Final Review

1) a) $$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h-5)-(x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \frac{1}{2\sqrt{x-5}}$$

b) $$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2x^2+2xh-2x+6x+6h-6-(2x^2+2xh+6x-2x-2h-6)}{(x+h+3)(x+3)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{2x^2}+\cancel{2xh}-\cancel{2x}+\cancel{6x}+6h\cancel{-6}-\cancel{2x^2}-\cancel{2xh}-\cancel{6x}+\cancel{2x}+2h+\cancel{6}}{(x+h+3)(x+3)}}{h}$$

$$= \lim_{h\to 0} \frac{8\cancel{h}}{(x+h+3)(x+3)} \cdot \frac{1}{\cancel{h}} = \boxed{\frac{8}{(x+3)^2}}$$

2) a) $\boxed{f'(x) = 6x+5}$

b) $f(x) = 3x^{-1/2}$

$f'(x) = \frac{-3}{2}x^{-3/2}$

$\boxed{f'(x) = \frac{-3}{2\sqrt{x^3}}}$

c) $f(x) = 2x^4 + x^{1/2}$

$\boxed{f'(x) = 8x^3 + \frac{1}{2\sqrt{x}}}$

d) $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3}$

$\boxed{f'(x) = \frac{2}{3\sqrt[3]{x}}}$

③ a) $y = (3x^2-2)(4x+5)$

$y' = (3x^2-2)(4) + (6x)(4x+5)$

$y' = 12x^2 - 8 + 24x^2 + 30x$

$\boxed{y' = 36x^2 + 30x - 8}$

b) $g(x) = (x^2-5x+2)(4x+1)$

$g'(x) = (x^2-5x+2)(4) + (2x-5)(4x+1)$

$g'(x) = 4x^2 - 20x + 8 + 8x^2 - 18x - 5$

$\boxed{g'(x) = 12x^2 - 38x + 3}$

④ a) $f(x) = \dfrac{2x^2+3}{3x-2}$

$f'(x) = \dfrac{(3x-2)(4x) - (2x^2+3)(3)}{(3x-2)^2}$

$f'(x) = \dfrac{12x^2 - 8x - 6x^2 - 9}{(3x-2)^2}$

$\boxed{f'(x) = \dfrac{6x^2 - 8x - 9}{(3x-2)^2}}$

b) $y = \dfrac{\sqrt{x}}{3+x^2}$

$y' = \dfrac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2})(2x)}{(3+x^2)^2}$

$y' = \dfrac{2\sqrt{x} \cdot \frac{3+x^2}{2\sqrt{x}} - 2x^{3/2} \cdot 2\sqrt{x}}{2\sqrt{x}\,(3+x^2)^2}$

$y' = \dfrac{3+x^2-4x^2}{2\sqrt{x}\,(3+x^2)^2}$

$\boxed{y' = \dfrac{3-3x^2}{2\sqrt{x}\,(3+x^2)^2}}$

⑤ $y = (x^2-3)^8$ at $x = 2$

$y = ((2)^2-3)^8$

$y = (4-3)^8$

$y = 1$

$\left.\begin{array}{l} x = 2 \\ y = 1 \end{array}\right\} \longrightarrow (2,1)$

$m = 32$

$y - 1 = 32(x-2)$

$y - 1 = 32x - 64$

$\boxed{0 = 32x - y - 63}$

$y' = 8(x^2-3)^7(2x)$

$y' = 16x(x^2-3)^7$

$m = y' = 16(2)(4-3)^7$

$m = 32$

Final Review

⑥ a) $f(x) = 3(2x^2-4)^4$

$f'(x) = 12(2x^2-4)^3(4x)$

$\boxed{= 48x(2x^2-4)^3}$

b) $y = 16(x-1)^{-1/2}$

$y' = -8(x-1)^{-3/2}(1)$

$= \dfrac{-8}{(x-1)^{3/2}}$

$\boxed{= \dfrac{-8}{\sqrt{(x-1)^3}}}$

⑦ a) $f(x) = \left[\dfrac{2x+1}{x-1}\right]^5$

$f'(x) = 5\left[\dfrac{2x+1}{x-1}\right]^4\left[\dfrac{(x-1)(2)-(2x+1)(1)}{(x-1)^2}\right]$

$f'(x) = 5\left[\dfrac{(2x+1)^4}{(x-1)^4}\right]\left[\dfrac{-3}{(x-1)^2}\right]$

$\boxed{f'(x) = \dfrac{-15(2x+1)^4}{(x-1)^6}}$

Final Review

⑦ b) $y = (x^2-1)^3(3x-2)^2$

$y' = (x^2-1)^3(2)(3x-2)(3) + 3(x^2-1)^2(2x)(3x-2)^2$

$y' = 6(x^2-1)^3(3x-2) + 6x(x^2-1)^2(3x-2)^2$

$y' = 6(x^2-1)^2(3x-2)[x^2-1 + x(3x-2)]$

$y' = 6(x^2-1)^2(3x-2)(4x^2-2x-1)$

c) $y = \frac{(2x+1)^2}{(x^4-x+1)^2}$

$$y' = \frac{(x^4-x+1)^2(2)(2x+1)(2) - (2x+1)^2(2)(x^4-x+1)(4x^3-1)}{[(x^4-x+1)^2]^2}$$

$$y' = \frac{4(2x+1)(x^4-x+1)^2 - 2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(x^4-x+1)[2(x^4-x+1) - (4x^3-1)(2x+1)]}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(x^4-x+1)(2x^4-2x+2 - (8x^4+4x^3-2x-1))}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)\cancel{(x^4-x+1)}(-6x^4-4x^3+3)}{(x^4-x+1)^{\cancel{4}}}$$

$$y' = \frac{2(2x+1)(-6x^4-4x^3+3)}{(x^4-x+1)^3}$$

⑧ a) $f(x) = \sin^3 x + \cos^3 x$

$= (\sin x)^3 + (\cos x)^3$

$f'(x) = 3(\sin x)^2(\cos x) + 3(\cos x)^2(-\sin x)$

$= 3\sin^2 x\cos x - 3\sin x\cos^2 x$

$= 3\sin x\cos x[\sin x - \cos x]$

b) $y = 3\sec(2x^2+1)$

$y' = 3\sec(2x^2+1)\tan(2x^2+1)\cdot 4x + \cancel{0(\sec(2x^2+1))}$

$y' = 12x\sec(2x^2+1)\tan(2x^2+1)$