Questions from Quiz

$$F'(x) = f(x)g'(x) + f'(x)g(x) \qquad F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

① $f(x) = \sqrt{3x+1} \qquad f(x+h) = \sqrt{3x+3h+1}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{3x+3h+1} - \sqrt{3x+1}}{h} \cdot \frac{\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)}{\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)}$$

$$= \lim_{h\to 0} \frac{3x+3h+1 - (3x+1)}{h\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3x+3h+1} + \sqrt{3x+1}\right)}$$

$$= \frac{3}{\sqrt{3x+1} + \sqrt{3x+1}} = \boxed{\frac{3}{2\sqrt{3x+1}}}$$

② b) $f(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$

$$= x^{1/2} + x^{1/3} + x^{1/4}$$

$$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-3/4}$$

$$= \frac{1}{2x^{1/2}} + \frac{1}{3x^{2/3}} + \frac{1}{4x^{3/4}}$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{4\sqrt[4]{x^3}}$$

⑥ Find the points (x, y) on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to $x + 4y = 1$

"Same slope"

① $4y = -x + 1$

$y = \frac{-x}{4} + \frac{1}{4}$

$\boxed{m = -\frac{1}{4}}$

② $y = \frac{x}{x-1}$

$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$

$y' = \frac{x - 1 - x}{(x-1)^2}$

$\boxed{y' = \frac{-1}{(x-1)^2}}$

③ $\frac{-1}{4} = \frac{-1}{(x-1)^2}$

$-(x-1)^2 = -4$

$(x-1)^2 = 4$

$x^2 - 2x + 1 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad | \quad x = -1$

④ $x = 3$

$y = \frac{3}{3-1}$

$y = \frac{3}{2}$

$\boxed{(3, 3/2)}$

⑤ $x = -1$

$y = \frac{-1}{-1-1}$

$y = \frac{-1}{-2}$

$y = \frac{1}{2}$

$\boxed{(-1, 1/2)}$

Chain Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Work from the outside to the inside

Examples:

$$f(x) = (5x^3 + 1)^{10}$$

$$f'(x) = 10(5x^3+1)^9(15x^2)$$

$$\boxed{= 150x^2(5x^3+1)^9}$$

$$F(x) = \sqrt{2x^2 + 3}$$

$$= (2x^2+3)^{1/2}$$

$$f'(x) = \frac{1}{2}(2x^2+3)^{-1/2}(4x)$$

$$= 2x(2x^2+3)^{-1/2}$$

$$= \frac{2x}{(2x^2+3)^{1/2}}$$

$$\boxed{= \frac{2x}{\sqrt{2x^2+3}}}$$

$$h(x) = \sqrt[3]{5 - 3x^4}$$

$$= (5-3x^4)^{1/3}$$

$$h'(x) = \frac{1}{3}(5-3x^4)^{-2/3}(-12x^3)$$

$$= -4x^3(5-3x^4)^{-2/3}$$

$$= \frac{-4x^3}{(5-3x^4)^{2/3}}$$

$$\boxed{= \frac{-4x^3}{\sqrt[3]{(5-3x^4)^2}}}$$

Homework

$$g(x) = 9x^{-3}\left(5x^3 - 1\right)^6$$

$$g(x) = \frac{\left(x^2 - 5x + 1\right)^8}{\left(1 - x^{-7}\right)^{20}}$$