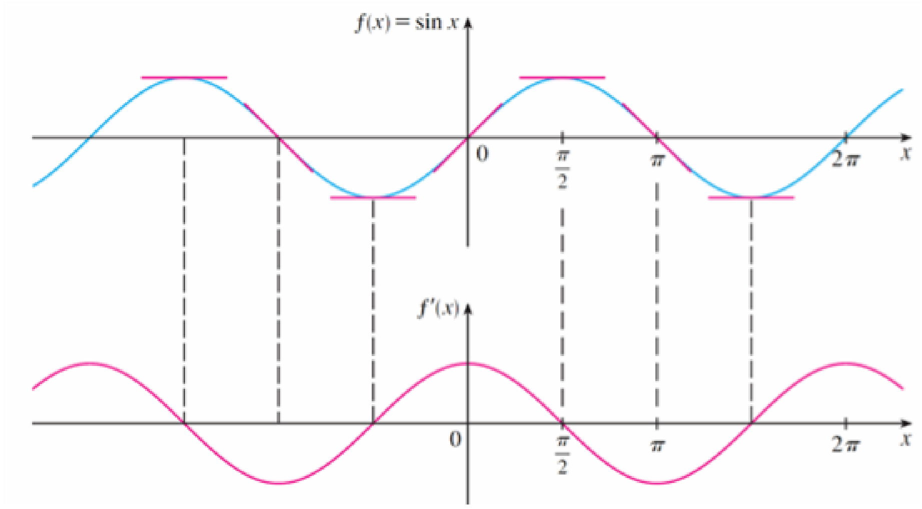
Correct Homework Sheet

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

* $\lim_{h \to 0} \frac{\sin h}{h} = 1$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$u = 3x$

$y = \sin 3x$

$y' = \cos 3x \cdot 3$

$y' = 3\cos 3x$

$u = x + 2$

$y = \sin(x + 2)$

$y' = \cos(x+2) \cdot (1)$

$y' = \cos(x+2)$

$u = kx + d$

$y = \sin(kx + d)$

$y' = \cos(kx+d) \cdot k$

$y' = k\cos(kx+d)$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$

$y' = \cos(x^3) \cdot 3x^2$

$y' = 3x^2 \cos(x^3)$

b) $y = \sin^3 x = (\sin x)^3$

$y' = 3(\sin x)^2 \cdot \cos x \cdot (1)$

$y' = 3\sin^2 x \cos x$

c) $y = \sin^3(x^2 - 1) = [\sin(x^2-1)]^3$

$y' = 3[\sin(x^2-1)]^2 \cdot \cos(x^2-1) \cdot (2x)$

$y' = 6x \sin^2(x^2-1) \cos(x^2-1)$

Ex #3.

Differentiate:

$$y = x^2 \cos x$$

$$y' = x^2(-\sin x)\cdot(1) + 2x(\cos x)$$

$$y' = -x^2 \sin x + 2x \cos x$$

$$y' = x(-x\sin x + 2\cos x)$$

$$y' = x(2\cos x - x\sin x)$$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc