

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

In words, *the Product Rule* says that the *derivative of a product of two functions is: the first function times the derivative of the second function, plus the derivative of the first function times the second function*

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Differentiate the following function and simplify your answer:

$$\begin{aligned}h(t) &= (t^3 - 5t)(6\sqrt{t} - t^{-5}) \\ &= (t^3 - 5t)(6t^{1/2} - t^{-5})\end{aligned}$$

$$\begin{aligned}h'(t) &= (t^3 - 5t)(3t^{-1/2} + 5t^{-6}) + (3t^2 - 5)(6t^{1/2} - t^{-5}) \\ &= 3t^{5/2} + 5t^{-3} - 15t^{1/2} - 25t^{-5} + 18t^{5/2} - 3t^{-3} - 30t^{1/2} + 5t^{-5} \\ &= 21t^{5/2} - 45t^{1/2} + 2t^{-3} - 20t^{-5} \\ &= 21\sqrt{t^5} - 45\sqrt{t} + \frac{2}{t^3} - \frac{20}{t^5}\end{aligned}$$

$$f(x) = (7x^3 - x^2 + 5)(x^9 + 3x - 5)$$

$$\textcircled{5} \quad y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x) \quad ; \quad (1, 5)$$

$$= (2 - x^{1/2})(1 + x^{1/2} + 3x)$$

$$\textcircled{1} \quad y' = (2 - x^{1/2})\left(\frac{1}{2}x^{-1/2} + 3\right) + \left(-\frac{1}{2}x^{-1/2}\right)(1 + x^{1/2} + 3x)$$

$$\textcircled{2} \quad y'(1) = (2 - (1)^{1/2})\left(\frac{1}{2}(1)^{-1/2} + 3\right) + \left(-\frac{1}{2}(1)^{-1/2}\right)(1 + (1)^{1/2} + 3(1))$$

$$y'(1) = (1)\left(\frac{7}{2}\right) + \left(-\frac{1}{2}\right)(5)$$

$$y'(1) = \frac{7}{2} - \frac{5}{2}$$

$$y'(1) = 1 \quad \leftarrow \text{slope of the tangent "m"}$$

$$\textcircled{3} \quad y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1)$$

$$y - 5 = x - 1$$

$$\boxed{0 = x - y + 4}$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, *the Quotient Rule* says that the *derivative of a quotient is: the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.*

Examples:

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1}$$

$$F'(x) = \frac{(x^3 + 1)(2x + 2) - (x^2 + 2x - 3)(3x^2)}{(x^3 + 1)^2}$$

$$= \frac{2x^4 + 2x^3 + 2x + 2 - (3x^4 + 6x^3 - 9x^2)}{(x^3 + 1)^2}$$

$$= \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3 + 1)^2}$$

$$F(x) = \frac{\sqrt{x}}{1 + 2x} = \frac{x^{1/2}}{1 + 2x}$$

$$F'(x) = \frac{(1 + 2x)(\frac{1}{2}x^{-1/2}) - (x^{1/2})(2)}{(1 + 2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2} + x^{1/2} - 2x^{1/2}}{(1 + 2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2} - x^{1/2}}{(1 + 2x)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} - \sqrt{x}}{(1 + 2x)^2 \cdot 2\sqrt{x}}$$

$$= \frac{1 - 2x}{2\sqrt{x}(1 + 2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x) = \frac{8 - 9x^7}{3x - 7}$$

$$F'(x) = \frac{(3x-7)(-63x^6) - (8-9x^7)(3)}{(3x-7)^2}$$

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$F'(x) = \frac{(x^8 - 4x^5)(3x^2 - 14x) - (x^3 - 7x^2 + 2)(8x^7 - 20x^4)}{(x^8 - 4x^5)^2}$$

Homework

$$f(x) = \frac{(10x^{-5} + x)(3x^3 + 5)}{(-2x^6 + \sqrt[3]{x})}$$

$$f(x) = \frac{(x-7)(2x^6 - x^4 + 5)}{(6x - x^5)(4x^3 + 2)}$$