

Warm Up

Solve the following system of equations and identify the type of system...

$$\begin{array}{l} 3x + 2y + z = 3 \\ x - 3y + z = 4 \\ -6x - 4y - 2z = 1 \end{array} \quad \begin{array}{l} 3x + 2y + z = 3 \\ \Leftrightarrow x - 3y + z = 4 \end{array} \quad \begin{array}{l} 2x - 6y + 2z = 8 \\ \Leftrightarrow -6x - 4y - 2z = 1 \end{array}$$
$$\boxed{\begin{array}{l} 2x + 5y = -1 \\ -4x - 10y = 9 \end{array}}$$

$$\begin{array}{l} 4x + 10y = -2 \\ \Leftrightarrow -4x - 10y = 9 \\ \hline 0 \neq 7 \end{array}$$

No Solution
 \hookrightarrow Inconsistent System

Solving Equations Using Matrices

or 3x3

Matrix Elimination involves taking the coefficients from a 2X2 system, placing them in a matrix, and working to make a new matrix by multiplying, dividing, and combining rows.

The combination of the coefficients from a system of equations and their solutions in an equivalent form is called an **augmented matrix**.

$$\begin{array}{l} \text{Ex. } 2x + y + 3z = 0 \\ x + y - 2z = -1 \\ x - 2y - z = 3 \end{array} \longrightarrow \begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & -2 & -1 \\ 1 & -2 & -1 & 3 \end{array}$$

$$\begin{array}{l} x + 3y = 4 \\ 3x + 4y = 2 \end{array} \longrightarrow \begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array}$$

Row Reduced Echelon Form

The goal in solving a system of equation using matrices is to obtain a new matrix - **row reduced echelon** form of a matrix. It takes the form:

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \end{pmatrix} \quad \text{Or} \quad \begin{pmatrix} 1 & \# & \# & x \\ 0 & 1 & \# & y \\ 0 & 0 & 1 & z \end{pmatrix}$$

To reduce a matrix to its row echelon form, we can:

- Multiply or divide a row by a constant.
- Add or subtract one row from another.
- Interchange rows.

Solve the following system of equations using an augmented matrix reduced to its row echelon form...

$$x + 3y = 4$$

$$3x + 4y = 2$$

$$\begin{array}{l} \textcircled{3R_1} - \textcircled{R_2} \\ \underline{ - \textcircled{R_2}} \end{array} \left[\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 4 & 2 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & 5 & 10 \end{array} \right] \begin{array}{l} \rightarrow x + 3y = 4 \\ \rightarrow x + 3(\underline{2}) = 4 \\ \rightarrow x + 6 = 4 \\ \boxed{x = -2} \end{array}$$

$5y = 10$
 $\boxed{y = 2}$

Try this one on your own...

$$3x + 2y = 12$$

$$2x + 3y = 13$$

1. Express system in the form of an augmented matrix
2. Eliminate "x" in equation 2 and 3.
3. Eliminate "y" in equation 3 (must add/subtract 2 and 3)
4. Create triangle of zeroes and solve.

Ex. $2x + y - z = -1$
 $3x - y + 2z = 8$
 $2x + 2y - 3z = -6$

$$\begin{array}{l} \textcircled{2} \textcircled{3} - 3\textcircled{1} \\ \textcircled{3} - \textcircled{1} \end{array} \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 3 & -1 & 2 & 8 \\ 2 & 2 & -3 & -6 \end{array} \right] \begin{array}{l} \textcircled{3} + \textcircled{2} \\ \textcircled{3} + \textcircled{2} \end{array} \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -8 & 13 & 34 \\ 0 & 1 & -2 & -5 \end{array} \right] \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 0 & -8 & 13 & 34 \\ 0 & 0 & -3 & -6 \end{array} \right]$$

$$-3z = -6$$

$$\boxed{z = 2}$$

$$-8y + 13z = 34$$

$$-8y + 13(2) = 34$$

$$-8y + 26 = 34$$

$$-8y = 8$$

$$\boxed{y = -1}$$

$$2x + y - z = -1$$

$$2x + (-1) - (2) = -1$$

$$2x - 3 = -1$$

$$2x = 2$$

$$\boxed{x = 1}$$

$$\boxed{(1, -1, 2)}$$

Homework

