Warm Up

Solve the following system of equations:

$$4x + 9 = 3y - 6z$$
$$3z = 10 + 2x + 4y$$
$$2y = 4z - 11 - 3x$$

① $4x - 3y + 6z = -9$
② $2x + 4y - 3z = -10$
③ $3x + 2y - 4z = -11$

$$\begin{aligned} &4x - 3y + 6z = -9 \\ (+)\ &4x + 8y - 6z = -20 \\ \hline ④\ &8x + 5y = -29 \end{aligned} \qquad \begin{aligned} &8x + 16y - 12z = -40 \\ (-)\ &9x + 6y - 12z = -33 \\ \hline ⑤\ &-x + 10y = -7 \end{aligned}$$

$$\begin{aligned} &16x + 10y = -58 \\ (-)\ &-x + 10y = -7 \\ \hline &17x = -51 \\ &\boxed{x = -3} \end{aligned}$$

$$\begin{aligned} 8(-3) + 5y &= -29 \\ -24 + 5y &= -29 \\ 5y &= -5 \\ \boxed{y &= -1} \end{aligned}$$

$$\begin{aligned} 4(-3) - 3(-1) + 6z &= -9 \\ -12 + 3 + 6z &= -9 \\ -9 + 6z &= -9 \\ 6z &= 0 \\ \boxed{z &= 0} \end{aligned}$$

$$\boxed{(-3, -1, 0)}$$

Questions from Homework

⑤
$$x-6y+7z=-39$$
$$\boxed{3x-2y=6}$$
$$5x-9y+5z=-36$$

$$5x-30y+35z=-195$$
$$(-)\ 35x-63y+35z=-252$$
$$\boxed{-30x+33y=57}$$

$$30x-20y=60$$
$$(+)\ -30x+33y=57$$
$$13y=117$$
$$\boxed{y=9}$$

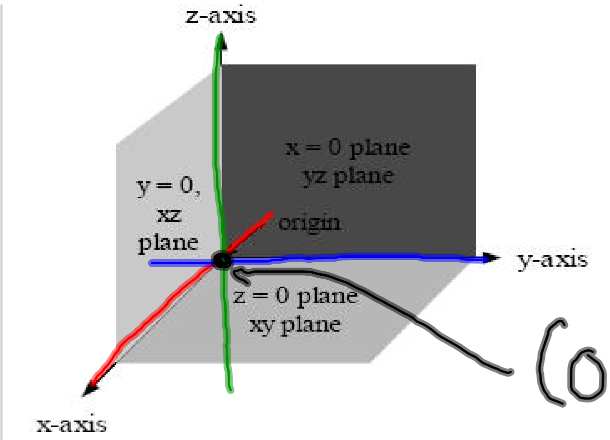
$$3x-2y=6$$
$$3x-2(9)=6$$
$$3x-18=6$$
$$3x=24$$
$$\boxed{x=8}$$

$$x-6y+7z=-39$$
$$8-6(9)+7z=-39$$
$$8-54+7z=-39$$
$$7z=7$$
$$\boxed{z=1}$$

$$\boxed{(8,9,1)}$$

ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet **(x, y, z)**
- Three planes exist: **xy** plane, **xz** plane, **yz** plane



x axis -axis coming "out of the page"

