① a) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$

$a = 1$

$r = \frac{1}{3}$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1-\frac{1}{3}}$$

$$= \frac{1}{\frac{3}{3}-\frac{1}{3}}$$

$$= \frac{1}{\frac{2}{3}}$$

$$= 1 \times \frac{3}{2} = \boxed{\frac{3}{2}}$$

c) $\frac{1}{4} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \ldots$

$a = \frac{1}{4}$

$r = \frac{-5}{4}$

Diverging

$$\lim_{n\to\infty} \left(\frac{1}{4}\right)\left(\frac{-5}{4}\right)^{n-1} = DNE$$

$t_n = \left(\frac{1}{4}\right)\left(\frac{-5}{4}\right)^{n-1}$

② b) $\sum_{n=1}^{\infty} \left(\frac{-2}{5}\right)^n$

Series

$$\frac{-2}{5} + \frac{4}{25} - \frac{8}{125}$$

$a = \frac{-2}{5}$

$r = \frac{-2}{5}$

$$S_n = \frac{\frac{-2}{5}}{1+\frac{2}{5}} = \frac{\frac{-2}{5}}{\frac{7}{5}} = \frac{-2}{\cancel{5}} \cdot \frac{\cancel{5}}{7}$$

$$= \frac{-10}{35}$$

$$= \boxed{\frac{-2}{7}}$$

⑤ c) $4 + 8 + 12 + \ldots + 400$

$a = 4$

$d = 4$

$t_n = 400$

$S_n = ?$

Solve for "n"

$t_n = a + (n-1)d$

$400 = 4 + (n-1)(4)$

$400 = 4 + 4n - 4$

$400 = 4n$

$100 = n$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{100} = \frac{100}{2}(4 + 400)$$

$$S_{100} = 50(404)$$

$$S_{100} = 20\,200$$

⑤ a) $\sum_{n=1}^{5} 2n + 1$

$= 3 + 5 + 7 + 9 + 11$

$= 35$

7

$t_5 = 48$

$t_5 = ar^{5-1}$

$t_5 = ar^4$

$t_8 = 384$

$t_8 = ar^{8-1}$

$t_8 = ar^7$

$$\frac{ar^7 = 384}{ar^4 = 48}$$

$r^3 = 8$

$\boxed{r = 2}$

$\boxed{ar^4 = 48 \qquad ar^7 = 384}$

$ar^4 = 48$

$a(2)^4 = 48$

$16a = 48$

$\boxed{a = 3}$

$t_n = ar^{n-1}$

$t_n = (3)(2)^{n-1}$