Warmup

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = 60.000$	$t_2 = 57.000$	$t_3 = 54.150$	$t_4 = 51.4425$

a) Does the data follow an arithmetic or geometric pattern?

$r = \frac{t_2}{t_1} = \frac{57}{60} = 0.95$

b) Write down a formula for t_n?

$t_n = ar^{n-1}$

$a = 60$

$r = 0.95$

$t_n = 60(0.95)^{n-1}$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L?

$t_n = 17.5$

$a = 60$

$r = 0.95$

$n = ?$

$t_n = ar^{n-1}$

$\frac{17.5}{60} = \frac{\cancel{60}(0.95)^{n-1}}{\cancel{60}}$

$0.291\overline{6} = (0.95)^{n-1}$

$(\cancel{0.95})^{24} = (\cancel{0.95})^{n-1}$

$24 = n - 1$

$25 = n$

In 25 years the tree will produce 17.5L

Questions from homework

⑨ $1+4+9+16$

$$\sum_{n=1}^{4} n^2$$

$$= (1)^2+(2)^2+(3)^2+(4)^2$$

$$= 1+4+9+16$$

$$= 30$$

⑩ $3+6+12+24+48$

$a=3$

$r=2$

$t_n=(3)(2)^{n-1}$

$$\sum_{n=1}^{5} 3(2)^{n-1}$$

Limit (of a sequence - t_n)

A finite number L that the value of t_n gets closer and closer to, or "approaches," as n becomes very large, or "approaches infinity."

The value of t_n can be made as close as you like to L by using a sufficiently large value for n.

The notation for a limit is $\boxed{\lim_{n\to\infty} \boxed{t_n} = \boxed{L}}$

Converging Sequence $\rightarrow$ Has a Limit

A seqence in which the terms approach a limit

For example, $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \ldots$ converges to 1

0.25, 0.40, 0.50, 0.57, ...

The above sequence was generated using the following general term.	$t_n = \frac{n}{n+3}$

What happens if *"n"* is a very large number?

$$t_{100} = \frac{100}{103} = 0.97$$

$$t_{1000} = \frac{1000}{1003} = 0.997$$

$$\boxed{\lim_{n \to \infty} \boxed{\frac{n}{n+3}} = \boxed{1}}$$

Diverging Sequence → Has no Limit

A seqence in which the terms do not approach a limit

For example, *1, 2, 3, 4,...* diverges. (no limit exists)

The above sequence was generated using the following general term.	$t_n = n$

What happens if *"n"* is a very large number?

$t_{100} = 100$

$t_{9999} = 9999$

$\lim_{n \to \infty} n = \infty$ → Limit does not exist

No Limit

DNE

Decide whether each sequence *converges* or *diverges* then state the limit.

2, 4, 8, 16, 32,... *diverges*

$a = 2$

$r = 2$

$t_n = (2)^1(2)^{n-1}$

$t_n = (2)^n$

$\lim_{n\to\infty} 2^n = \infty \rightarrow$ DNE

3, 1.5, 0.75, 0.375,... *converges*

$a = 3$

$r = \frac{1}{2}$

$t_n = (3)(\frac{1}{2})^{n-1}$

$\lim_{n\to\infty} (3)(\frac{1}{2})^{n-1} = 0$

$t_{10} = 3\left(\frac{1}{2}\right)^9$

$= 0.0058$

Infinite Sequences

Suppose we have a sequence defined by $t_n = \dfrac{n}{2n+1}$, $\boxed{n \in N}$

Generate the first 4 terms of the sequence

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots \quad \frac{100}{201}$$

You may notice that as "n" increases "t_n" approaches ½

Symbolically this is written $\boxed{\displaystyle\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}}$

and is read "The limit as n approaches infinity of n over $(2n + 1)$ is ½."

Algebraically we solve by dividing the numerator and the denominator by the highest power of n. (degree)

$$\frac{1n}{2n+1}$$

$$\lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{2n}{n}+\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{1}{2+\boxed{\frac{1}{n}}} \longrightarrow 0$$

$$\lim_{n\to\infty} \frac{1}{2+0} = \boxed{\frac{1}{2}}$$

Find the limit if it exists

Diverging

$$t_n = \frac{n+5}{1}$$

$$\lim_{n\to\infty} \frac{\frac{n}{n} + \boxed{\frac{5}{n}} \to 0}{\boxed{\frac{1}{n}} \to 0}$$

$$\lim_{n\to\infty} \frac{1+0}{0}$$

$$\lim_{n\to\infty} \frac{1}{0} = \text{DNE}$$

Converging

$$t_n = \frac{3n+1}{4n-2}$$

$$\lim_{n\to\infty} \frac{\frac{3n}{n} + \frac{1}{n}}{\frac{4n}{n} - \frac{2}{n}}$$

$$\lim_{n\to\infty} \frac{3+0}{4-0} = \frac{3}{4}$$

Homework

#1 b)
#2
#3
#4

$$t_n = \frac{1-n^2}{n^3+2}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n^3} - \frac{n^2}{n^3}}{\frac{n^3}{n^3} + \frac{2}{n^3}}$$

$$\lim_{n\to\infty} \frac{\boxed{\frac{1}{n^3}} - \boxed{\frac{1}{n}}}{1 + \boxed{\frac{2}{n^3}}}$$

$$\lim_{n\to\infty} \frac{0-0}{1+0} = 0$$