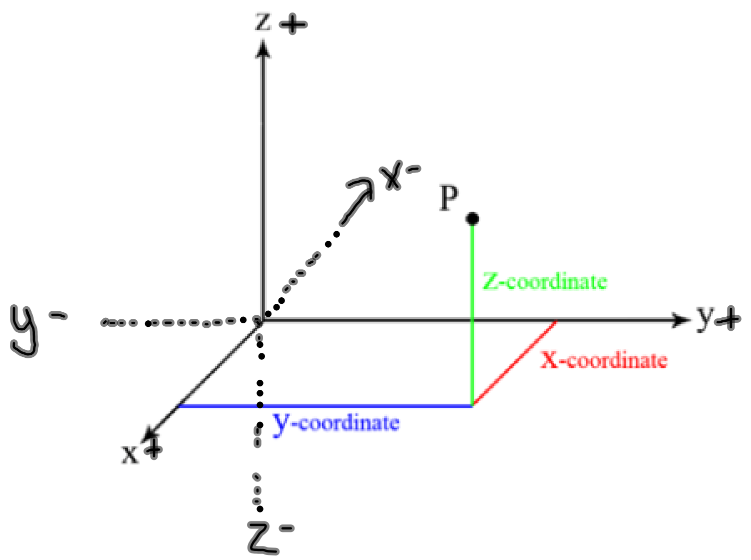
y axis - horzontal axis

z axis - vertical axis

(0,0,0) → origin

xz plane　　**xy** plane　　**yz** plane

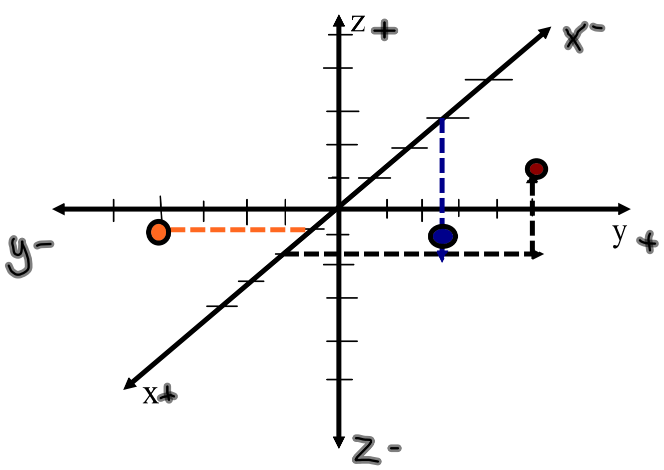
Plotting Points in 3-Space



Plotting points in 3-space...

x, y, z

Ex: a) (2, 6, 3) b) (-3, 0, -4) c) (1, -4, 0)

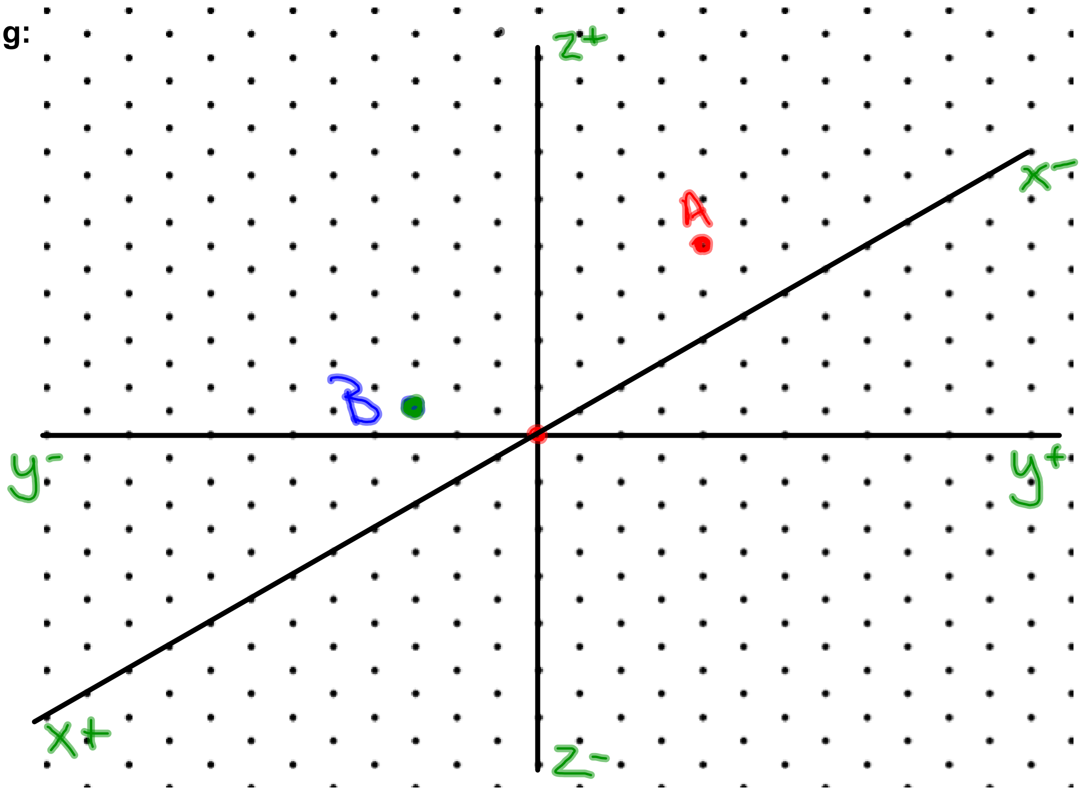


Plot the following:

A (-2, 1, 3)

B (3, 0, 2)

C (-1, -2, 0)



Finding Intercepts in 3D

As in two dimensions...

$\boldsymbol{x}$ intercept can be found when $\boldsymbol{y} = 0$ and $\boldsymbol{z} = 0$

$$(x,y,z) \longrightarrow (x,y,z)$$

$\boldsymbol{y}$ intercept can be found when $\boldsymbol{x} = 0$ and $\boldsymbol{z} = 0$

$$(x,y,z) \longrightarrow (0,y,0)$$

$\boldsymbol{z}$ intercept can be found when $\boldsymbol{x} = 0$ and $\boldsymbol{y} = 0$

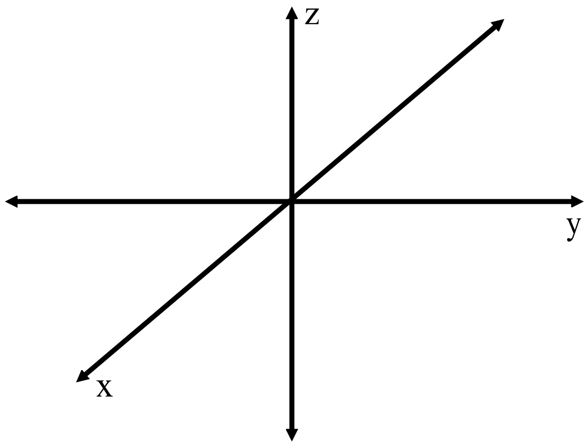
$$(x,y,z) \longrightarrow (0,0,z)$$

Plotting Planes in 3-Space

- Use the **intercept method** to plot the x, y, and z intercepts to form a triangle
- The triangle is part of the plane being sketched

Ex. $2x - y + 3z = 6$

[redacted]



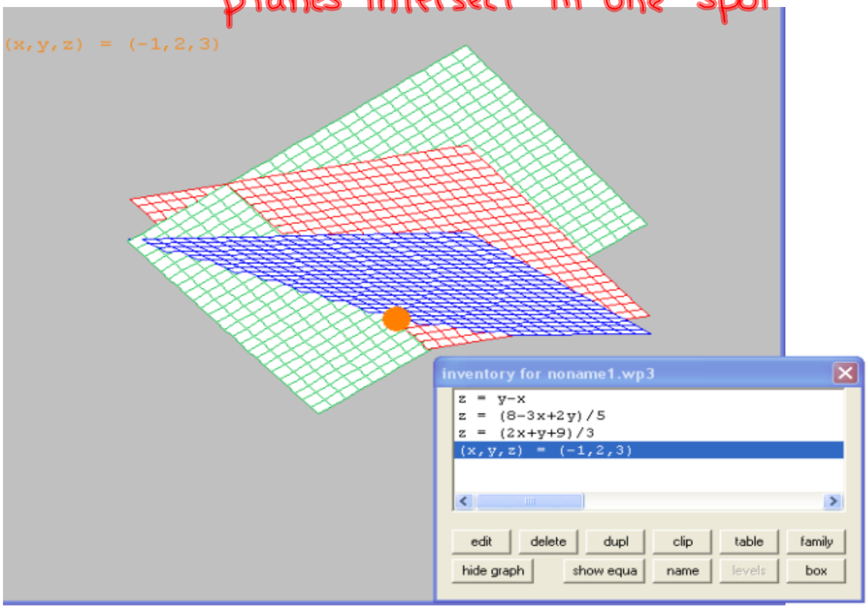
Types of Systems

Remember: Looking at **intersecting planes**!

