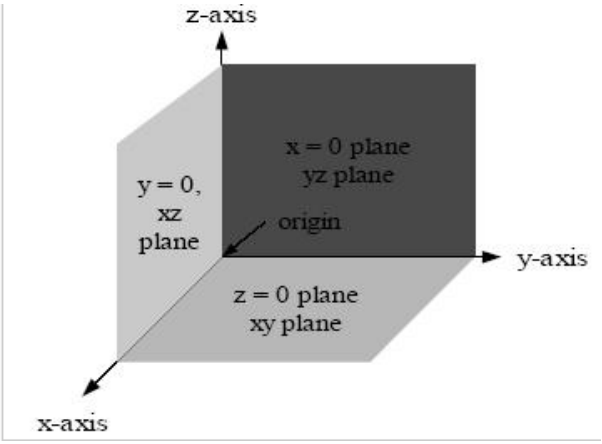
ALGEBRA OF 3-SPACE

- Coordinate geometry that represents space in **three** dimensions
- Coordinates are in the form of an ordered triplet **(x, y, z)**
- Three planes exist: **xy** plane, **xz** plane, **yz** plane

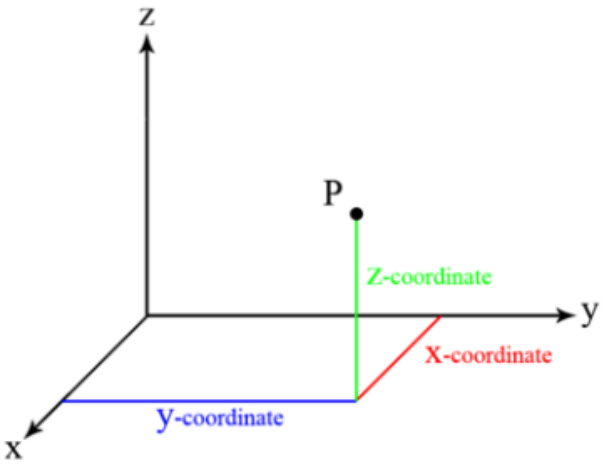


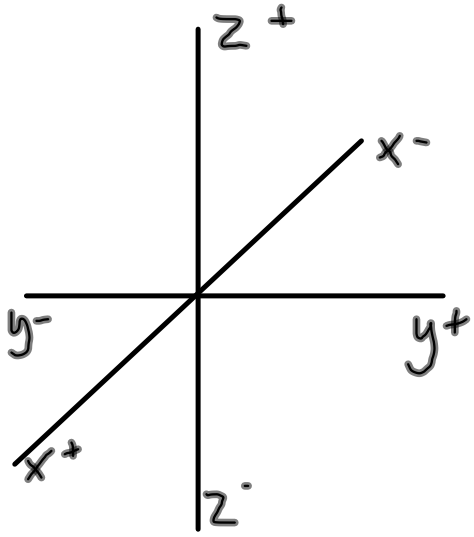
x axis -axis coming "out of the page"

