Series & Sequence

Sequence: A list of numbers with a pattern. The list can be finite: 1, 2, 3, 4, 5. or infinite: 1, 2, 3, 4, 5... The numbers have a rule or formula that defines them.

Series: The sum of the terms of a sequence. The sum is usually finite: 1+2+3+4+5. However it could be infinite: 2+4+8+16+... You can find the sum of many finite series and certain types of infinite series by using formulas.

Sequences

"common difference"
Arithmetic
2, 5, 8, 11, 14, ⎵

$d = 3$

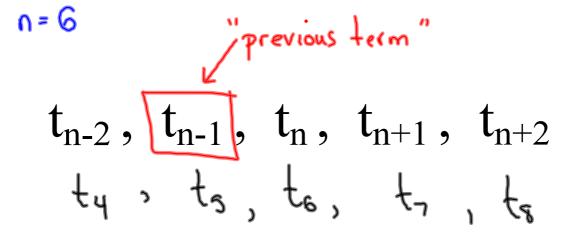"common ratio"
Geometric
2, 4, 8, 16, 32, ⎵

$r = 2$

Other
1, 2, 4, 7, 11, 16.
1, 4, 9, 16, 25.

Sequences

Arithmetic and Geometric sequences each have their own **EXPLICIT** formulas but many types of sequences can be defined by a **RECURSIVE** formula.

A *recursive formula* can be used to define a sequence. A recursive formula defines each term with reference to the previous term or terms

In any sequence, t_n is the given term, t_{n-1} is the term before it, t_{n+1} is the term after it. It looks like this:

$n = 6$

"previous term"

$$t_{n-2}, \quad \boxed{t_{n-1}}, \quad t_n, \quad t_{n+1}, \quad t_{n+2}$$

$$t_4, \quad t_5, \quad t_6, \quad t_7, \quad t_8$$

Recursive Rules

Ex: 2, 5, 8, 11, 14, $\underline{\underline{17}}$ t_6

- The first term is 2
- Any given term is equal to *the term before it plus 3*

Therefore:

$t_1 = 2$

$\boxed{t_n = t_{n-1} + 3}$ $\rightarrow$ recursive rule

Find the recursive formula for the following

#1. 13, 7, 1, -5, -11..., $-\underline{\underline{17}}$

$t_1 = 13$

$$\boxed{t_n = t_{n-1} - 6}$$

$$t_6 = t_5 - 6$$

$$t_6 = -11 - 6$$

$$t_6 = -\underline{\underline{17}}$$

#2. 4, 9, 14, 19, 24...

$t_1 = 4$

$$\boxed{t_n = t_{n-1} + 5}$$

#3. 27, 18, 9, 0, -9...

$t_1 = 27$ $t_n = t_{n-1} - 9$

#4. 7, 26, 45, 64, 83....

$t_1 = 7$ $t_n = t_{n-1} + 19$

What About???

#5. 2, 4, 8, 16, 32.., $\underline{64}$

$t_1 = 2$

$$t_n = 2(t_{n-1})$$

$t_6 = 2(t_5)$

$t_6 = 2(32) = \underline{64}$

#6. 1, 4, 13, 40, 121...

$t_1 = 1$

$$t_n = 3(t_{n-1}) + 1$$

$t_6 = 3(t_5) + 1$

$t_6 = 3(121) + 1$

$t_6 = 363 + 1$

$t_6 = \underline{364}$

Homework

$f{>}$ $t_1 = 5$ $t_n = t_{n-1} + t_{n-2}$

$t_2 = 6$ $t_3 = t_2 + t_1$

$t_3 = 5 + 6$

$t_3 = 11$

Arithmetic Sequences

Ex: 2, 5, 8, 11, 14

- The difference between each term is constant.
- In the sequence 2, 5, 8, 11, 14. the difference between each term is 3.
- The difference is called **"d".**
- The first term is called **"a"** or **"t_1".**
- The second term is called **"t_2".**
- The last term or an indicated term is called **"t_n".**
- The position of a term or the number of terms is called **"n".**