Consistent: At least one solution

Independent: One Unique solution

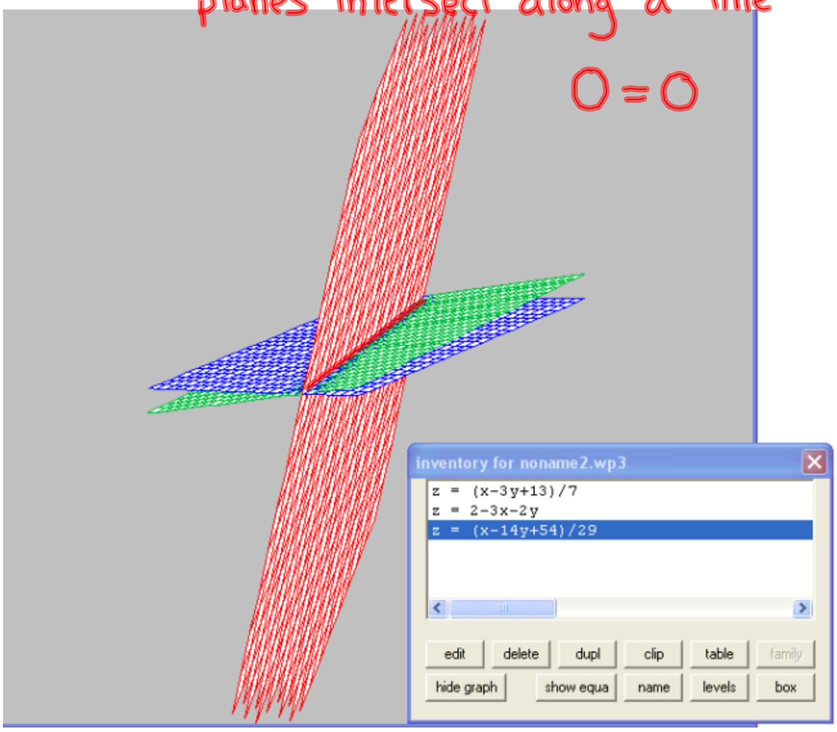
planes intersect in one spot



Dependent: Infinite number of solutions

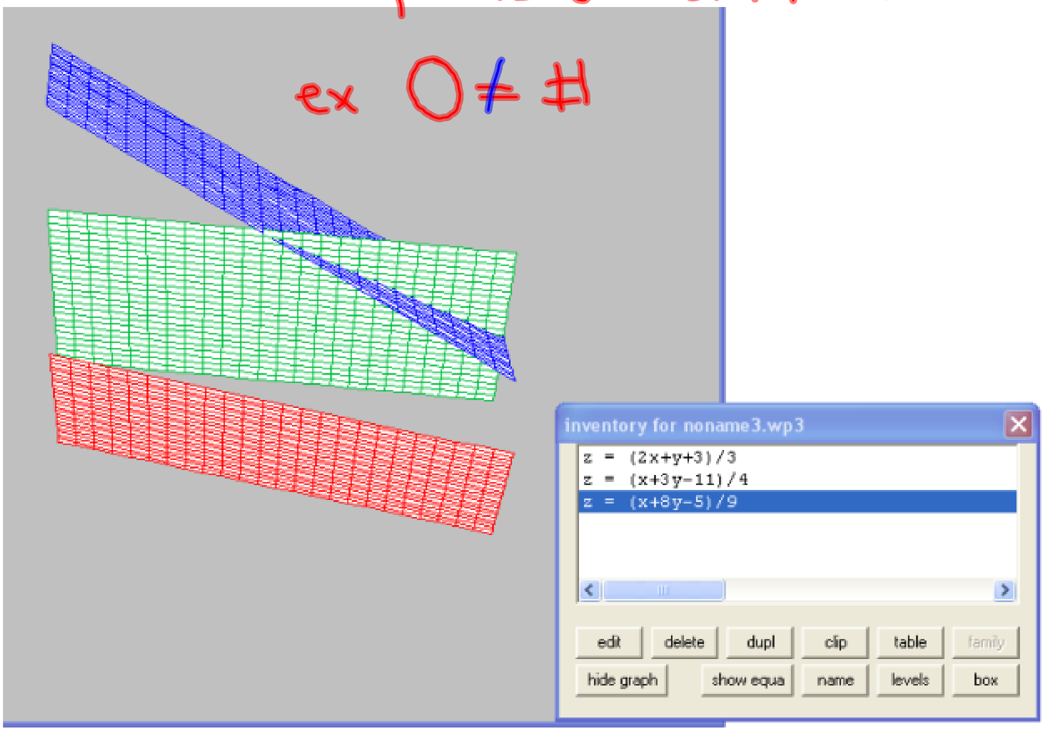
planes intersect along a line

$0=0$



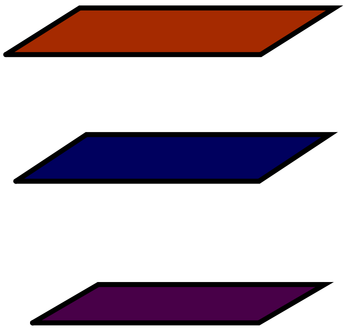
<u>Inconsistent:</u> No Solutions

planes do not intersect

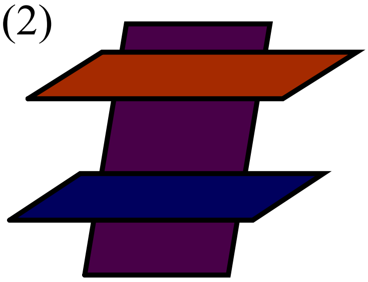


3 Possible Orientations That Give No Solution...

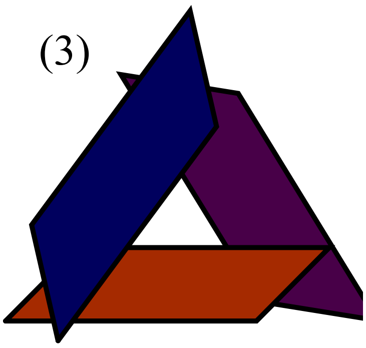
(1)



(2)



(3)



I. Consistent System with an Independent Solution

$x - y + z = 0$
$3x - 2y + 5z = 8$
$2x + y - 3z = -9$

$$\begin{array}{r} 2x - 2y + 2z = 0 \\ (-)\ 3x - 2y + 5z = 8 \\ \hline -x - 3z = -8 \end{array} \qquad \begin{array}{r} 3x - 2y + 5z = 8 \\ (+)\ 4x + 2y - 6z = -18 \\ \hline 7x - z = -10 \end{array}$$

$$\begin{array}{r} -x - 3z = -8 \\ (-)\ 21x - 3z = -30 \\ \hline -22x = 22 \\ \boxed{x = -1} \end{array}$$

$7x - z = -10$
$7(-1) - z = -10$
$-7 - z = -10$
$-z = -3$
$\boxed{z = 3}$

$x - y + z = 0$
$-1 - y + 3 = 0$
$-y + 2 = 0$
$-y = -2$
$\boxed{y = 2}$

II. Consistent System with a Dependent Solution

x - 3y - 7z = -13
3x + 2y + z = 2
x - 14y - 29z = -54

$$\begin{array}{rl} & 3x - 9y - 21z = -39 \\ (-) & \underline{3x + 2y + z = 2} \\ & \boxed{-11y - 22z = -41} \end{array}$$

$$\begin{array}{rl} & 3x + 2y + z = 2 \\ (-) & \underline{3x - 42y - 87z = -162} \\ & \boxed{44y + 88z = 164} \end{array}$$

$$\begin{array}{rl} & -44y - 88z = -164 \\ (+) & \underline{44y + 88z = 164} \\ & \boxed{0 = 0} \end{array}$$

Infinite # of Solutions

Write a general solution in terms of a parameter (i.e. z = t). For each value assigned to the parameter there will be one distinct solution.

III. Inconsistent System

(planes do not intersect)

$3x + 2y + z = 3$
$x - 3y + z = 4$
$-6x - 4y - 2z = 1$

$$\begin{array}{r} 3x+2y+z=3 \\ (-)\quad x-3y+z=4 \\ \hline 2x+5y=-1 \end{array} \qquad \begin{array}{r} 2x-6y+2z=8 \\ (+)\quad -6x-4y-2z=1 \\ \hline -4x-10y=9 \end{array}$$

$$\begin{array}{r} 4x+10y=-2 \\ (+)\quad -4x-10y=9 \\ \hline \boxed{0\neq 7} \end{array}$$

No Solution

Homework

Handout: Solving Systems of Equations in 3-Space

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