Warm Up

Given the following matrices...

$$X = \begin{pmatrix} -1 & -2 \\ 3 & 1 \\ 2 & 5 \end{pmatrix} \qquad Y = \begin{pmatrix} 4 & -2 & 6 \\ 1 & 3 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & -5 & 6 \\ -3 & 0 & 2 \\ 2 & -1 & 7 \end{pmatrix}$$

Determine the value of $2XY - 5Z$

Check your work on a TI - 83 calculator when finished

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2*[A]*[B]-5[C]
   [[-17 17  -42]
    [41   -6 26 ]
    [16   27  -11]]
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SPECIAL MATRICES

(1) Zero Matrix

- all entries are 0's.

(2) Identity Matrix - "I" (unit matrix)

Ex.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- square matrix with 1's along the diagonal and 0's everywhere else
- behaves like the number "1" in multiplication (any matrix multiplied by I is the same matrix)

$$\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} \text{ x } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$$

"identity matrix"

(3) Inverses - two matrices whose product is a unit matrix are called inverses

i.e. A and B are inverses if $AB = I$ and $BA = I$

Ex. Find AB & BA

$$\underset{A}{\begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}} \cdot \underset{B}{\begin{pmatrix} 9 & -4 \\ -2 & 1 \end{pmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$\underset{B}{\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}} \cdot \underset{A}{\begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

Finding the Inverse of a Matrix "Identity Matrix Method"

Ex. $\left(\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 2 & 9 & 0 & 1 \end{array}\right)$ $\begin{array}{l} 9R_1 - 4R_2 \\ 2R_1 - R_2 \end{array}$ Perform elementary operations to make left side unit matrix

$\left(\begin{array}{cc|cc} 1 & 0 & 9 & -4 \\ 0 & -1 & 2 & -1 \end{array}\right)$ $R_2 \div -1$

$\left(\begin{array}{cc|cc} 1 & 0 & 9 & -4 \\ 0 & 1 & -2 & 1 \end{array}\right)$ Inverse

Determinants

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Det = ad - bc

Det = (product of major diagonal) - (product of other diagonal)

Ex.

$$\begin{pmatrix} 2 & 7 \\ 5 & 18 \end{pmatrix}$$

$\text{Det} = (2)(18) - (7)(5)$

$= 36 - 35$

$= 1$

$$\begin{pmatrix} 12 & 10 \\ 7 & 6 \end{pmatrix}$$

$\text{Det} = (12)(6) - (7)(10)$

$= 72 - 70$

$= 2$

$$\begin{pmatrix} -2 & -1 \\ 4 & 1 \end{pmatrix}$$

$\text{Det} = -2 - (-4)$

$= 2$

Finding an inverse of a 2 x 2 matrix using determinants...

Inverse

New Matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

1. Find the Determinant ($D = ad - bc$)
2. Form New Matrix ("a" and "d" switch positions "b" and "c" change signs.)
3. $\frac{1}{D} \times NewMatrix = Inverse$

Ex.

$$A = \begin{pmatrix} 0 & -3 \\ 4 & 5 \end{pmatrix} \qquad A^{-1} = \text{Find Inverse}$$

① Det = ad − bc
$= (0)(5) - (-3)(4)$
$= 0 - (-12)$
$= 12$

② New Matrix:
$$\begin{bmatrix} 5 & 3 \\ -4 & 0 \end{bmatrix}$$

③ $A^{-1} = \frac{1}{\text{Det}A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 5 & 3 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 5/12 & 3/12 \\ -4/12 & 0 \end{bmatrix} = \begin{bmatrix} 5/12 & 1/4 \\ -1/3 & 0 \end{bmatrix}$

Find the inverse

$$\begin{pmatrix} 12 & 10 \\ 7 & 6 \end{pmatrix}$$

① Det = $(12)(6) - (10)(7)$
$= 72 - 70$
$= 2$

② New Matrix:
$$\begin{bmatrix} 6 & -10 \\ -7 & 12 \end{bmatrix}$$

③ Inverse $= \frac{1}{2}\begin{bmatrix} 6 & -10 \\ -7 & 12 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -7/2 & 6 \end{bmatrix}$

Homework