y axis - horzontal axis

z axis - vertical axis

Plotting Points in 3-Space



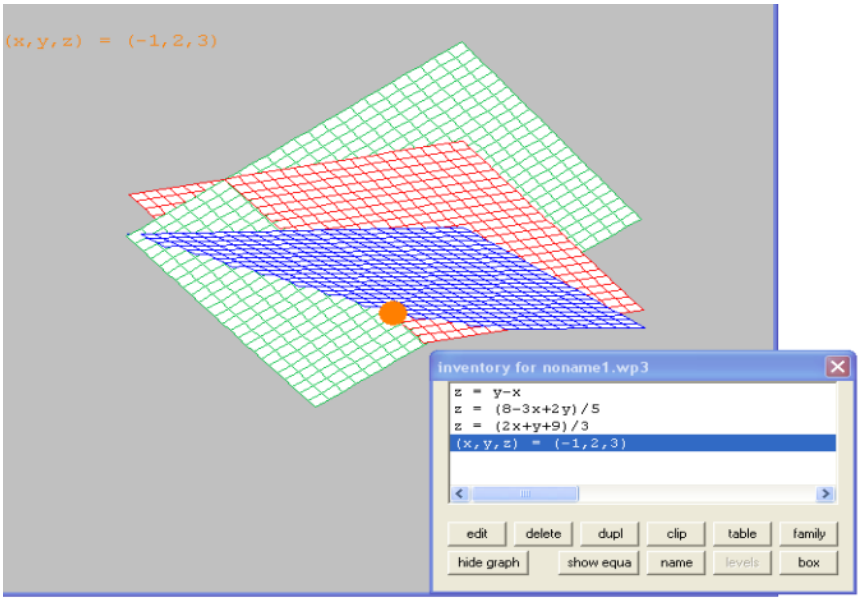


Types of Systems

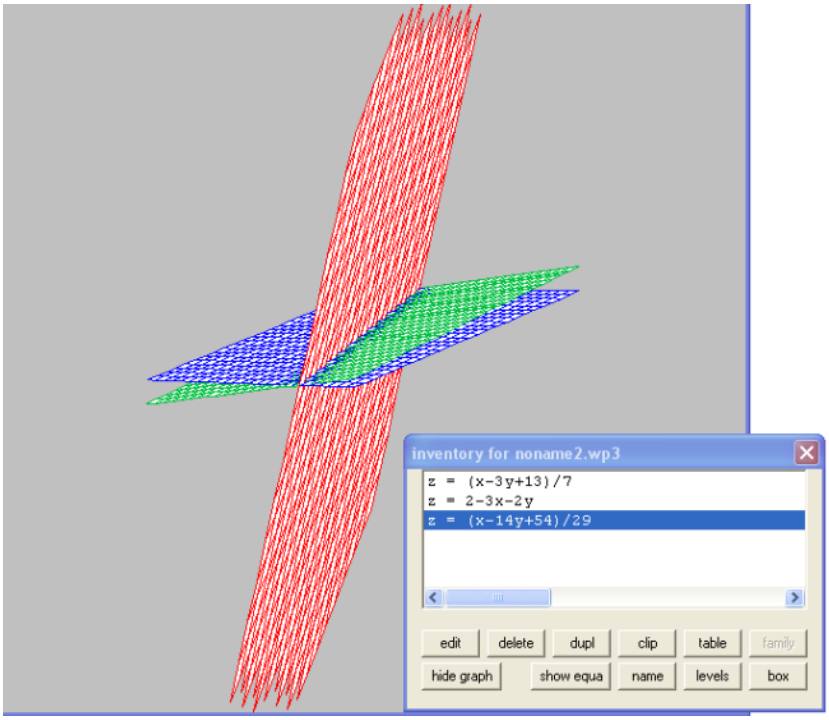
Remember: Looking at **intersecting planes**!

Consistent:

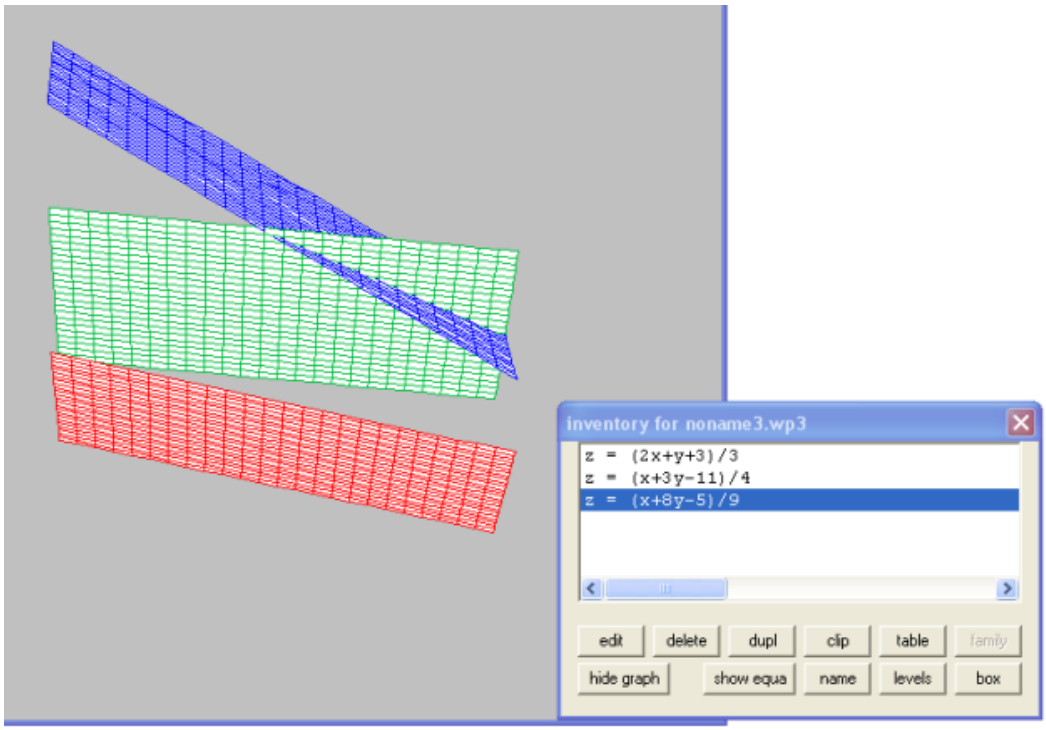
Independent: one unique solution



Dependent: Infinite number of solutions

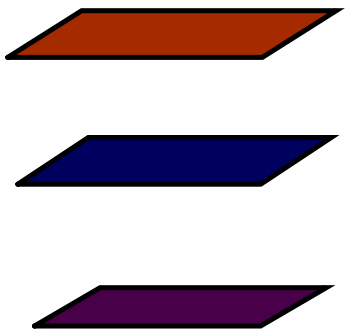


Inconsistent: No Solutions

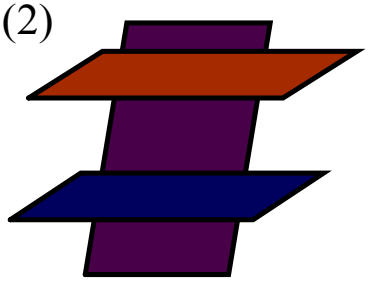


3 Possible Orientations That Give No Solution...

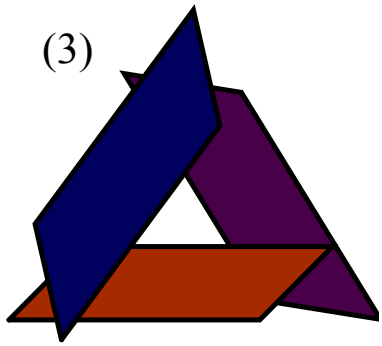
(1)



(2)



(3)



Solving 3 x 3 Systems

REMEMBER:

- you can multiply equations by a constant
- can add & subtract 2 equations to get a new equation
- you can rearrange the order of equations

STEPS:

1) Eliminate one of the variables
2) Solve the 2 x 2 system
3) Use "backward substitution" to obtain a solution

