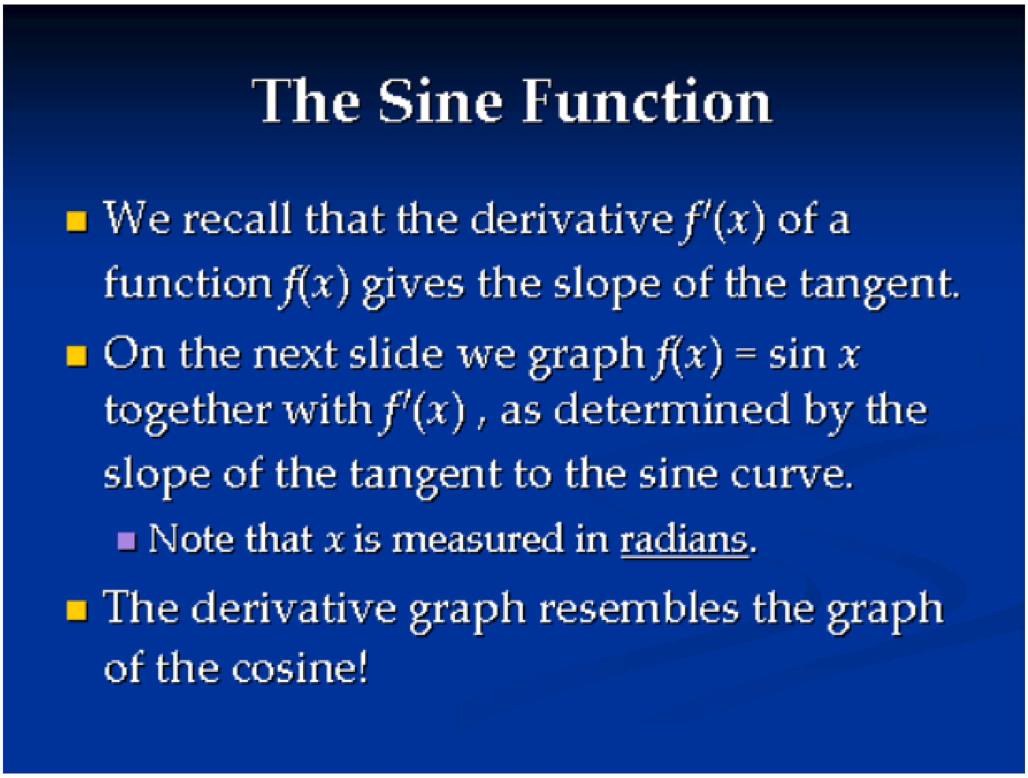
$$f'(x) = \frac{\left[12x^{1/2}(5x^3-8)^7\right]\left[6\left[x^5-x(4-x^2)^{1/2}\right]^5\left[5x^4-\left[x\left(\frac{1}{2}\right)(4-x^2)^{-1/2}(-2x)+(4-x^2)^{1/2}\right]\right]\right]-\left[x^5-x(4-x^2)^{1/2}\right]^6\left[(12x^{1/2})(7)(5x^3-8)^6(15x^2)+(6x^{-1/2})(5x^3-8)^7\right]}{\left[12x^{1/2}(5x^3-8)^7\right]^2}$$

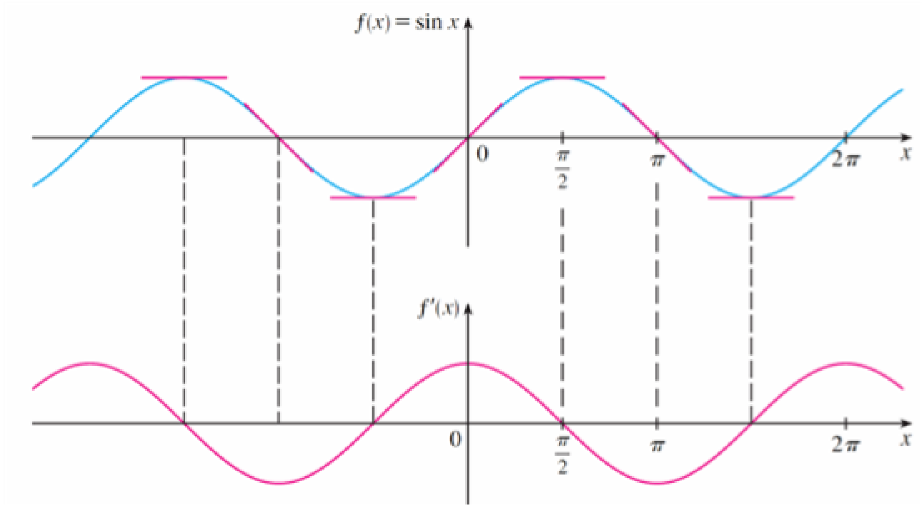
To be handed in today

Differentiate the following (do not simplify)

$$f(x)=\sqrt[7]{\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}}$$

$$=\left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{1/7}$$


The Sine Function
We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
Note that x is measured in radians.
The derivative graph resembles the graph of the cosine!


$f(x) = \sin x$
0
$\frac{\pi}{2}$
π
2π
x
$f'(x)$
0
$\frac{\pi}{2}$
π
2π
x

Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $\quad$ $y = \sin(x + 2)$ $\quad$ $y = \sin(kx + d)$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$ b) $y = \sin^3 x$ c) $y = \sin^3(x^2 - 1)$

Ex #3.

Differentiate:

$$y = x^2 \cos x$$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc