Matrices

Matrix - a rectangular array of numbers enclosed in parentheses

Example:
$$\begin{pmatrix} 2 & 1 & 13 \\ 6 & -2 & 8 \end{pmatrix} \quad 2 \times 3$$

Each number in a matrix is called an "entry".

A matrix is made up of "rows" and "columns".

The dimensions of a matrix state the size of it.

ROWS X COLUMNS

$$\begin{pmatrix} 2 & 3 & -4 \\ 1 & -2 & \textcircled{5} \\ 0 & 4 & 1 \end{pmatrix}$$

The dimensions of the above matrix are: 3×3

The entries in row 2 are: $1, -2, 5$

The entries in column 3 are: $-4, 5, 1$

The entry in row 2 column 3 is: 5

MATRIX OPERATIONS

Adding & Subtracting Matrices

To add and subtract matrices the _**dimensions**_ of each matrix _**must**_ be the _**same**_.

$$\begin{pmatrix} -3 & 5 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 7 & 0 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -10 & 5 \\ -1 & -8 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 7 \\ -1 & 11 \end{pmatrix}$$

$$X + \begin{pmatrix} -3 & 4 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -8 \\ 3 & -2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -8 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 4 \\ 5 & 7 \end{pmatrix}$$

$$X = \begin{pmatrix} 5 & -12 \\ -2 & -9 \end{pmatrix}$$

Scalar Multiplication

$$3 \begin{pmatrix} 5 & -2 \\ 4 & 1 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 15 & -6 \\ 12 & 3 \\ -21 & 0 \end{pmatrix}$$

Multiply through the matrix!

$$A = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix} \quad C = \begin{pmatrix} -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Find 3A + 2 B

$$= 3 \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 \\ 0 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -1 \\ 6 & 12 \end{bmatrix}$$

Matrix Multiplication

Steps:

1. **State the dimensions of each matrix.**

2. **Determine if it is possible to multiply them.**

3. **Set up the "Template".**

4. **Multiply "Row x Column"**

5. **Simplify**

Matrix Multiplication

In order to multiply matrices, the number of columns in the 1st matrix must equal the number of rows in the 2nd matrix.

Product Dimensions:

$$(\text{\# rows } 1\text{st}) \ \times \ (\text{\# columns } 2\text{nd})$$

Ex.
$$\begin{pmatrix} 2 & 7 \\ 3 & 5 \end{pmatrix} \times \begin{pmatrix} 6 & -2 & 0 & -1 \\ 7 & 1 & 5 & 4 \end{pmatrix} = \begin{bmatrix} - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$(2 \times 2) \qquad (2 \times 4)$$

Always multiply a row through a column, adding the products as you go.

Ex.

$$\begin{pmatrix} 5 & 1 & -1 \\ 6 & 2 & 4 \end{pmatrix} \times \begin{pmatrix} 4 & -1 \\ 2 & -5 \\ -3 & 0 \end{pmatrix} = \begin{bmatrix} 25 & -10 \\ 16 & -16 \end{bmatrix}$$

$$2 \times 3 \qquad\qquad 3 \times 2$$

Matrix Multiplication

2.
$$\begin{pmatrix} 2 & 1 \\ 3 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 1 & 4 \end{pmatrix} = \begin{bmatrix} \underline{1} & \underline{8} \\ \underline{0} & \underline{6} \\ \underline{1} & \underline{8} \end{bmatrix}$$

$$3 \times \underline{2} \qquad \underline{2} \times 2$$

3.
$$(2 \quad 3 \quad 5 \quad 7)\begin{pmatrix} 5 \\ 0 \\ 2 \\ 0 \end{pmatrix} = \begin{bmatrix} \underline{20} \end{bmatrix}$$

$$\underline{1} \times 4 \qquad 4 \times \underline{1}$$

What would the entry for r3c4 be in the product?

4.
$$\begin{pmatrix} 2 & 3 & 0 & 6 \\ 0 & 2 & 8 & 2 \\ 1 & 0 & 3 & 1 \\ 8 & 4 & 4 & 8 \\ 4 & 7 & 1 & 9 \end{pmatrix} \begin{pmatrix} 6 & 2 & 0 & 2 & 0 \\ 2 & 8 & 8 & 1 & 8 \\ 1 & 3 & 9 & 5 & 2 \\ 5 & 0 & 4 & 3 & 6 \end{pmatrix}$$

5 x 4   4 x 5

$$= \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & 20 & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

r3c4 = 20

Homework

Finish Sheet & Study for Quiz