I. Consistent System with a Unique Solution

Solve using algebraic techniques

$3x - 2y = 6$

$5x - 9y + 5z = -36$

$x - 6y + 7z = -39$

$35x - 63y + 35z = -252$

$(-)\ 5x - 30y + 35z = -195$

$30x - 33y = -57$

$3x - 2y = 6$

$30x - 33y = -57$

$30x - 20y = 60$

$(-)\ 30x - 33y = -57$

$13y = 117$

$y = 9$

$3x - 2(9) = 6$

$3x - 18 = 6$

$3x = 24$

$x = 8$

$x - 6y + 7z = -39$

$8 - 6(9) + 7z = -39$

$8 - 54 + 7z = -39$

$-46 + 7z = -39$

$7z = 7$

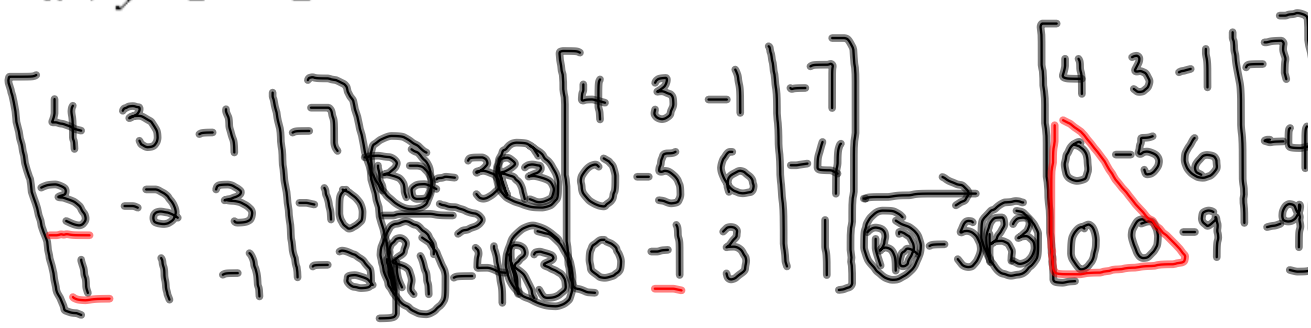
$z = 1$

$(8, 9, 1)$

I. Consistent System with a Unique Solution

Solve the following system of equations using a matrix reduced to its row echelon form.

$$4x + 3y - z = -7$$
$$3x - 2y + 3z = -10$$
$$x + y - z = -2$$



$-9z = -9$

$z = 1$

$-5y + 6z = -4$

$-5y + 6(1) = -4$

$-5y + 6 = -4$

$-5y = -10$

$y = 2$

$4x + 3y - z = -7$

$4x + 3(2) - 1 = -7$

$4x + 6 - 1 = -7$

$4x = -12$

$x = -3$

$(-3, 2, 1)$

Word Problems

The San Diego Chargers football team uses three brands of cleats each year: Nike, Adidas, and Reebok. Last year the team went through a total of 410 pairs of cleats. Nike's cost $84/pair, Adidas $72/pair and Reeboks $65/pair and they spent $31 050 on cleats last season. If Nike's cleats were used twice as much as Reeboks, how many pairs of each brand of football cleat did they use?

(Declare variables, write a system of equations and an augmented matrix to model the problem then use your TI-84 to solve.)

Let x = Nike
Let y = Adidas
Let z = Reebok

$x = 2z$

$x - 2z = 0$

$x + y + z = 410$

$84x + 72y + 65z = 31050$

$x - 2z = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 410 \\ 84 & 72 & 65 & 31050 \\ 1 & 0 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 180 \\ 0 & 1 & 0 & 140 \\ 0 & 0 & 1 & 90 \end{array}\right]$$

∴ They used 180 pairs of Nike cleats, 140 pairs of Adidas and 90 pairs of Reebok.

II. Consistent System with a Dependent Solution
(must create a parametric solution)

① $x - 3y - 7z = -13$
② $3x + 2y + z = 2$
③ $x - 14y - 29z = -54$

$3x - 9y - 21z = -39$
② $3x + 2y + z = 2$
④ $-11y - 22z = -41$

$3x + 2y + z = 2$
③ $3x - 42y - 87z = 162$
⑤ $44y + 88z = 164$

④ $-44y - 88z = -164$
⑤ $44y + 88z = 164$
$0 = 0$

let $z = t$

$-11y - 22t = -41$
$-11y = -41 + 22t$
$y = \frac{41 - 22t}{11}$

$x - 3y - 7z = -13$

$x - 3\left(\frac{41 - 22t}{11}\right) - 7t = -13$

$11\left(x - \frac{(123 + 66t)}{11} - 7t = -13\right)11$

$11x - 123 + 66t - 77t = -143$

$11x = -20 + 11t$

$x = \frac{-20 + 11t}{11}$

Don't forget about Matrices:

- Basic operations
- Determinants
- Identity Matrix
- Inverse Matrices
- Operations with TI-83
- *Row Reduced Echelon Form*

① $\begin{bmatrix} 2x+y & 5 \\ -1 & y-x \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ -1 & 1 \end{bmatrix}$

$2x+y=-2$
$y-x=1$

(-)
$2x+y=-2$
$-x+y=1$
$3x=-3$
$x=-1$

$2(-1)+y=-2$
$-2+y=-2$
$y=0$